

**ΑΠΑΝΤΗΣΕΙΣ**

50.  $\sqrt{1+x^2} + \sqrt{1+y^2} = C$ . 51.  $\sqrt{1-x^2} + \sqrt{1-y^2} = 1$ ,  
 $y = 1$ . 52.  $e^x = C(1 - e^{-y})$ .  
 53.  $y = 1$ . 54.  $a^x + a^{-y} = C$ . 55.  $1 + e^y = C(1 + x^2)$ .  
 56.  $y = \sin [C + \ln(1 + x^2)]$ . 57.  $\arctan e^x = \frac{1}{2 \sin^2 y} + C$ .  
 58.  $y = (1 + Cy + \ln y) \cos x$ . 59.  $x + C = \cot \left( \frac{y-x}{2} + \frac{\pi}{4} \right)$ .  
 60.  $b(ax + by + c) + a = Ce^{bx}$ .  
 61.  $x + y = a \tan \left( C + \frac{y}{a} \right)$ . 62.  $y = -\frac{1}{x}$ .  
 63.  $y = a \tan \sqrt{\frac{a}{x} - 1}$ . 64.  $\tan \frac{y}{2} = e^{2 \sin x}$ .  
 65.  $y' = 3y$ ,  $y = -2e^{3x}$ .  
 66.  $\int_0^x y dt = a^2 \ln \frac{y}{a}$ ;  $y = \frac{a^2}{a-x}$  (the hyperbola).  
 67.  $\frac{dv}{dt} = 20 \frac{t}{v}$ ,  $v = 50 \sqrt{29}$  cm/s.  
 69.  $m \frac{dv}{dt} = kv^2$ ,  $t = \frac{h(v_1 - v_0)}{v_0 v_1 \ln \frac{v_1}{v_0}} = \frac{3}{40 \ln 2.5}$  s.  
 70.  $m \frac{dv}{dt} = -kv$ ;  $t = -\frac{5 \ln 10}{\ln 0.8}$  s. 72.  $\frac{dT}{dt} = k(T - T_0)$ ;  
 $T = 20 + 80 \left( \frac{1}{2} \right)^{t/20}$ ;  $t = 60$  min.  
 73.  $y' = n \frac{y}{x}$ ;  $y = Cx^n$ . 74.  $\frac{dS}{dt} = kS$ ;  $S = 25 \times 2^{t/5}$ .

99.  $y = \arccot \frac{1}{2x} + \frac{9}{4} \pi$ . 100.  $\tan \frac{y}{x} = \ln Cx$ .  
 101.  $y = x(C - \ln x)$ . 102.  $y = xe^{1+Cx}$ .  
 103.  $(x-y) \ln Cx = x$ . 104.  $y + \sqrt{y^2 - x^2} = Cx^2$ ,  $y = x$ ,  
 $y = -x$ . 105.  $2x = (x-y) \ln Cx$ .  
 106.  $y^2 - 3xy + 2x^2 = C$ . 107.  $y^2 + 2xy - x^2 = C$ .  
 108.  $y = 1 + (x-1) \ln C(x-1)$ .  
 109.  $(x+y-1)^5 (x-y-1)^2 = C$ .  
 110.  $y^2 - 2xy - x^2 - 8y + 4x = C$ .  
 111.  $y^2 - 2xy - x^2 + 4y = C$ .  
 112.  $y^2 + 3xy + x^2 - 5x - 5y = C$ .  
 113.  $(4x + 2y + 1)^2 = 4x + C$ .  
 114.  $x + 3y - \ln |x - 2y| = C$ .  
 115.  $(x + y - 1)^2 + 2x = C$ . 116.  $y^2 = x \ln Cy^2$ .

124.  $x^2 + y^2 = Cx^4$ . 125.  $y = Ce^{-2x} + e^{-x}$ .  
 126.  $y = x - x^2$ . 127.  $y = (C + x^2)e^{x^2}$ .  
 128.  $y = (C + x)e^{-x^2}$ . 129.  $y = \frac{x^2}{\cos x}$ .  
 130.  $y = Cx^2 + x^2 \sin x$ . 131.  $y = \frac{\sin x}{\cos^2 x}$ .  
 132.  $y = (C + x^3) \ln x$ . 133.  $x = Cy - \frac{y^2}{2}$ . 134.  $y = 1$ .  
 135.  $x = \frac{C}{y} + y \ln y$ . 136.  $x = (C + y)e^{-\frac{y^2}{2}}$ .  
 137.  $y = (C + x^2)e^{e^x}$ . 138.  $y = (C + x)e^{(1-x)e^x}$ .  
 139.  $i(t) = \frac{E_0}{R^2 + (2n\pi L)^2} [R \sin 2n\pi t + 2n\pi L (e^{-\frac{Rt}{L}} - \cos 2n\pi t)]$   
 $+ I_0 e^{-\frac{Rt}{L}}$ . 140.  $q = QE(1 - e^{-t/QR})$ ;  $R \frac{dq}{dt} = E - \frac{q}{Q}$ .  
 141.  $v = \frac{k_1}{k_2} \left( t - \frac{m}{k_2} + \frac{m}{k_2} e^{-\frac{k_2 t}{m}} \right)$ ;  $m \frac{dv}{dt} = k_1 t - k_2 v$ ,  $v(0) = 0$ .  
 181.  $x^3 + y^3 - x^2 - xy + y^2 = C$ . 182.  $y \sqrt{1 + x^2}$   
 $+ x^2 y - y \ln x = C$ . 183.  $\sqrt{x^2 + y^2} + \frac{y}{x} = C$ .  
 184.  $x \sin y - y \cos x + \ln |xy| = C$ . 185.  $\tan xy - \cos x$   
 $- \cos y = C$ .  
 186.  $y = x$ . 187.  $(x^2 + y^2)^2 + 2a^2(y^2 - x^2) = C$ .  
 188.  $xy(x^2 + y^2) = C$ . 189.  $xy^2 - 2x^2y - 2 = Cx$ ;  $\mu = 1/x^2$ .  
 190.  $x - \frac{y}{x} = C$ ;  $\mu = \frac{1}{x^2}$ . 191.  $x \ln |x| - y^2 = Cx$ ;  $\mu = 1/x^2$ .  
 192.  $5 \arctan x + 2xy = C$ ;  $x = 0$ ;  $\mu = \frac{1}{1 + x^2}$ .  
 193.  $y^3 + x^3(\ln x - 1) = Cx^2$ ;  $\mu = 1/x^4$ .  
 194.  $2e^x \sin y + 2e^x(x - 1) + e^x(\sin x - \cos x) = C$ ;  $\mu = e^x$ .  
 195.  $x^2 - \frac{7}{y} - 3xy = C$ ;  $\mu = 1/y^2$ . 196.  $(x + y^2)^2 C = x - y^2$ ;  
 $\mu = \frac{1}{(x + y^2)^3}$ . 197.  $1 + y^2 - x^2 = Cx$ ;  $\mu_2 = 1/x^2$ ;  
 $\mu_1 = \frac{1}{(1 + y^2 - x^2)^2}$ . 198.  $y - 1 = C \sqrt{x^2 + y^2}$ ;

338.  $y = \frac{\sqrt{2}}{5} x^{5/2}$ . 339.  $y = C_3 + C_2 x - \sin(x + C_1)$ .  
 340.  $y = C_1 x^3 + C_2 x + C_3$ . 341.  $y = \cosh(x + C_1) + C_2$ .  
 342.  $y = C_2 - \ln|C_1 - x|$ . 343.  $y = C_2 - \cos(C_1 + x)$ .  
 344.  $y = C_2 - \ln|\cos(C_1 + x)|$ . 345.  $y = \frac{(x + C_1)^3}{12} - x + C_2$ .  
 346.  $y = x$ . 347.  $y = -2x$ . 348.  $y = C_2 - \ln|1 - e^{x+C_1}|$ .  
 349.  $(x + C_1)^2 + (y + C_2)^2 = 9$ .  
 350.  $y = (x + C_1) \ln|x + C_1| + C_2 x + C_3$ . 351.  $y = C_2 e^{C_1 x}$ .  
 352.  $y = \frac{1}{1-x}$ . 353.  $y = \left(1 + \frac{x}{3}\right)^3$ .  
 354.  $y = \frac{4}{(x+4)^2}$ . 355.  $y^2 = C_1 x + C_2$ .  
 356.  $y = \frac{1}{C_1} (1 + C_2 e^{C_1 x})$ . 357.  $y = C_1 \cosh \frac{x+C_2}{C_1}$ .  
 358.  $y = \frac{1}{C_1} \left[1 + \frac{(C_1 x + C_2)^2}{4}\right]$ . 359.  $y = \sqrt{2x - x^2}$ .  
 360.  $C_1 x + C_2 = \ln \left| \frac{y}{y+C_1} \right|$ . 361.  $y = -\ln|x-1|$ .  
 362.  $y \cos^2(x + C_1) = C_2$ . 363.  $y = \frac{4}{(x-2)^2}$ .  
 364.  $(x - C_1)^2 - C_2 y^2 + k C_2^2 = 0$ .

432.  $y = C_1 e^x + C_2 e^{-x}$ . 433.  $y = C_1 e^{2x} + C_2 e^{-\frac{4}{3}x}$ .  
 434.  $y = e^x (1 + x)$ . 435.  $y = e^{-x} (C_1 + C_2 x)$ .  
 436.  $y = 4e^x + 2e^{3x}$ . 437.  $y = C_1 e^{-x} + C_2 e^{-2x} + C_3 e^{-3x}$ .  
 438.  $y = C_1 e^{(1-\sqrt{3})x} + C_2 e^{(1+\sqrt{3})x}$ .  
 439.  $y = C_1 + C_2 x + C_3 x^2 + C_4 x^3 + e^{-x} (C_5 + C_6 x)$ .  
 440.  $y = e^x \left( C_1 \cos \frac{x}{2} + C_2 \sin \frac{x}{2} \right)$ . 441.  $y = C_1 e^{2x}$   
 $+ e^{-x} (C_2 \cos \sqrt{3}x + C_3 \sin \sqrt{3}x)$ .  
 442.  $y = e^{-x} (C_1 + C_2 x) + e^{-x} (C_3 \cos 2x + C_4 \sin 2x)$ .  
 443.  $y = e^x \sin x$ . 444.  $y = e^x (\cos \sqrt{2}x + \sqrt{2} \sin \sqrt{2}x)$ .  
 445.  $y = C_1 e^x + C_2 e^{-x} + e^{-x} (C_3 \cos 2x + C_4 \sin 2x)$ .  
 446.  $y = C_1 e^x + C_2 e^{-2x} + e^{-x} (C_3 + C_4 \cos x + C_5 \sin x)$ .  
 447.  $y = C_1 e^x + C_2 e^{-x} + C_3 e^{-2x}$ .  
 448.  $y = C_1 + e^x (C_2 \cos x + C_3 \sin x)$ . 449.  $y = C_1 e^x$   
 $+ C_2 e^{-x} + C_3 \cos x + C_4 \sin x$ . ~~450.  $y = C_1 + C_2 x$~~

510.  $y = (C_1 + C_2 x) e^{-x} - 2$ . 511.  $y = C_1 + C_2 e^{-2x} - x$ .  
 512.  $y = C_1 \cos 3x + C_2 \sin 3x + 1$ . 513.  $y = C_1 + C_2 x$   
 $+ C_3 e^{-x} + \frac{x^2}{2}$ . 514.  $y = C_1 + C_2 x + C_3 e^{\frac{7}{5}x} - \frac{3}{14} x^2$ .  
 515.  $y = C_1 + C_2 x + C_3 x^2 + C_4 e^{6x} + \frac{1}{6} x^3$ .  
 516.  $y = C_1 + C_2 x + C_3 x^2 + C_4 e^{-\frac{x}{3}} + \frac{x^3}{3}$ .  
 517.  $y = C_1 \cos x + C_2 \sin x + (C_3 + C_4 x) e^x + 1$ .  
 518.  $y = (C_1 + C_2 x) e^{2x} + \frac{x^2}{4} + \frac{x}{2} + \frac{3}{8}$ .  
 519.  $y = C_1 + C_2 e^{-8x} + \frac{x^2}{2} - \frac{x}{8}$ .  
 520.  $y = (C_1 + C_2 x) e^{kx} + \frac{e^x}{(k-1)^2}$ . 521.  $y = (C_1 + C_2 x) e^{-2x}$   
 $+ 4x^2 e^{-2x}$ . 522.  $y = C_1 e^{-3x} + C_2 e^{-x} - \frac{9}{2} x e^{-3x}$ .  
 523.  $y = C_1 + C_2 e^{\frac{x}{7}} - 7x^2 - 98x$ . 524.  $y = C_1 + C_2 e^{-3x}$   
 $- \left( \frac{x^2}{x} + \frac{x}{3} \right) e^{-3x}$ . 525.  $y = C_1 e^{-3x} + C_2 e^{-2x}$   
 $+ (20x - 5x^2) e^{-2x}$ . 526.  $y = (C_1 \cos x + C_2 \sin x) e^{-x} + \frac{x}{2}$ .  
 527.  $y = \left( C_1 \cos \frac{\sqrt{3}}{2} x + C_2 \sin \frac{\sqrt{3}}{2} x \right) e^{-\frac{x}{2}} + \frac{1}{3} (x^2 - x + 1) e^x$ .  
 528.  $y = C_1 e^{-(\sqrt{6}+2)x} + C_2 e^{(\sqrt{6}-2)x} - \frac{16 \cos 2x + 12 \sin 2x}{25}$ .

529.  $y = C_1 \cos x + C_2 \sin x + x \cos x + x^2 \sin x.$
530.  $y = (C_1 + C_2 x) e^{mx} + \frac{2mn \cos nx + (m^2 - n^2) \sin nx}{(m^2 + n^2)^2}.$
531.  $y = (C_1 \cos 2x + C_2 \sin 2x) e^{-x} - \frac{1}{4} x e^{-x} \cos 2x.$
532.  $y = C_1 \cos ax + C_2 \sin ax + \frac{2 \cos mx + 3 \sin mx}{a^2 - m^2} (|a| \neq |m|).$
533.  $y = C_1 + C_2 e^x - \frac{1}{2} (\cos x + \sin x) e^x.$
534.  $y = C_1 + C_2 e^{-2x} + \frac{1}{5} (6 \sin x - 2 \cos x) e^x.$
535.  $y = (C_1 \cos x + C_2 \sin x) e^{-2x} + 5x e^{-2x} \sin x.$
536.  $y = C_1 + C_2 e^{-2x} - \left( \frac{x}{10} + \frac{1}{50} \right) \cos x + \left( \frac{7}{50} - \frac{x}{20} \right) \sin x.$
537.  $y = C_1 e^{2x} + C_2 e^x - \left( \frac{x^2}{2} + x \right) e^x.$
538.  $y = C_1 e^x + C_2 e^{-2x} + \frac{1}{18} \left( x^2 - x + \frac{7}{18} \right) e^{4x}.$
539.  $y = C_1 e^x + C_2 e^{2x} + \left( \frac{x^2}{2} - x + 1 \right) e^{3x}.$
540.  $y = C_1 e^x + C_2 \cos x + C_3 \sin x - (x^2 + 3x + 1).$
541.  $y = (C_1 + C_2 x) e^x + C_3 \cos x + C_4 \sin x + \frac{x^2}{4} e^x.$
542.  $y = (C_1 + C_2 x) e^x + x^3 + 6x^2 + 18x + 24.$
543.  $y = C_1 + C_2 x + C_3 \cos x + C_4 \sin x + \frac{x^4}{12} + \frac{x^3}{6} - x^2.$
544.  $y = \left( C_1 + \frac{x}{4} - \frac{x^3}{6} \right) \cos x + \left( C_2 + \frac{x^2}{4} \right) \sin x.$
545.  $y = (C_1 + C_2 x) e^{-x} + [(6 - x^2) \cos x + 4x \sin x] e^{-x}.$
546.  $y = C_1 e^x + \left( C_2 \cos \frac{\sqrt{3}}{2} x + C_3 \sin \frac{\sqrt{3}}{2} x \right) e^{-\frac{x}{2}}$   
 $+ \frac{1}{2} (\cos x - \sin x).$  547.  $y = (C_1 + C_2 x) e^x$   
 $+ (C_3 + C_4 x) e^{-x} + \frac{1}{4} \cos x.$
548.  $y = (C_1 + C_2 x + C_3 x^2) e^x - \frac{e^x}{8} \sin 2x.$
549.  $y = \left( C_1 \cos x + C_2 \sin x - \frac{x}{2} \cos x + x \sin x \right) e^{2x}.$

- $+ B_2 \sin 3x$ . 558.  $y_{p.n} = A_1 x + B_1 \cos 8x + B_2 \sin 8x$ .  
 559.  $y = C_1 e^{-x} + C_2 e^{2x} - 2x + 1 + e^x$ .  
 560.  $y = C_1 + C_2 e^{3x} - 3x^2 - 2x + \cos x + 3 \sin x$ .  
 561.  $y = 2 + e^x (C_1 + C_2 x - \sin x)$ . 562.  $y = (C_1 \cos x + C_2 \sin x) e^{-x} + x e^x + e^{-x}$ . 563.  $y = (C_1 \cos 2x + C_2 \sin 2x) e^{-x} + e^{-x} - 4 \cos 2x + \sin 2x$ .  
 564.  $y = C_1 e^{2x} + C_2 e^{-\frac{x}{2}} + \frac{1}{4} (2x e^{2x} - 5)$ .  
 565.  $y = C_1 \cos 2x + C_2 \sin 2x + \frac{x}{8} \left( 1 + \frac{\cos 2x}{4} - \frac{x}{2} \sin 2x \right)$ .  
 566.  $y = (C_1 + C_2 x) e^{-x} + C_3 \cos x + C_4 \sin x - \frac{x}{8} \cos x + \frac{1}{4} \left( \frac{x}{2} - 1 \right) e^x$ . 567.  $y = C_1 + C_2 e^{-x} + \frac{1}{2} e^x - \frac{1}{10} \cos 2x + \frac{1}{20} \sin 2x + \frac{x^3}{3} - x^2 + 2x$ .  
 568.  $y = C_1 + C_2 x + C_3 x^2 + C_4 \cos 2x + C_5 \sin 2x + \frac{e^x}{5} + \frac{x^3}{24} + \frac{3x \sin 2x}{32}$ . 569.  $y = (C_1 \cos 2x + C_2 \sin 2x) e^x + \cos x + 2 \sin x + 4 \cos 2x + \sin 2x$ .  
 570.  $y = C_1 + C_2 e^{-x} + x e^{-x} + \frac{1}{2} e^x + \frac{x^3}{3} - x^2 + 2x$ .  
 571.  $y = C_1 e^{-x} + C_2 e^{3x} - \frac{2}{3} x + \frac{4}{9} - \frac{1}{4} x e^{-x} - \frac{1}{2} x e^{3x}$ .  
 572.  $y = C_1 \cos 2x + C_2 \sin 2x + x \left( \frac{1}{4} \sin 2x - \cos 2x \right) + \frac{1}{5} e^x$ .  
 573.  $y = C_1 e^{-x} + C_2 e^{-2x} + 3(x^2 - 2x) e^{-x} + 3(x^2 + 2x) e^{-2x}$ .  
 574.  $y = C_1 \cos x + C_2 \sin x - \frac{1}{3} \cos 4x - \frac{x}{4} \sin x + 1$ .  
 575.  $y = (C_1 \cos x + C_2 \sin x) e^{2x} + \cos x - \sin x + e^{2x} + \frac{1}{5}$ .

$$\begin{array}{l}
794. \begin{cases} x = C_1 t, \\ y = C_2 e^t. \end{cases} \\
796. \begin{cases} x^2 + y^2 = C_1^2, \\ p^2 + q^2 = C_2^2, \\ xp + yq = C_3. \end{cases} \\
798. \begin{cases} x^2 + y^2 = C_1 x - t^2, \\ y = C_2 x. \end{cases} \\
800. \begin{cases} x = \frac{t}{3} + \frac{C_2}{t^2}, \\ y = C_1 e^t - \frac{t}{3} - \frac{C_2}{t^2}. \end{cases} \\
802. \begin{cases} x = 2C_1 e^{3t} - 4C_2 e^{-3t}, \\ y = C_1 e^{3t} + C_2 e^{-3t}. \end{cases} \\
804. \begin{cases} x \equiv 0, \\ y \equiv 0. \end{cases} \\
806. \begin{cases} x = -5e^{2t} \sin t, \\ y = e^{2t} (\cos t - 2 \sin t). \end{cases} \\
807. \begin{cases} x = \frac{1}{3} C_1 e^t - C_2 e^{-2t}, \\ y = \frac{1}{3} C_1 e^t + 2C_2 e^{-2t}, \\ z = \frac{1}{3} C_1 e^t - C_2 e^{-2t}. \end{cases}
\end{array}$$

$$\begin{array}{l}
795. \begin{cases} t^2 - x^2 = C_1, \\ x^2 - y^2 = C_2. \end{cases} \\
797. \begin{cases} xy = C_1, \\ \ln x = C_2 + \frac{t^2}{2C_1}. \end{cases} \\
799. \begin{cases} 2x + 3y + 4t = C_1, \\ x^2 + y^2 + t^2 = C_2. \end{cases} \\
801. \begin{cases} x^2 + y^2 + t^2 = C_1, \\ x^2 - 2xy - y^2 = C_2. \end{cases} \\
803. \begin{cases} x = C_1 + C_2 e^{2t}, \\ y = C_1 - C_2 e^{2t}. \end{cases} \\
805. \begin{cases} x = e^{2t} - e^{3t}, \\ y = e^{2t} - 2e^{3t}. \end{cases}
\end{array}$$