A Two-Layered Optimisation-Based Control Strategy for Multi-Echelon Supply Chain Networks  

P. Seferlis and N. F. Giannelos  
Chemical Process Engineering Research Institute (CPERI)  
PO Box 361, 57001 Thessaloniki, Greece, email: seferlis@cperi.certh.gr  

Abstract  
A new two-layered optimisation-based control approach is developed for multi-product, multi-echelon supply chain networks. The first layer employs simple feedback controllers to maintain inventory levels at all network nodes within pre-specified targets. The feedback controllers are embedded as equality constraints within an optimisation framework that incorporates model predictive control principles for the entire network. The optimisation problem aims at adjusting the resources and decision variables of the entire supply chain network to satisfy the forecasted demands with the least required network operating cost over a specified receding operating horizon. The proposed control strategy is applied to a multi-product supply chain network consisting of four echelons (plants, warehouses, distribution centres, and retailers). Simulated results exhibit good control performance under various disturbance scenarios (stochastic and deterministic demand variation) and transportation time lags.  

1. Introduction  
A supply chain network is commonly defined as the integrated system encompassing raw material vendors, manufacturing and assembly plants, and distribution centres. The network is characterised by procurement, production, and distribution functions. Leaving aside the procurement function (purchasing of raw materials), the supply chain network becomes a multi-echelon production/distribution system (figure 1). The operational planning and direct control of the network can in principle be addressed by a variety of methods, including deterministic analytical models, stochastic analytical models, and simulation models, coupled with the desired optimisation objectives and network performance measures (Beamon, 1998; Riddalls et al., 2000). Operating network cost, average inventory level, and customer service level are commonly employed performance measures (Thomas and Griffin, 1996; Perea et al., 2001).  
In the present work, we focus on the operational planning and control of integrated production/distribution systems under product demand uncertainty. For the purposes of our study and the time scales of interest, a discrete time difference model is developed. The model is applicable to networks of arbitrary structure. To treat demand uncertainty within the deterministic supply chain network model, a receding horizon, model predictive control approach is suggested. The two-level control algorithm relies on a decentralised safety inventory policy, coupled with the overall optimisation-based control approach.
2. Supply Chain Model

Let $DP$ denote the set of desired products (or aggregated product families) of the system. These can be manufactured at plants, $P$, by utilising various resources, $RS$. The products are subsequently transported to and stored at warehouses, $W$. Products from warehouses are transported upon customer demand, either to distribution centres, $D$, or directly to retailers, $R$. Retailers receive time-varying orders from different customers for different products. Satisfaction of customer demand is the primary target in the supply chain management mechanism. Unsatisfied demand is recorded as back-orders for the next time period. A discrete time difference model is used to describe the supply chain network dynamics. The duration of the base time period depends on the dynamic characteristics of the network.

The inventory balance equation, valid for warehouses and distribution centres, is:

$$y_{i,k}(t) = y_{i,k}(t-1) + \sum_{k'} x_{i,k',k}(t - L_{k,k'}) - \sum_{k'} x_{i,k',k}(t) \quad \forall \ k \in \{W,D\}, t \in T, i \in DP$$  \hspace{1cm} (1)

$y_{i,k}$ is the inventory of product $i$ stored at node $k$. $x_{i,k',k}$ and $x_{i,k',k'}$ denote the amounts of the $i$-th product transported through routes $(k',k)$ and $(k,k')$, respectively, where $k'$ supply $k$ and $k''$ are supplied by $k$. $L_{k,k'}$ denotes the transportation lag for route $(k',k)$.

The transportation lag is assumed to be an integer multiple of the base time period. For retailer nodes, the inventory balance considers the actual delivery of product $i$ attained, denoted by $d_i$:

$$y_{i,k}(t) = y_{i,k}(t-1) + \sum_{k'} x_{i,k',k}(t - L_{k,k'}) - d_i(t) \quad \forall \ k \in R, t \in T, i \in DP$$  \hspace{1cm} (2)

The balance equations for unsatisfied demand (e.g., back-orders) take the form:
\[ BO_{i,k}(t) = BO_{i,k}(t-1) + R_{i,k}(t) - d_{i,k}(t) - LO_{i,k}(t) \quad \forall \ k \in R, t \in T, i \in DP \] (3)

where \( R_{i,k}(t) \) denotes the demand for product \( i \) at retailer \( k \) and time period \( t \). \( LO_{i,k}(t) \) denotes the amount of cancelled back-orders (lost orders).

At each node capable of carrying inventory (nodes of type \( W, D, \) and \( R \)), capacity constraints are in effect that account for a maximum allowable inventory level:

\[ Y_{i}(t) = \sum \alpha_{i} \cdot Y_{i,k}(t) \leq V_{k}^{\max} \quad \forall \ k \in \{W, D, R\}, t \in T \] (4)

where \( Y_{i} \) denotes the actual inventory of the node, \( \alpha_{i} \) the storage volume factor for each product, and \( V_{k}^{\max} \) the maximum capacity of the node.

A maximum allowable transportation capacity, \( T_{k,k}^{\max} \), is defined for each permissible transportation route within the supply chain network:

\[ \sum \beta_{i} \cdot Y_{i,k}(t) \leq T_{k,k}^{\max} \quad \forall \ k \in \{P,W,D\}, t \in T \] (5)

where \( \beta_{i} \) denotes the transportation volume factor for each product.

For each manufacturing resource \( RS_{j} \), a maximum level of availability, \( C_{j,k}^{\max} \), in each plant node is specified:

\[ \sum \sum \kappa_{i,j} \cdot Y_{i,k}(t) \leq C_{j,k}^{\max} \quad \forall \ k \in P, t \in T, j \in RS \] (6)

where \( \kappa_{i,j} \) denotes the usage factor of the \( j \)-th resource for the \( i \)-th product.

### 3. Control Strategy for Supply Chain Management

Supply chain management is performed within a two-layered approach. The first layer aims at keeping inventory levels around pre-specified targets. A single dedicated controller is used for each inventory node. Disturbances are generated by demand fluctuations at downstream nodes.

The second level of control is a model predictive optimisation-based scheme that considers the entire network dynamics, embedding the inventory controllers of the first layer and a stochastic model for demand variation.

#### 3.1 Inventory Control

Proportional-integral-derivative (PID) controllers are derived for the feedback control of inventory levels. The PID control law in discrete velocity form is given by the following relationship (Marlin, 1995):
\[
\Delta mv_i(t) = K_c \frac{\Delta Y_{SP,k}}{\tau_i} - K_i \left[ \left( 1 + \frac{\Delta Y_{SP,k}}{\tau_i} + \tau_D \right) Y_i(t) - \left( 1 + 2 \frac{\tau_D}{\Delta t} \right) Y_i(t-1) + \frac{\tau_D}{\Delta t} Y_i(t-2) \right]
\]

(7)

where \( \Delta mv_i(t) \) is the change for the manipulated variable of the inventory controller, \( Y_{SP,k} \) the target value for \( Y_i \), \( \Delta t \) the discrete control interval, \( K_c \) the proportional gain, \( \tau_i \) the integral time, and \( \tau_D \) the derivative time for the inventory controller.

The manipulated variable for the inventory control of each node is the total amount of the products transferred from all supplying nodes to node \( k \):

\[
mv_i(t) = \sum_{k'} \sum_i x_{i,k',t} (t - L_{i,k})
\]

(8)

The present choice of the manipulated variables imposes a constraint on all incoming material to a particular node. The PID controllers are tuned to allow for fast set-point tracking and good disturbance rejection dynamics, taking into consideration the transportation delay between nodes.

### 3.2 Optimisation-based model predictive control

Supply chain management requires a number of decisions to be determined at every time period. The main objectives of the supply chain network can be summarised as follows: (i) maximise customer satisfaction, and, (ii) keep operating supply chain costs low. The first target is attained by the minimisation of back-orders for a period of time, while the second target is achieved by minimising transportation and inventory (storage) costs associated with the supply chain network.

Based on the fact that past and present control actions affect the future response of the system, a receding-time horizon is selected. The trajectory of the system is predicted and compared to the desired trajectory. The control actions are then determined from the minimisation of a performance index over the given time horizon, \( t_h \):

\[
J = \sum_{i=1}^{t_{i_1}} \sum_{k \in [R]} \sum_i \left[ w_{BO,i,k} BO_i(t) \right] + \sum_{i=1}^{t_{i_2}} \sum_{k \in [W, D, R]} \sum_i \left[ w_{T,i,k} x_{i,k,t} \right]
\]

(9)

The performance index, \( J \), includes a term penalizing back-orders at all retailer nodes, and a term accounting for the transportation costs. The weighting factors \( w_{BO} \) and \( w_T \) reflect the relative importance of the controlled (back-orders) and manipulated (transportation of products) variables.

The overall optimisation model-based predictive controller for the supply chain network takes the following form:

\[
\text{Min } J
\]

s.t.  
Supply chain model Eq. 1 – 6
Feedback inventory controllers Eq. 7 – 8
Stochastic disturbance model
All variables in the supply chain network are assumed to be continuous. This is definitely valid for bulk commodities and products. For unit products, continuous variables can still be utilised, with the addition of a post-processing rounding step to identify neighbouring integer solutions. This approach, though not formally optimal, may be necessary to retain computational tractability in systems of industrial relevance. The computational cost for solution of linear programme (P) increases with the size of the receding time horizon, which has to be carefully chosen in order to balance good control performance with robustness to external disturbances. The time horizon should be selected at least as large as the largest transportation delay in the system.

4. Simulated Results

A four-echelon supply chain system consisting of two production nodes, two warehouse nodes, four distribution centres, and 16 retailer nodes is studied. All possible connections between immediately successive echelons are permitted. There is a number of low-cost routes between successive echelons that carry the bulk of the supply, and a number of more expensive alternative routes that will be used periodically when the cost of accumulated back-orders becomes significant. Five product families are being distributed. The size of the network model is 650 variables per time period with 358 product quantities along transportation routes and 80 back-order quantities as optimisation variables.

4.1 Deterministic disturbance in demand

The given network can accommodate step changes in the order of 50% for each product family. Retailer inventories resume their set-points within typically 10 time periods, whereas 15 time periods are required in the larger warehouse nodes. The maximum deviation from inventory set-points is kept below 20% (figure 2).

4.2 Stochastic disturbance in demand

A stationary stochastic model is used to describe the demand variation over the simulated time span. Typical simulation results are reported in table 1 and figure 3. Demand satisfaction is optimised at high levels in most cases. The performance index, reflecting network operating costs, increases at higher demand variances and transportation delays. Larger transportation lags require a more conservative controller tuning in order to avoid controller-induced instability. However, there is a compromise between tight inventory control (small inventory variance) and achievable minimum performance index.

Cases 4-5 and 7-8 exemplify the impact of proper receding horizon selection to supply chain network performance. In case 4, a time horizon equal to 3 time periods achieves a higher demand satisfaction rate at the expense of higher transportation costs than case 5, where a time horizon equal to 5 is selected. In case 7, the control horizon is equal to the maximum transportation lag in the system (5 time periods), resulting in a relatively high performance index and lower customer satisfaction. By increasing the control horizon to 6 time periods, a significant improvement is observed. In general, shorter time horizons lead to more aggressive control actions, but larger time horizons may render the control scheme more susceptible to demand variations.
Table 1. Results of simulated cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>Transport. lags</th>
<th>Receding horizon $t_h$</th>
<th>Demand satisfaction %</th>
<th>Variance in prod. demand</th>
<th>$J$/time period</th>
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5. Conclusions
A two-layered control strategy was described for supply chain management purposes. The strategy combines feedback controllers to account for the fast dynamics at the inventory nodes, while utilising the power of a fully-centralised optimisation-based model predictive controller to achieve an optimal operating policy for the supply chain network over a selected time horizon.

6. References