

Four Questions on the History of Proof

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The Historical and Epistemological Studies are referred to as a point of view in the introductory text of the Group 4 (on Argumentation and Proof) of the Fourth Conference of the European Society for Research on Mathematics Education [CERME 4] that will take place in Spain in February 2005, {<http://www.lettredelapreuve.it/Newsletter/04 Ete/Cerme4.html>}. In this way a strong motivation is given in order to create a relevant problématique to develop some ideas and questions on the historical aspects of the proof. These ideas or questions can improve the historical understanding of the proof and its pedagogical approach. In that context let us present a relevant problématique consisting of 4 questions.

1. What is the historical evolution of proof in algebra? And what is its evolution in school algebra?

The conceptions of proof in school algebra have been the object of a study by L. Healy and C. Hoyles {*Journal for Research in Mathematics Education*, 31(4), 2000, pp.396- 428}. The historical dimension is absolutely absent in that remarkable paper. So the meaning of proof in the different epistemological contexts of the history of Algebra and of School Algebra cannot be seen. This absence can also be detected in the collective studies on the Didactic of school Algebra that have been published in recent years. However, it is easy to see that proofs have entered impetuously in school Algebra under the influence of the New Math movement and the structural epistemology underlying it. Before that movement, proofs were absolutely absent in school Algebra, or if present they used to have a false form. On the other hand the relevant historiography of the proof in Algebra is not empty. There are some publications on special cases, for example:

- a. Abdeljahouad, Mahdi: Proof in Arabian Algebra, *Preuve, International Newsletter on the Teaching and Learning of Mathematical Proof*, Hiver, 2000.
- b. Borowczyk, J.: Sur l' histoire des démonstrations de la règle des variations de signe de Descartes, in *La Démonstration Mathématique dans l' Histoire*, IREM de Besançon et IREM de Lyon, 1989, pp. 275-312.
- c. Pensivy, M. : Les démonstrations de la formule du binôme au XVIIIème siècle, *ibid*, pp. 325-338.

Never the less there seems to be poverty and a lack of contextual and cognitive studies on history (or historical mentalities) of proofs in Algebra.

2. What is the place of experimentation, in particular of thought experiments in the history of proof?

The emphasis posed on the logical aspect of proof usually obscures all its other aspects. But the digital revolution produces radical changes. So we read: “The use of computers gives the mathematicians another view of mathematical reality” like “experimentation and proof in mathematics” {*Notices of the AMS*, 42(6), 1995, p. 670} or “thought experiments: proofs in a computer environment” {*Micromath*, 10(3), 1994, p. 21}. From this point of view a historical interest emerges for the relation of the proof with the experiments and thought experiments in particular. Relevant comments can be found in the following papers:

- a. Mueller, E.: Euclid's *Elements* and Axiomatic Method, *The British Journal for the Philosophy of Science*, 20, 1969, pp.289-309.
- b. Bettinelli, B.: Intuition et démonstration chez Archimède, in *La Démonstration Mathématique dans l' Histoire*, IREM de Besançon et IREM de Lyon, 1989, pp. 181-195.
- c. Lacatos, I.: *Proofs and Refutations. The Logic of Mathematical Discovery*, Cambridge Univ. Press, 1976.
- d. Hanna, G./Jahnke, H.N.: Another Approach to Proof: Arguments from Physics, *ZDM*, 34(1), 2002, pp. 1-8.
- e. Hattiangadi, J.: Algebra as Thought Experiment, <http://www.bu.edu/wcp/Papers/Scie/Scie Hatt.htm>.
- f. Reiner, M/Leron, U.: Physical Experiments, Thought Experiments, Mathematical Proofs, *Model-Based Reasoning Conference (MBR' 01)*, 2001, Pavia, Italy.
- g. Glass, E.: Thought- Experimentation and Mathematical Innovation, *Studies in History and Philosophy of Science, Part A*, 30(1), 1999, pp.1-19.
- h. Tall, D.: The Chasm between Thought Experiment and Formal Proof, in *Kadunz, G. et al (eds): Mathematische Bildung und neue Technologien*, Teubner, 1999, pp. 319-343.

3. Why did the notion of proof appear at all?

Until now the dominant point of view is that proof was generated in the Ancient Greek Civilization. However there is not very much that has been written on the reasons have generated it. The two most well-known justifications are those advanced by Szabó and, respectively, Loyd. According to the first, the origins of

the idea of proof are to be found in the philosophy of Eleatics. He suggested that this was due to the genius of those pioneer philosophers of the 5th century {Szabó, A.: *The Beginning of Greek Mathematics*, D. Reidel, 1978, pp. 185-329}. On the other hand Lloyd regards proof to spring out of the argumentation of the rhetoric which, being an element of democracy, gives proof its social matrix (Lloyd, G.E.R.: *Magic, Reason and Experience*, Cambridge Univ. Press, 1979, pp. 59-125).

It is useful to note that without the concept of equality, together with its transitive property, it would be impossible to shape the process of proof. Equality was interwoven with the coinage system and democracy and both of them emerged and evolved within the Greek Civilization from 7th to 6th century. In other words the concept of equality was a cultural medium that played a decisive role in the evolution of the function of proof. This is a matter of fact that has escaped the attention of the aforementioned historical researches. The following paper could be useful:

Harvey, F. D.: Two Kinds of Equality, *Classica et Mediaevalia*, 26, 1965, pp. 101-146.

On the other hand, we also see an interest in the development of proof through illustrative example, as in Mesopotamia and China. Two relevant papers are the following:

a. Chemla, K.: What is at Stake in Mathematical Proofs from Third-Century China, *Science in Context*, 10, 1997, pp. 227-251.

b. Høystrup, J.: Mathematical justification as non-conceptualized practice: the Babylonian example, in *Proceedings of HPM 2004*, Uppsala, 2004, pp. 28-41.

4. What are the historical developments and characteristics of proof in the context of absolutist philosophies? And how did they appear in a non-absolutist context?

The historical evolution of proof is not independent of its epistemological. Thus, looking for the cognitive changes in the thought of people who got involved in logical proofs during the historical development of mathematics, can help a deeper understanding of the notion of proof itself. Accordingly, being aware of the philosophical influences on the nature of proof can contribute to its better didactic treatment. There are rather few such publications. We might indicate the following:

a. Lee, Joong Kwoen: Philosophical Perspectives on Proof in Mathematics Education, *Philosophy of Mathematics Education Journal*, July 2002, <http://www.ex.ac.uk/~PErnest/pome16/docs/lee.pdf>

b. Guichard, J.: Arrière-plans philosophiques de la démonstration, in *La Démonstration Mathématique dans l' Histoire*, IREM de Besançon et IREM de Lyon, 1989, pp. 39-52.

c. Bkouche, R. : Quelques remarques sur la démonstration (Autour de la philosophie de Gonseth), *ibid*, pp. 115-127.

d. Brown, J. R.: *Philosophy of Mathematics. An Introduction to the World of Proofs and Pictures*, Routledge, 1999.

e. Roulet, G.: The Legacy of Piaget: Some Negative Consequences for Proof and Efforts to Address Them, *Preuve, International Newsletter on the Teaching and Learning of Mathematical Proof*, May/June, 2000.

f. Bassler, O. B.: The Surveyability of Mathematical Proof: A Historical Perspective, *Synthese*, forthcoming.