DISCUSSION PAPER

0401

DEBT, EQUITY AND WARRANTS

Kostas KOUFOPOULOS
The Roles of Debt, Equity and Warrants
Under Asymmetric Information

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Abstract

This paper considers project financing under adverse selection and moral hazard and makes three contributions. First, the issue of combinations of debt and equity is explained as the outcome of the interaction between adverse selection and moral hazard. Second, it shows that, in the presence of moral hazard, adverse selection may result in the conversion of negative into positive NPV projects leading to an improvement in social welfare. Third, it provides two rationales for the use of warrants. It also shows that, under certain conditions, a debt-warrant combination can implement the optimal contract as a competitive equilibrium.

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1. Introduction

Following the famous irrelevance proposition of Modigliani and Miller (1958), a vast literature has developed trying to explain the financial choices of firms when they seek outside funds.\(^1\) Despite this research effort, important puzzles remain. Some recent empirical studies find that neither of the two dominant theories of capital structure, the trade-off theory and the pecking-order theory, provides a satisfactory explanation for the observed financing patterns.\(^2\) Firms appear to issue surprisingly large amounts of equity, even after controlling for the various costs (due to financial distress, bankruptcy and agency problems between debtholders and shareholders) associated with debt issues.\(^3\) Moreover, although equity issue announcements are associated with stock price drops (due to adverse selection),\(^4\) equity dominates debt as a source of external financing.\(^5\) Myers and Majluf (1984) show that this adverse-selection problem may lead to underinvestment and so a loss in social welfare.

Furthermore, a considerable fraction of the securities with option features issued by firms are debt-warrant (or equity-warrant) combinations rather than convertible debt.\(^6\) Existing models offer various explanations of why firms issue convertible debt.\(^7\) However, none of them justifies the necessity for the issue of warrants.

This paper abstracts from taxes, financial distress, bankruptcy and other agency costs and focuses on asymmetric information. We consider a model involving both adverse selection and (effort) moral hazard. There are two types of firms (projects): risky (R), safe (S). Given identical effort levels, the success probability of the safe project is higher but its return in case of success is lower. In the event of failure the return of both types is zero. The entrepreneur can increase the success probability by exerting costly effort. Regardless of the project’s type, if the entrepreneur exerts effort the net present value (NPV) of his project exceeds the cost of effort whereas if he shirks the project has negative NPV. That is, exerting effort is socially efficient for both types. Both the project’s type and the entrepreneur’s action are unobservable.

In this setting, we analyse the roles of debt, equity and warrants and make three contributions. First, we explain the issue of combinations of debt and equity as the outcome of the interaction between adverse selection and moral hazard. Some firms (the risky ones) issue equity even if under pure adverse selection they would have issued just debt. Second, we show that, in the presence of moral hazard, adverse selection may result in the conversion of a negative into a positive NPV project and an improvement in social welfare. Third, we provide two rationales for the use of

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\(^1\) See Harris and Raviv (1991) for a survey.

\(^2\) See, for example, Helwege and Liang (1996), Lemmon and Zender (2001) and Frank and Goyal (2003).

\(^3\) The tax benefits of debt are significant and firms’ decisions about financial policies appear to be affected by tax considerations (see, MacKie-Mason (1990) and Graham (1996) and (2000)). On the other hand, although financial distress and other agency costs are important, they are not large enough to explain these conservative debt policies (see Andrade and Kaplan (1998) and Parrino and Weisbach (1999) and Lemmon and Zender (2001)).

\(^4\) See Lemmon and Zender (2001).

\(^5\) Frank and Goyal (2003) report that net equity issues follow the financing deficit more closely than debt issues.

\(^6\) For example, de Roon and Veld (1998) report that about 30 percent of the convertible securities issued by Dutch companies from 1976 to 1996 were debt-warrant combinations.

\(^7\) Convertible debt is a special case of a debt-warrant combination that obtains when the exercise price of the warrant equals the face value of debt. Green (1984), Constantinides and Grundy (1989), and Stein (1992) provide three different rationales for the use of convertible debt.
warrants. We also show that, under certain conditions, a debt-warrant combination can implement the optimal contract as a competitive equilibrium.

Two cases are considered: i) pure adverse selection and ii) adverse selection cum moral hazard. In the former case, a combination of securities is only used to convey socially costless information about the type of the project. In the latter case, in addition to transmitting information, the securities issued are the means of providing the appropriate effort incentives. Because of this second role, the introduction of moral hazard into an adverse selection framework has significant effects both on the combinations of securities issued in equilibrium and their pricing.

Regarding the pure adverse selection case, if firms have more information about the quality of their projects than their financiers, then they have an incentive to issue overpriced securities. To the extent that firms cannot credibly signal their type, the resulting adverse-selection problem may lead firms to forego a positive NPV project. Following Myers and Majluf (1984), a great deal of research effort has been devoted to exploring the extent to which this problem can be overcome if firms use different combinations of financial instruments to transmit information. It has been shown that debt or equity repurchases in conjunction with the issue of some other security (e.g. equity or convertible debt respectively) may allow for the existence of fully revealing equilibria where the securities issued are correctly priced.

In this paper, we do not allow for debt or equity repurchases. Firms try to reveal their type by issuing debt-equity or debt-warrant combinations. Equity (warrant) is a convex claim and so its value increases with the variability of returns. On the other hand, debt is a concave claim and so its value falls with risk. That is, in relative terms, debt is more valuable for the safe type and equity (warrant) for the risky one. Thus, by issuing more of the less valuable for him security, an entrepreneur can credibly signal his type and reduce the underpricing of his securities. However, the existence of an equilibrium where the securities issued are fairly priced requires that debt is more valuable for the S-type and equity (warrant) for the R-type not only in relative but also in absolute terms. Otherwise, the type whose securities are more valuable can only minimise the underpricing of his securities by issuing just the relatively less valuable for him security.

If the risky projects are mean-increasing or mean-preserving spreads of the safe ones or the risky projects dominate the safe ones by first-order stochastic dominance both conditions are met. In the first case, separation requires the issue of both debt and equity (Heinkel 1982). In the two remaining cases, the adverse-selection problem can be solved (mitigated) by issuing either just equity (mean-preserving spreads) or just debt (first-order stochastic dominance). On the other hand, if the risky projects are

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8 In this case, we seek methods of financing that result in nondissipative equilibria (Bhattacharya (1980)). That is, equilibria that imply no deadweight losses relative to the full information equilibrium.

9 See, for example, Brennan and Cras (1987), Constantinides and Grundy (1989) and Heider (2001). However, models that use debt (or equity) repurchases to obtain fully revealing equilibria have a serious shortcoming. They do not explain why firms issued debt (equity) in the past.

10 Provided the returns of the two types in case of success are different, this (the single-crossing) condition is satisfied regardless of the distributional assumption or the combination of the securities issued. On the other hand, if the safe projects dominate the risky ones by first-order stochastic dominance (the returns of both types in case of success are equal), both conditions are violated. In this case, the equality of returns prevents us from extracting any information about the type of the project. There can exist only pooling equilibria where the S-type provides the R-type with the same amount of subsidy regardless of the securities issued. Notice that if firms have assets in place, the use of collateral could be a solution. However, this solution may not be costless, it may imply deadweight losses (e.g. Besanko and Thakor (1987) and Bester (1987)).

11 These results are well-known (see de Meza and Webb (1987) and Nachman and Noe (1994)).
mean-reducing spreads of the safe ones, both the debt and equity issued by the S-type are more valuable than those issued by the R-type. Thus, the S-type cannot reveal his type and inevitably subsidises the R-type through the mispricing of the relatively less valuable for him security (equity) at individual level.

The use of warrants, through the appropriate choice of their exercise price,\(^\text{12}\) allows for the achievement of full separation even in this case. Since the return of the R-type in case of success is greater, a given increase in the exercise price of the warrant implies that the project’s return constitutes a smaller proportion of the total payment to the financier if the warrant is issued by the S-type. That is, as the exercise price rises, the value of the warrant issued by the S-type falls faster. As a result, for a sufficiently high exercise price, the warrant issued by the R-type can be more valuable than that of the S-type even if the S-type equity is more valuable.

This mechanism provides a rationale for the use of warrants. Warrants are issued because they can serve as separation devices when other standard securities (debt, equity and/or convertible debt) cannot.\(^\text{13}\)

The introduction of moral hazard into an adverse selection framework has significant effects both on the combinations of the securities issued in equilibrium and their pricing. The distinguishing feature of this part of the paper is the existence of pooling equilibria involving cross subsidisation across types and the issue of both debt and equity (warrants). These pooling equilibria reflect a trade-off between information revelation and effort incentives. The securities issued by the R- and S-type are priced as a pool. Although, because of perfect competition, debt and equity (warrants) are fairly priced collectively, at individual level they are mispriced. In fact, it is precisely this mispricing that provides the more prone to shirking type with the subsidy necessary to induce him to choose the socially efficient high effort level.\(^\text{14}\)

Consider, for example, the case where the S-type is more prone to shirking and we restrict ourselves to debt and equity. In this case, in the pooling equilibrium the R-type subsidises the S-type through the mispricing of equity. In the absence of moral hazard, the R-type would have issued more debt and less equity. Since, in doing so, he would reduce the subsidy and increase his expected return. However, in the presence of moral hazard, the S-type always mimicks the R-type and such a deviation would destroy his effort incentives. As a result, both the collective and the R-type’s net expected return would fall. Since he cannot reveal his type, the R-type accepts to issue just enough equity to induce the S-type to exert effort because the resulting increase in his net expected return (due to the lower interest rate on debt) more than offsets the cost of the incremental subsidy (the adverse selection cost of issuing equity). That is, this pooling equilibrium involves the minimum subsidy consistent with the S-type exerting effort. In any pooling equilibrium involving more than this minimum subsidy, the R-type can still profitably deviate by issuing more debt and less equity.

That is, in the presence of both adverse selection and moral hazard, in addition to being communication devices, debt and equity play a second role. That of

\(^{12}\) It is set such that, in case of success, the option is exercised regardless of the issuer type. Also, the proceeds (from the exercise of the option) are distributed as dividends to the shareholders.

\(^{13}\) Notice that convertible debt cannot play this role. The mechanism described above works only if the exercise price of the warrant increases while the face value of debt is fixed (in equilibrium, the exercise price of the warrant is strictly greater than the face value of debt). If the two coincide, the increase in the exercise price is exactly offset by the increase in the face value of debt. Hence, the value of convertible debt is strictly greater if it is issued by the S-type regardless of the face value of debt or whether conversion takes place.

\(^{14}\) If funds are offered at fair terms, the more prone to shirking type chooses the low effort level. Hence, his project NPV is negative and so, if his type is revealed, no rational financier offers funds to him.
incentivising the more prone to shirking type through their mispricing at individual level. This double role stems from the interaction between adverse selection and moral hazard and provides an explanation for the issue of combinations of debt and equity even if the issue of equity implies an adverse selection cost. What is more, in contrast with the pure adverse selection case, the cross-subsidisation is socially beneficial. It converts a negative into a positive NPV project and improves social welfare.

However, if firms can only issue debt and equity, it may be the case that, at any given debt level, the proportion of equity issued consistent with exerting effort is strictly lower for the S-type. That is, the pooling equilibrium where both types exert effort may collapse although the R-type would have exerted effort even if a higher proportion of equity was issued (more subsidy was given to the S-type). Because the warrant value falls with the exercise price faster for the S-type, the S-type is willing to increase faster the proportion of equity offered to the financier than the R-type while still exerting effort. As a result, through the appropriate choice of their exercise price, warrants allow for the implementation of the socially efficient outcome even if this is not possible when we restrict ourselves to debt, equity and/or convertible debt. This result provides a second rationale for their use. Finally, we show that, under certain conditions, a debt-warrant combination can implement the optimal contract as a competitive equilibrium.

1.1 Related Literature

This paper is related to three strands in the literature: agency models, pure adverse selection models and models combining adverse selection and moral hazard.

In the celebrated Jensen and Meckling (1976) paper firms issue both debt and equity to minimise the sum of agency costs of these two securities. The agency cost of equity arises from the conflict of interest between management and outside shareholders. The agency cost of debt stems from the conflict of interest between existing shareholders (managers) and would-be debtholders. The issue of debt induces the managers to undertake riskier projects that reduce the value of debt and transfer wealth from debtholders to shareholders (asset substitution problem).15

Related to that is the debt overhang problem described by Myers (1977). The existence of risky debt implies that shareholders (managers) may not undertake positive NPV projects because they will incur the total cost of the project but obtain only part of the returns. In fact, if the increase in the value of the outstanding risky debt exceeds the NPV of a project, investment in the project would result in a fall in the shareholders net return.

Therefore, in agency models, the reduction in the agency cost of equity resulting from the issue of debt is offset at the margin by the increase in the agency costs of debt. This trade-off determines the optimal debt-equity ratio (capital structure).

In this context, Green (1984) focuses on the asset substitution problem and develops a rationale for the use of convertible debt (warrants). Convertible debt reverses the convex shape of levered equity over the upper range of the firm’s returns (where conversion takes place). As a result, it alters the incentives of the shareholders to take risk and so mitigates the asset substitution problem.

15 Notice that in our model there is no conflict of interests between shareholders and debtholders (no asset substitution problem) which, given the agency cost of equity, is the driving force of the coexistence of debt and equity in Jensen and Meckling. In our case, moral hazard concerns the choice between different effort levels rather than the choice between a safe and a risky project (the source of the asset substitution problem).
More recently, Biais and Casamatta (1999) consider a model similar to Jensen and Meckling but they completely endogenise the contractual form. Nevertheless, they show that if the risk-shifting problem is more severe, a debt-equity combination (or convertible debt) can implement the optimal contract whereas if the effort problem is more severe stock options are also needed.

On the other hand, pure adverse selection models emphasise the signalling role of the financing decisions of the firm. If firms have no assets in place and there are no bankruptcy or financial distress costs, by using debt and equity, we can obtain fully revealing equilibria when the risky projects are mean-increasing or mean-preserving spreads of the safe ones or they dominate the safe projects by first-order stochastic dominance.

Moreover, if firms have debt and equity outstanding and debt and equity repurchases are allowed, there potentially exist fully separating equilibria under a wider range of distributional assumptions. Brennan and Kraus (1987) allow only for debt repurchases and consider two cases: first-order stochastic dominance and mean-preserving spreads. They show that fully revealing equilibria can be obtained by issuing equity and repurchasing debt in the first case and by issuing convertible debt in the second. Constantinides and Grundy (1989) allow only equity repurchases and prove that, under first-order stochastic dominance, the issue of convertible debt coupled with equity repurchases leads to full information revelation.

Stein (1992) introduces financial distress costs and provides another justification for the use of convertible debt as well as the issue of debt and equity. In a three-type model, he obtains a fully separating equilibrium where the good type issues debt, the medium type issues convertible debt that is always converted into equity, and the bad type issues equity directly to avoid incurring the distress costs. In this separating equilibrium all firms invest and no distress costs are borne in equilibrium. If convertible debt were not used, this separating equilibrium would not, in general, exist and a situation similar to that described in Myers and Majluf (1984) would arise.

The justifications provided by Green (1984), Brennan and Kraus (1987), and Constantinides and Grundy (1989) for the use of convertible debt rely on the fact that its payoff is concave in the firm’s returns for low values of returns and convex for higher values. On the other hand, in Stein (1992), the usefulness of convertible debt stems from the presence of financial distress costs and the inability of a bad firm to force conversion. In our paper, the mechanism at work is different. First, it does not depend on financial distress costs. Second, in our case, convertible debt does not improve on a debt-equity combination. Our mechanism relies on the fact that the warrant exercise price can be greater than the face value of debt. By appropriately choosing the exercise price, we can exploit the difference between the returns of the two types of projects and satisfy the revelation or effort incentive constraints under weaker conditions than if warrants were not available.

Our paper is most directly linked to models involving both adverse selection and (effort) moral hazard. Darrough and Stoughton (1986) provide such a model where entrepreneurs are risk averse and can issue combinations of debt and equity. However,

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16 The pure adverse selection part of this paper belongs to a class of models that seek methods of financing that lead to nondissipative equilibria (Bhattacharya (1980)). Other examples include Heinkel (1982), Brennan and Kraus (1987), and Constantinides and Grundy (1989). Early examples of signalling models in the corporate finance literature are: Leland and Pyle (1977), Ross (1977), Bhattacharya (1979).

17 If both debt and equity repurchases are allowed, there potentially exist fully separating equilibria under any assumption about the ordering of the distributions of returns as demonstrated by Heider (2001) in a two-type model.
they only consider separating equilibria where the securities issued are fairly priced. As a result, neither cross-subsidisation across types occurs nor the issue of equity when it implies adverse selection costs can be explained. In contrast, in Vercammen (2002) firms cannot signal their type. Because of his distributional assumption and the fact that firms are restricted to issue only debt a unique pooling equilibrium arises. He shows that the cross-subsidisation that takes place through the mispricing of debt at individual level raises aggregate surplus. Because the low-quality firms are more severely affected by the moral hazard, the cross-subsidisation results in a higher overall effort level and so a lower average failure probability.\textsuperscript{18}

In our model, we allow for a wider range of distributional assumptions and firms can use combinations of securities to reveal information about their type. In our case, the pooling equilibrium involves the issue of both debt and equity (warrants) and the minimum subsidy consistent with S-type (the more prone to shirking) exerting effort. In any pooling equilibrium involving more than this minimum subsidy, the R-type (the subsidiser) can profitably deviate by issuing more debt and less equity. However, because the S-type always mimics him, the R-type issues just enough equity to induce him to exert effort because the resulting increase in the R-type's net expected return more than offsets the cost of the incremental subsidy. In other words, if both types exert effort the collective expected return increases so much that both are strictly better off than the case where just debt is issued and the S-type shirks.\textsuperscript{19}

Notice that if types were observable, the S-type would not receive financing and so both investment and social welfare would be lower. These results contrast with those of pure adverse-selection models. In Myers and Majluf (1984) adverse selection leads firms to forego positive NPV projects whereas in de Meza and Webb (1987) it encourages firms to undertake negative NPV projects.\textsuperscript{20} Hence, in either case social welfare is lower than under full information about types. The key to this difference is that in the presence of (effort) moral hazard the cross-subsidisation taking place in the pooling equilibrium relaxes this additional constraint and so it can be beneficial. On the contrary, in these two pure adverse selection models there is no channel through which the cross-subsidy can have positive effects but it may have negative ones.

However, as de Meza (2002) argues, assuming that agents are risk averse, hidden types may result in an improvement in social welfare even in the absence of moral hazard. If types are observable, the low-quality agents have lower income and so higher expected marginal utility in all states. Therefore, if adverse selection leads to a pooling equilibrium where the high-quality agents subsidise the low-quality ones, the welfare gains of the subsidisees more than offset the welfare losses of the subsidisers and so aggregate welfare rises.

\subsection*{1.2 Plan of the Paper}

This paper is organised as follows. Next section describes the basic framework and develops the analytical tools. Section 3 provides some general results about the existence and the type of the equilibria where funds are offered. Section 4 analyses the roles of debt and equity under pure adverse selection and adverse selection cum moral hazard. The roles and the usefulness of warrants are explored in Section 5. In Section 6, we show that, in the adverse selection cum moral hazard case, a debt-warrant

\textsuperscript{18} Although our mechanism is similar, it was independently discovered.
\textsuperscript{19} If just debt is issued the S-type does not receive enough subsidy to induce him to exert effort.
\textsuperscript{20} De Meza and Webb (1999, 2000) also demonstrate that hidden types may lead to socially excessive lending.
combination can implement the optimal contrast as a competitive equilibrium. Some brief concluding remarks are provided in Section 7.

2. The Model

We consider a simple static (one-period) model of financing involving both adverse selection and effort moral hazard. There are two dates, 0 and 1, and one homogeneous (perishable) good which can be used either for consumption or investment purposes. There are also two groups of agents: entrepreneurs (henceforth Es) and financiers (henceforth Fs). Both the Es and the Fs consume only at date 1.

Each E has an indivisible project but no initial wealth. All projects require the same fixed initial investment $I$, at date 0. Since the Es have no initial wealth, they need to raise (at least) $I$ from the market.

Each F has a very large amount of initial wealth and can lend at zero interest rate. For simplicity, I assume that there are just two Fs involved in Bertrand competition. Both the Es and the Fs are risk neutral. The Fs are only interested in the monetary returns of the project. The Es, however, care not only about the pecuniary returns but also about a private benefit $B_i$. Also, there are no taxes, no bankruptcy or financial distress costs. Finally, there is no conflict of interest between managers and entrepreneurs. In fact, firms are run by entrepreneurs.

Investment takes place at date 0. Returns are realised at date 1 and are observable and verifiable. There are two states of nature: Success, Failure. If a project succeeds it yields $X_i$. In case of failure, all projects yield 0 regardless of the type of the E.

The probability of success of a project, denoted by $\pi(B_i)$, is related to both the type of the E (project) and the effort level that each E chooses privately. There are two types of Es (projects), R (risky) and S (safe), with respective proportions in the population $\lambda$ and $1 - \lambda$, $0 \leq \lambda \leq 1$. At any given identical effort level, the success probability of the safe project is higher but its return in case of success is lower: $X_R > X_S$ and $\pi^S_j \geq \pi^S_i$. There are two effort levels: Low (shirking), High (working). $B_i$ is a binary variable which denotes the private benefit, in terms of utility, corresponding to each effort level. If the E chooses to shirk, then $B_i = 0$ and $\pi(B_i) = \pi^0_i$, if the high effort level is chosen, then $B_i = 1$ and $\pi(B_i) = \pi^C_i$ where $B > b \geq 0$ and $1 \geq \pi^C_i > \pi^0_i > 0$. The difference $B - b = C$ can be interpreted as the cost of effort.

If the high effort level is chosen, the NPV of both types of projects exceeds the cost of effort. On the other hand, if shirking is chosen, neither project is economically viable (both types of projects have strictly negative NPV). That is,

Assumption 1: $\pi^C_i X_i - I > C > 0 > \pi^0_i X_i - I$, $i = R, S$

Assumption 1 also implies $(\pi^C_i - \pi^0_i)X_i > C$, $(i = R, S)$. That is, the choice of the high effort level by either type leads to an increase in the net social surplus and so is

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21 It is implicitly assumed that the amount of initial wealth held by the Fs is strictly greater than the amount demanded by the Es even if all of them wish to invest.

22 The remaining case where $X_R = X_S$ and $\pi^C_i > \pi^0_i$ (the safe projects dominate the risky ones by first-order stochastic dominance) is not considered explicitly below. See Footnote 10.
socially efficient. Notice also the importance of the first inequality in Assumption 1. If
the project NPV were positive when an E exerted effort but it were less than the cost
of effort, both types would always shirk and so no funds would be offered.

For expository purposes, we begin by restricting the contract space to debt and
outside equity. That is, the Es can borrow by issuing a combination of debt and
equity. Debt claims are zero-coupon bonds that are senior to equity.

A contract \( Z = (\alpha, D) \) provides the E with the required amount of funds, \( I \), in
return for a combination of debt of face value \( D \) and a proportion of equity of the
project \( \alpha \), \( 0 \leq \alpha \leq 1 \), \( D \geq 0 \).

Therefore, given risk neutrality and limited liability, the Es seek to maximise:

\[
U_i(X_i, \alpha_i, D_i, B_i) = \pi(B_i) \max[(1 - \alpha_i)(X_i - D_i), 0] + B_i, \quad i = R, S
\]

where \( U_i \) is the expected utility of an E of type \( i \) when choosing the contract
\( Z_i = (\alpha_i, D_i) \). That is, the expected utility of an E consists of two components: i) the
expected monetary return and ii) the private benefit, represented by the first and
second term respectively in Eq. (1).

At date 0, when the contract is signed, the Es know their own type but the Fs
cannot observe either the type of each individual E or verify the actions (choice of
effort level) of the Es applying for funds. The Fs do, however, know the proportion
of each type in the population of Es and the nature of the investment and moral hazard
technology. The Fs also wish to maximise their expected profit. The expected
profit, \( P_F \), of an F offering a contract \( (\alpha, D) \), given limited liability, is given by:

\[
P_F = \pi(B_i) \{\max[\alpha(X_i - D), 0] + \min(X_i, D)\} - I, \quad i = R, S
\]

2.1 Effort Incentive Constraints

Let us first consider the moral hazard problem an E of type \( i \) faces. A given contract
\( (\alpha, D) \) will induce the high effort level if

\[
(\pi_C^i - \pi_0^i)(1 - \alpha)(X_i - D) \geq C, \quad i = R, S
\]

or \( (1 - \alpha)(X_i - D) \geq c_i, \quad \text{where} \quad c_i = \frac{C}{\pi_C^i - \pi_0^i}, \quad i = R, S \) (3')

The left-hand side of (3) is the increase in the E’s net expected return from exerting
effort and the right-hand is the cost of effort. The contracts \( (\alpha, D) \) satisfying (3) or
(3') are called (effort) incentive compatible. Let \( IC_i \) be the set of effort incentive
compatible contracts and \( ICF_i \) its frontier. The equation of \( ICF_i \) is:

\[
(1 - \alpha)(X_i - D) = c_i, \quad i = R, S
\]

23 Whenever the Max or Min operators are irrelevant they will be suppresed.
The constant $c_i$ tells us how much it costs an E, in utility terms, to increase his success probability by a given amount \((\pi_C - \pi_0^i)\). Notice that this cost depends, in general, on the E’s type and is inversely related to the “productivity” of effort \((\pi_C - \pi_0^i)\). However, the fact that, at any given identical effort level, the safe type’s success probability is higher does not imply that it changes more when another effort level is chosen. It may well be true that the risky type is more “productive” in this sense. Thus, $c_s$ can be greater, equal or less than $c_r$. In combination with $X_i$, $c_i$ describes the moral hazard “technology”. Lemma 1 summarises its key features.

**Lemma 1:** In the \((\alpha,D)\) space:

a) $ICF_i$ are downward sloping and strictly concave with slope
\[
\left(\frac{d\alpha}{dD}\right)_{ICF_i} = -\frac{1 - \alpha}{X_i - D} < 0.
\]
That is, at any \((\alpha,D)\) pair, $ICF_R$ is flatter than $ICF_S$.

b) $ICF_R$ and $ICF_S$ intersect at some \((1 \geq \alpha \geq 0, D \geq 0)\) if $c_r/X_r \geq c_s/X_s$ and $X_r - c_r \geq X_s - c_s$. Otherwise, either $IC_R \subset IC_S$ or $IC_S \subset IC_R$.

c) Neither $IC_R$ nor $IC_S$ is empty.

**Proof:** See Appendix A.

Figure 1 illustrates the case where $ICF_R$ and $ICF_S$ intersect.
2.2 Indifference Curves and Revelation Constraints

The family of indifference curves of type i can be derived from Eq. (1). It should be noted that the shape of the indifference curves is independent of the probability of success.24 As a result, no indifference curve of type i crosses $IFC_i$ and therefore the indifference curves do not exhibit kinks in the $(\alpha, D)$ space. For each type, one of the indifference curves coincides with the corresponding $ICF$.

**Lemma 2**: Let $U_i$ denote the family of indifference curves of type i, and $u_i$ denote a member of this family. In the $(\alpha, D)$ space, for $0 < \alpha < 1$ and $0 < D < X_i$

a) $u_i$ are downward sloping and concave with slope $\left( \frac{d\alpha}{dD} \right)_{u_i} = -\frac{1 - \alpha}{X_i - D} < 0$

b) The indifference curves of R and S cross only once.

**Proof**: See Appendix A.

That is, the marginal rate of substitution of debt for equity of the R-type is greater than that of the S-type. Intuitively, regardless of the assumption about the ordering of the distributions of returns, at any given $(\alpha, D)$ pair, equity is more valuable for the R-type and debt for the S-type (even if, in absolute terms, both debt and equity issued by the S-type are more valuable).25 As a result, the R-type is willing to accept a greater increase in $D$ in exchange for a given reduction in $\alpha$ than the S-type. Technically, the single-crossing condition is satisfied.

**Figure 2**

---

24 This is due to fact that in case of failure the return is zero.

25 Equity is a convex claim and so its value increases, whereas debt is a concave claim and its value falls with the variability of returns, given the expected PV of the project.
Notice also that, due to limited liability, any contract \((\alpha, D)\) above (to the right of) \(u^0_S (u^0_R)\) provides the (R)-S-type with the same level of expected utility as those on \(u^0_S (u^0_R)\). Clearly, the closer to the origin an indifference curve, the higher the expected utility (see Figure 2).

For any given pair of contracts \((Z_R, Z_S)\) the revelation constraints are:

\[
U_R(Z_R) \geq U_R(Z_S) \quad (5)
\]
\[
U_S(Z_S) \geq U_S(Z_R) \quad (6)
\]

where \(U_i, i = R, S\), is given by Eq. (1).

### 2.3 Zero-profit Lines

The expected profit of an F offering a contract \((\alpha, D)\) is given by Eq. (2). It is clear that the expected profit depends crucially on the effort level chosen (through the success probability of the project). Thus, if a zero-profit line crosses the corresponding effort incentive frontier ICF, it will exhibit a discontinuity because the success probability changes discontinuously when the Es change their effort level. However, given limited liability and the assumption that both types of projects have negative NPV when the low effort level is chosen \((\pi^0_i X_i - I < 0)\), the zero-profit lines corresponding to shirking \((\pi^i = \pi^0_i)\) do not exist. Any contract \((\alpha, D)\) financing a shirking E is loss-making and no rational F will offer it. Therefore, zero-profit lines can exist only if the high effort level is chosen (by at least one of the two types of Es).

More specifically, the zero-profit line corresponding to the i-type \((ZP_i)\) exists only if the i-type chooses the high effort level (his effort incentive constraint is satisfied) when he receives funds at fair terms.\(^{26}\) In other words, the existence of a zero-profit line \((ZP_i)\) requires that it belong to the corresponding set of effort incentive compatible contracts \((IC_i)\). Given the investment and moral hazard technology, if both types receive funds at fair terms three different cases may arise: i) the effort incentive constraint is not binding for either type, ii) it is not binding for the one type but is violated for the other, and iii) it is violated for both types. Conditional on the choice of the high effort level there exist three zero-profit lines: that corresponding to the R-type \((ZP_R)\), to the S-type \((ZP_S)\), and the pooling zero-profit line \((PZP_H)\).\(^{27}\)

Lemma 3 summarises the key properties of the zero-profit lines and their relationship with the corresponding indifference curves and effort incentive frontiers. Subsequently, Lemma 4 provides the conditions for the existence of the individual zero-profit lines \(ZP_R, ZP_S\) and \(ZP_S\).

\(^{26}\) By assumption 1, both types of projects have strictly positive NPV when the high effort level is chosen and negative NPV when the Es opt for shirking.

\(^{27}\) There can also exist another pooling zero-profit line corresponding to the case in which one type opts for the high effort level and the other shirks \((PZP_L)\).
Lemma 3: In the \((\alpha, D)\) space,

a) All \(ZP_i, PZP_w\) are downward sloping and strictly concave with slopes:

\[
\left( \frac{d\alpha}{dD} \right)_{ZP_i} = -\frac{1 - \alpha}{X_i - D} < 0
\]

\[
\left( \frac{d\alpha}{dD} \right)_{PZP_{w}} = \frac{(1 - \alpha)[\lambda \pi^{R}_{C} + (1 - \lambda)\pi^{S}_{C}]}{\lambda \pi^{R}_{C} (X_{R} - D) + (1 - \lambda)\pi^{S}_{C} (X_{S} - D)} < 0
\]

where

\[
\left| \left( \frac{d\alpha}{dD} \right)_{ZP_i} \right| \geq \left| \left( \frac{d\alpha}{dD} \right)_{PZP_{w}} \right| \geq \left| \left( \frac{d\alpha}{dD} \right)_{ZP_{w}} \right|
\]

b) \(ICF_i, u_i\), and \(ZP_i\) never cross each other, \(i = R, S\).

Proof: See Appendix A.

Since all three, zero-profit lines, indifference curves, and effort incentive frontiers corresponding to type \(i\) have the same slope, they never cross. One of the indifference curves coincides with the corresponding zero-profit line. However, the location of the zero-profit line relative to the corresponding effort incentive frontier is the key determinant for the existence of the former.

Lemma 4: Suppose both types obtain funds at fair terms, then

a) If \(\pi_{C}^{i} X_{i} - I \geq \pi_{C}^{i} c_{i},\) \(i = R, S\), then both \(ZP_{S}\) and \(ZP_{R}\) exist.

b) If \(\pi_{C}^{R} X_{R} - I \geq \pi_{C}^{R} c_{R}, \pi_{C}^{S} X_{S} - I < \pi_{C}^{S} c_{S}\), then only \(ZP_{R}\) exists.

c) If \(\pi_{C}^{S} X_{S} - I \geq \pi_{C}^{S} c_{S}, \pi_{C}^{R} X_{R} - I < \pi_{C}^{R} c_{R}\), then only \(ZP_{S}\) exists.

d) If \(\pi_{C}^{i} X_{i} - I < \pi_{C}^{i} c_{i},\) \(i = R, S\), then neither \(ZP_{S}\) nor \(ZP_{R}\) exists.

Proof: If \(\pi_{C}^{i} X_{i} - I \geq \pi_{C}^{i} c_{i}\), then the intersection point of \(ZP_{i}\) with the vertical axis, \((1/\pi_{C}^{i} X_{i})\), lies (weakly) below that of \(ICF_{i}, (1-c_i/X_i)\). By Lemma 3, \(ZP_{i}\) and \(ICF_{i}\) never intersect (they may coincide). Therefore, \(ZP_{i}\) belongs to \(IC_{i}\) and hence it exists. Conversely, if \(\pi_{C}^{i} X_{i} - I < \pi_{C}^{i} c_{i}\), then \(ZP_{i}\) lies outside \(IC_{i}\) and so it does not exist. In the latter case, if the \(i\) type obtains funds at fair terms, his effort incentive constraint is violated and so he opts for shirking contradicting the condition \((\pi^{i} = \pi^{i}_{C})\) on which \(ZP_{i}\) is constructed. Q.E.D.

Case (a) corresponds to pure adverse selection. Although, moral hazard is present, because for both types the NPV \((\pi_{C}^{i} X_{i} - I)\) exceeds the “effective” cost of effort \((\pi_{C}^{i} c_{i})\), it has no bite. If either type obtains funds at fair terms, he exerts effort and so the corresponding zero-profit line exists. In Case (b), financing at fair terms implies
that the effort incentive constraint of the R-type is satisfied but that of the S-type is violated ($ZP_R$ belongs to $IC_R$ but $ZP_S$ lies outside $IC_S$). As a result, the R-type exerts effort and so $ZP_R$ exists whereas the S-type opts for shirking and $ZP_S$ does not exist. In the third case the reverse is true ($ZP_S$ belongs to $IC_S$ but $ZP_R$ lies outside $IC_R$). In Case (d), the NPV of the project falls short of the “effective” cost of effort for both types. Thus, both types opt for shirking and so no zero-profit line exists. Figure 3 provides an illustration for Case (b).

2.4 Equilibrium

It is well-known that, in most cases, the equilibrium outcome in competitive markets with asymmetric information depends crucially on the game-theoretic specification of the strategic interaction between the informed and uninformed agents. Yet, no agreement has been reached on which game structure is the most appropriate. It is a difficult task to determine the game specification that fits best the case at hand. Here, I assume that the Fs and the Es play the following three-stage game due to Hellwig (1987):

Stage 1: The two Fs simultaneously offer contracts $(\alpha, D)$. Each F may offer any number of contracts.

Stage 2: Given the offers made by the Fs, the Es apply for (at most) one contract from one F. If an E’s most preferred contract is offered by both Fs, the E chooses each F’s offer with probability $1/2$. In the light of the contract chosen, the E decides whether to work or shirk.

Stage 3: After observing the contracts offered by his rival and those chosen by the Es, each F decides which applications will accept or reject. If an application is rejected, the applicant does not receive funds.

This game structure rationalises a Wilson equilibrium (1977) as a perfect Bayesian equilibrium. Unlike the two-stage screening game, it allows for the existence of a (interior) Nash pooling equilibrium when this pooling equilibrium Pareto-dominates any other equilibrium. That is, this equilibrium concept allows agents to exploit all the
gains from trade and is a necessary condition for the implementation of the optimal contract as a competitive equilibrium in the adverse selection cum moral hazard case.

I only consider pure-strategy perfect Bayesian equilibria. A pair of contracts \( Z_R = (\alpha_R, D_R) \) and \( Z_S = (\alpha_S, D_S) \) is an equilibrium if the following conditions are satisfied:\(^{28}\)

- No contract in the equilibrium pair implies negative (expected) profits for the F. In other words, the Fs’ participation or IR constraints are satisfied:

\[
\pi(B_i)\{\text{Max}[\alpha(X_i - D),0] + \text{Min}(X_i,D)\} \geq I, \quad i = R, S
\] (7a)

- Revelation constraints:

\[
U_R(Z_R) \geq U_R(Z_S)
\] (7b)

\[
U_S(Z_S) \geq U_S(Z_R)
\]

- Effort incentives constraints:

\[
B_i = b \quad \text{if} \quad (1 - \alpha)(X_i - D) \geq c_i
\]

\[
B_i = B \quad \text{if} \quad (1 - \alpha)(X_i - D) < c_i
\] (7c)

\[
B_i = 0 \quad \text{if the project is not undertaken.}
\]

- Profit maximisation: No other set of contracts, if offered alongside the equilibrium pair at Stage 1, would increase an F’s expected profit.

To begin with, because of Bertrand competition, any equilibrium involves zero profits for the Fs. Lemma 5 formalises this argument.

**Lemma 5:** In any equilibrium whether pooling or separating, both Fs must have zero expected profits.

**Proof:** Let \( (\alpha_R, D_R) \) and \( (\alpha_S, D_S) \) be the contracts chosen by the R and S-type respectively (they could be the same contract). Suppose that the two Fs’ aggregate expected profits are \( P_F > 0 \). Then the expected profit of one of the Fs must be no more than \( P_F / 2 \). This F has an incentive to deviate and offer contracts \( (\alpha_R - \varepsilon, D_R) \) and \( (\alpha_S - \varepsilon, D_S) \), or alternatively \( (\alpha_R, D_R - \varepsilon) \) and \( (\alpha_S, D_S - \varepsilon) \), for \( \varepsilon > 0 \). By doing so, he will attract all Es. Since \( \varepsilon \) can be chosen arbitrarily small, this deviation will yield the deviant F an expected profit arbitrarily close to \( P_F \). Thus, if \( P_F > 0 \) (at least) one of the Fs has an incentive to deviate and increase his expected profit. This implies that in any equilibrium it must be true that \( P_F \leq 0 \). However, since Fs have

\(^{28}\) Given limited liability and the strictly positive private benefit, the Es’ participation constraints are always satisfied.
always the option to offer no contracts (or reject all the applications) and make zero
profits, in any equilibrium, they cannot make (expected) losses. Therefore, in any
equilibrium, both Fs make zero expected profits. \textit{Q.E.D.}

3. Types of Equilibria and Provision of Funds: General Results

An important implication of Lemma 5 is that any equilibrium contract must lie on one
of the zero-profit lines. This, in turn, implies the following result:

\textbf{Lemma 6:} A separating equilibrium can exist only if both $ZR$ and $ZS$ exist. If
either $ZR$ or $ZS$ or both do not exist, then no separating equilibrium exists.\(^{29}\)

\textbf{Proof:} First, given limited liability and the strictly positive private benefit, if funds
are offered (whatever the terms they are offered at) both types of Es will always
accept them and undertake their project. Thus, there cannot exist a separating
equilibrium where only one type invests. Suppose now there is a separating
equilibrium in which the R-type chooses contract $(\alpha^R, D^R)$ and the S-type chooses
contract $(\alpha^S, D^S)$. By Lemma 5, the contract chosen by the R-type must lie on the R-
zero-profit line $(ZR)$ and that chosen by the S-type on the S-zero-profit line $(ZS)$.
Therefore, a separating equilibrium can exist only if both zero-profit lines exist. If one
(or both) of the zero-profit lines does not exist, a separating equilibrium cannot exist.
\textit{Q.E.D.}

Lemma 6 implies that in cases where one (or both) of the zero-profit lines does not
exist, if there exists an equilibrium, it must be pooling. Proposition 1 summarises
these results.

\textbf{Proposition 1:} A separating equilibrium can exist only if $\pi^i c_i - I \geq \pi^i c_i$, $i = R, S$.
If $\pi^i c_i - I < \pi^i c_i$, for either $i = R$, or $i = S$, or $i = R, S$, then the resulting
equilibria must be pooling.

The next general result concerns the conditions under which funds are provided.\(^{30}\)

\textbf{Proposition 2:}

a) If $\pi^i c_i X_i - I \geq \pi^i c_i$, $i = R, S$, then both types of projects receive financing.
b) If $\pi^i c_i X_i - I \geq \pi^i c_k$, $\pi^k c_k X_k - I < \pi^k c_k$, $i = R, S$, $k = R, S$, then funds are offered
to both types only if (a part of) either $PZR$ or $PZS$ exists.
c) If $\pi^i c_i X_i - I < \pi^i c_i$, $i = R, S$, there exists a unique pooling equilibrium where no
E obtains funds (no project is undertaken).

\(^{29}\) Given that the Es’ participation constraints are always satisfied, the result in Lemma 6 holds true
regardless of the form of the contracts.

\(^{30}\) Under the proposed game structure, in all cases, there exists a pooling equilibrium where funds are
not offered.
Proof: By Lemma 5, in any equilibrium, funds are offered only along the zero-profit lines. Thus, in any equilibrium, the Fs will offer funds only if (a part of) a zero-profit line exists.

a) By Lemma 4, both $ZP_S$ and $ZP_R$ exist. As a result, $PZP_{H}$ also exists. Hence, regardless of the type of the equilibrium (separating or pooling) funds are offered.

b) By Lemma 6, in this case, only pooling equilibria can exist. However, the existence of pooling equilibria where funds are offered requires that (a part of) a pooling zero-profit line exist. Thus, (a part of) either $PZP_{H}$ or $PZP_{L}$ must exist.\(^{31}\)

c) By Lemma 4, neither $ZP_R$ nor $ZP_S$ exists. As a result, by Lemma 6, no separating equilibrium exists. Moreover, since neither $ZP_R$ nor $ZP_S$ exists, no pooling zero-profit line exists. If an F offers funds to any E, he will make losses. Therefore, no rational F will do so (the Fs’ participation constraints are violated). Q.E.D.

Notice that even in Case (c), if either type had chosen the high effort level he would have enjoyed a strictly positive expected utility (the sum of two positive components: i) the difference between the NPV of the project and the cost of effort, and ii) the private benefit) instead of zero. However, due to moral hazard, the inducement of this choice is not feasible.

4. Types of Equilibria and Methods of Financing: Specific Results

Thus far, no assumption has been made about the ordering of the distributions of returns. However, if an equilibrium exists where funds are provided, then both the type of the equilibrium (pooling or separating) and the method of financing depend, in general, on these assumptions. To proceed further with the analysis, we consider four different assumptions. The risky projects: i) dominate the safe ones by first-order stochastic dominance with respect to returns, ii) are mean-preserving spreads, iii) mean-reducing spreads, and iv) mean-increasing spreads of the safe projects. These distributional assumptions determine the location (intersection) of both the zero-profit lines $ZP_R$ and $ZP_S$ (if they exist) and the effort incentive frontiers $ICF_R$ and $ICF_S$ in the $(\alpha, D)$ space. This, in turn, determines the type of the equilibrium and the method of financing. Lemmas 7 and 8 describe analytically the location of the zero-profit lines and effort incentive frontiers respectively under each assumption.

Lemma 7: If the risky projects

a) dominate the safe projects by first-order stochastic dominance ($\pi^R_j = \pi^S_j$), then $ZP_R$ and $ZP_S$ intersect at $\alpha = 0$. For $\alpha > 0$, $ZP_R$ lies entirely below $ZP_S$.

b) are mean-preserving spreads of the safe ones ($\pi^R_j X_R = \pi^S_j X_S$), then $ZP_R$ and $ZP_S$ intersect at $D = 0$. For $D > 0$, $ZP_S$ lies entirely below $ZP_R$.

\(^{31}\) (A part of) $PZP_{H}$ exists if it belongs to the intersection of $IC_R$ and $IC_S$. Contracts offered along it are effort incentive compatible for both types. Thus, both types choose the high effort level and so it actually exists. If $PZP_{L}$ does not belong to the intersection of $IC_R$ and $IC_S$, it does not exist. In such a case, (at least) one of the two types shirks contradicting the condition on which $PZP_{H}$ is drawn. (A part of) $PZP_{L}$ exists if the following two conditions are satisfied: i) (A part of) it belongs to either $IC_S$ or $IC_R$ and ii) $(1-\lambda)\pi^R_c X_S + \lambda \pi^R_b X_R \geq I$ or $(1-\lambda)\pi^S_c X_S + \lambda \pi^R_c X_R \geq I$.  

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c) are mean-increasing spreads of the safe ones \((\pi_j^R X_R > \pi_j^S X_S)\), then \(ZP_R\) and \(ZP_S\) intersect at some \((\alpha > 0, D > 0)\).

d) are mean-reducing spreads of the safe projects \((\pi_j^R X_R < \pi_j^S X_S)\), then \(ZP_R\) and \(ZP_S\) do not intersect at any \((1 \geq \alpha \geq 0, D \geq 0)\). \(ZP_S\) lies entirely below \(ZP_R\).

**Proof:** See Appendix A.

Intuitively, in Part (a), since both have the same success probability, given its face value, the debt issued by both types is equally valuable. Thus, if both issue only debt, zero profit for Fs requires the issue of the same level of debt. However, if equity is also issued, since the R-type equity is more valuable, an F who just breaks even would ask for a lower proportion of equity if he offered funds to the R-type than to the S-type (given the face value of debt). That is, \(ZP_R\) lies below \(ZP_S\) at any strictly positive level of equity issued. Under mean-preserving spreads, the equity issued by both types is equally valuable but the debt issued by the S-type is more valuable for the financiers. Thus, the S-type would be asked for the same proportion of equity but a lower face value of debt. Under mean-increasing spreads, equity is more valuable if it is issued by the R-type (its expected return is higher) and debt of given face value if it is issued by the S-type. As a result, a lower proportion of equity and a higher face value of debt is demanded by the R-type. Finally, under mean-reducing spreads, since both debt and equity issued by the S-type are more valuable, a lower proportion of equity and face value of debt is demanded by the S-type.

**Lemma 8:** If the risky projects

a) dominate the safe projects by first-order stochastic dominance \((\pi_j^R = \pi_j^S)\), then \(ICF_R\) lies entirely above \(ICF_S\) \((IC_S \subset IC_R)\).

b) are mean-preserving spreads of the safe ones \((\pi_j^R X_R = \pi_j^S X_S)\), then \(ICF_R\) and \(ICF_S\) intersect at \(D = 0\). For \(D > 0\), \(ICF_R\) lies above \(ICF_S\) \((IC_S \subseteq IC_R)\).

c) are mean-increasing \((\pi_j^R X_R > \pi_j^S X_S)\) or mean-reducing \((\pi_j^R X_R < \pi_j^S X_S)\) spreads of the safe ones, then three cases may arise: i) \(ICF_R\) and \(ICF_S\) intersect at some \((\alpha > 0, D > 0)\), ii) \(ICF_S\) lies above \(ICF_R\) \((IC_S \supset ICF_R)\), and iii) \(ICF_R\) lies above \(ICF_S\) \((IC_S \subset IC_R)\).

**Proof:** See Appendix A.

That is, only the first two assumptions about the ordering of the distributions of returns restrict the location (intersection) of the effort incentive frontiers. These restrictions have the following implications:

**Corollary 1:** If the risky projects

a) dominate the safe projects by first-order stochastic dominance \((\pi_j^R = \pi_j^S)\) and the effort incentive constraint for the R-type is violated, then it is also violated for the S-type (but not necessarily vice versa).
b) are mean-preserving spreads of the safe ones \( \pi^R_j X_j = \pi^S_j X_j \) and one of the effort incentive constraints is violated, then the other one is also violated.

**Proof:** a) By Lemma 7, \( ZP_R \) lies (weakly) below \( ZP_S \). Also, by Lemma 8, \( ICF_R \) lies entirely above \( ICF_S \). Therefore, if \( ZP_R \) lies above \( ICF_R \) (the effort incentive constraint for the R-type is violated), then \( ZP_S \) lies necessarily above \( ICF_S \) (the effort incentive constraint for the S-type is also violated).

b) By Lemma 7, \( ZP_R \) and \( ZP_S \) intersect at \( D = 0 \) and for \( D > 0 \), \( ZP_S \) lies entirely below \( ZP_R \). By Lemma 8, \( ICF_R \) and \( ICF_S \) intersect at \( D = 0 \) and for \( D > 0 \), \( ICF_R \) lies above \( ICF_S \). Also, by Lemma 3, \( ZP_i \) and \( ICF_i, i = R, S \), have the same slope. Thus, if \( ZP_R \) \((ZP_S)\) lies above \( ICF_R \) \((ICF_S)\), then \( ZP_S \) \((ZP_R)\) lies also above \( ICF_S \) \((ICF_R)\). That is, if one effort incentive constraint is violated, the other one is also violated. *Q.E.D.*

Now that we have developed the analytical apparatus, we can go on to prove the main results of this paper. Subsection 4.1 examines the pure adverse selection case. In subsection 4.2 we consider the case where both adverse selection and moral hazard play a crucial part in the determination and nature of the equilibrium outcome.

**4.1. The Pure Adverse Selection Case**

We first consider the case where the NPV of the project exceeds the “effective” cost of effort for both types (Case (a) of Lemma 4). In this case, as we have seen, no effort incentive constraint is binding if funds are offered at fair terms and so both zero-profit lines \( ZP_R \) and \( ZP_S \) exist. This, in turn, implies that the pooling zero-profit line \( PZP_H \) also exists. Therefore, both separating and pooling equilibria can exist. Moreover, given that the single-crossing condition is satisfied, a “reasonable” pooling equilibrium where both debt and equity are issued can exist only if, in equilibrium, there is no cross-subsidisation across types.

Under pure adverse selection, debt and equity are only used to convey socially costless information about the type of the project. Hence, in any pooling equilibrium where cross-subsidisation takes place, the subsidiser has an incentive to deviate by issuing more of the less valuable for him security. By doing so, he can credibly signal his type, reduce the cross-subsidisation and increase his expected return (utility). As a result, no pooling equilibrium involving cross-subsidisation can sustain. Notice, however, that the breaking of such a pooling equilibrium is possible only if it involves the issue of either both debt and equity or only the more valuable for the deviant security. If only the less valuable for the subsidiser security is issued, such a deviation is not possible and so the equilibrium cannot be broken (corner solution). In such a case, the subsidy is simply minimised (this is the case under mean-reducing spreads).

Therefore, under pure adverse selection, debt and equity can coexist in a pooling equilibrium only if both securities are fairly priced not only collectively but also individually. This, in turn, can occur only if this pooling equilibrium lies at the intersection of the individual zero-profit lines \( ZP_R \) and \( ZP_S \) (when they intersect at some \( (\alpha > 0, D > 0) \)). That is, only under mean-increasing spreads. More formally,
Proposition 3: If \( \pi^i_c X_i - I \geq \pi^i_c c_i, \ i = R, S \), both types of projects obtain funds but the type of the equilibrium (separating or pooling) and the equilibrium method of financing depend on the ordering of the distributions of returns. In particular,

a) If the risky projects dominate the safe ones by first-order stochastic dominance, there exists a pooling equilibrium where both types issue only debt as well as a continuum of separating equilibria where the risky type issues only debt whereas the safe type issues a combination of debt and equity.

b) If the risky projects are mean-preserving spreads of the safe ones, there exists a pooling equilibrium where both types issue only equity as well as a continuum of separating equilibria where the safe type issues only equity whereas the risky type issues a combination of debt and equity.

c) If the risky projects are mean-increasing spreads of the safe ones, there exists a continuum of separating equilibria (as well as a pooling equilibrium) where both types issue a combination of debt and equity. The risky type issues (weakly) more debt and less equity.

d) If the risky projects are mean-reducing spreads of the safe ones, there exists a unique pooling equilibrium where both types issue only equity.

Proof: By Lemma 4, in this case, \( ICF_i \) lies above \( ZP_i, \ i = R, S \), and so we can proceed with the analysis ignoring the effort incentive constraints. Let \( (A_R, A_S) \) be the equilibrium pair of contracts (in a pooling equilibrium \( A_R = A_S = A \)). We test whether the pair \( (A_R, A_S) \) or \( A \) is an equilibrium by considering deviations.

We begin with the case of mean-increasing spreads (Part (c)). First, we have to show that there cannot exist a pooling equilibrium except that at the intersection of \( ZP_S \) and \( ZP_R \) (point \( A \)). By Lemma 5, if there exists a pooling equilibrium it must lie on the pooling zero-profit line \( (PZP_H) \). Suppose that the pooling equilibrium contract is contract B that lies on \( PZP_H \) to the left of point \( A \) (see Figure 4c). Consider now the following deviation. An F offers a contract just below B in the area between the indifference curve of the two types through B. Given contract B is still offered, the deviant contract will reasonably attract only an R-type and so is profitable (since it lies above \( ZP_R \)). At the same time, contract B becomes loss-making and so any application for it would be rejected at Stage 3. Thus, contract B (any contract on \( PZP_H \) to the left of point \( A \)) cannot be a pooling equilibrium. By a similar argument, any contract along \( PZP_H \) to the right of point \( A \) cannot be a pooling equilibrium. However, it is easy to see that there is no profitable deviation from contract A. Therefore, contract A is a pooling equilibrium.

We also have to show that no contract along \( ZP_R \) to the left of \( A \) and along \( ZP_S \) to the right of \( A \) can be an equilibrium. All these contracts attract both types and so are loss-making for the financiers (since they lie below \( PZP_H \)). Therefore, no rational financier will offer any of them.

Finally, any pair \( (A_R, A_S) \), where \( A_R \) lies on \( ZP_R \) to the right of \( A \) and \( A_S \) lies on \( ZP_S \) to the left of \( A \), is a separating equilibrium. Clearly, all these pairs satisfy the revelation and effort incentive constraints of both types as well as the zero-profit conditions. Furthermore, all separating pairs are equally preferred by both types of Es as well as the Fs and so there is no way to rule any of them out.
In Part (a), clearly, offers below $ZP_R$ are unprofitable. Also, any offer along $ZP_R$ (to the left of point $A$) would attract both types and so is loss-making. Thus, there cannot exist a separating equilibrium where the R-type issues equity. By an argument similar to that used in Part (c) above, in Part (a) there cannot exist a pooling equilibrium where equity is issued. Consider now an F who deviates by offering a contract in the area between $ZP_S$ and $ZP_R$. Given contract $A$ is still offered, at Stage 3, the deviant F will reasonably infer that his contract will be chosen by an S-type. As a result, the deviant contract is unprofitable (loss-making) and so any application for it will be rejected at Stage 3. Actually, anticipating the rejection of this application, no S-type would make it at Stage 2. Therefore, contract $A$ which involves both types issuing only debt is a pooling equilibrium (see Figure 4a). Finally, any pair $(A_R, A_S)$, where $A_R = A$ and $A_S$ lies on $ZP_S$ to the left of $A$, is a separating equilibrium. Because debt issued by both types at $A$ is fairly priced, the S-type is indifferent.
between issuing debt and any debt-equity combination along $ZP_s$. Also, given contract $A$, the financiers are equally well off by offering any contract along $ZP_s$ because, given contract $A$, any such an offer is going to be taken only by the S-type. Thus, none of these separating pairs can be ruled out.

By similar reasoning, we can show that under mean-preserving spreads (Part (b)) there exist a pooling equilibrium where only equity is issued as well as a continuum of separating equilibria where the S-type issues just equity whereas the R-type issues a debt-equity combination along $ZP_R$ (see Figure 4b). Similarly, under mean-reducing spreads (Part (d)) there exists a unique pooling equilibrium that involves both types issuing only equity (see Figure 4d).\textsuperscript{32} \textit{Q.E.D.}

Three remarks should be made here. First, regardless of the distributional assumption, the NPV of all projects (given the high effort level is chosen) exceeds the cost of effort, $C$, and all projects receive financing. That is, investment is at its optimal level. Second, although under most distributional assumptions a pooling equilibrium exists, with the exception of mean-reducing spreads, the securities issued by both types are fairly priced both collectively (because of perfect competition) and individually. That is, there is no cross-subsidisation across types. On the contrary, under mean-reducing spreads because, in absolute terms, both the debt and equity issued by the S-type are more valuable, in the resulting pooling equilibrium the S-type inevitably subsidises the R-type through the mispricing of equity at individual level (the S-type equity is underpriced and the R-type overpriced). However, because debt is relatively more valuable than equity for the S-type, in the resulting all-equity pooling equilibrium the cross-subsidisation is minimised. Third, full separation requires that, in absolute terms, debt issued by the S-type and equity issued by the R-type be weakly more valuable for the financiers.

4.2. The Adverse Selection cum Moral Hazard Case

In this subsection, we examine the case where the S-type NPV falls short of his “effective” cost of effort (Case (b) of Lemma 4). That is, if the S-type is offered funds at fair terms, his effort incentive constraint is violated and so the corresponding zero-profit line does not exist. Thus, only pooling equilibria can exist. Because the choice of the high effort level is socially efficient, here we focus on pooling equilibria where both types exert effort. These equilibria involve cross-subsidisation across types and Pareto-dominate any other equilibrium. Through the mispricing of equity at individual level, the S-type receives the subsidy necessary to induce him to work.

That is, in the presence of both adverse selection and (effort) moral hazard, in addition to conveying information, debt and equity play a second role. That of incentivising the more prone to shirking type. This double role stems from the interaction between adverse selection and moral hazard and provides an explanation for the issue of combinations of debt and equity even if the issue of equity implies an adverse selection cost. What is more, in contrast with the pure adverse selection case, the cross-subsidisation is socially beneficial. It converts a negative into a positive NPV project and improves social welfare.

To illustrate this point, we consider the case where, at any identical effort level, the risky projects dominate the safe ones by first-order stochastic dominance. The remaining cases are analysed in Appendix B.

\textsuperscript{32} These equilibria also obtain in a two-stage signalling or screening game.
**Proposition 4:** Suppose the risky projects dominate the safe ones by first-order stochastic dominance ($X_R > X_S$, $\pi^R_j = \pi^S_j = \pi_j$, $j = C, 0$) and $\pi^C_R X_R - I > \pi^C_S c_R$, $\pi^C_S X_S - I < \pi^C_S c_S$, $I/\pi^C_R X_R < 1 - c_S/X_S$. Then if $\lambda > \lambda_1$, there exists a unique pooling (funding) equilibrium where both types choose the socially efficient high effort level and obtain funds by issuing a combination of debt and equity.

where $\lambda_1 \equiv \frac{I - \pi^S_c (X_S - c_S)}{(\pi^C_R X_R - \pi^S_c X_S)(1 - c_S/X_S)}$

The equilibrium contract, $A = (\alpha^*, D^*)$, lies at the intersection of $ICF_S$ and $PZP_H$ with $\alpha^*$ and $D^*$ given by:

$$\alpha^* = \frac{I - \left[\lambda \pi^R_C + (1 - \lambda) \pi^S_C\right] X_S - c_S}{\lambda \pi^C_R (X_R - X_S)}$$

(8)

$$D^* = X_S - c_S / (1 - \alpha^*)$$

(9)

**Proof:** We test whether the contract at $A$ is an equilibrium by considering deviations. Offers below $ZP_R$ are clearly loss-making. Any offer in the area between $u^R_A$ (the R-type indifference curve through the equilibrium contract) and $ZP_R$ to the left of $ICF_S$ is going to be taken by both types and so is unprofitable. Thus, we only need to consider the following two deviations: i) Suppose that an F deviates by offering a contract, say $A'$, in the area between $u^R_A$ and $ZP_R$ to the right of $ICF_S$. Given that contract $A$ is still offered, the deviant contract, contract $A'$, will reasonably attract only the R-type. This, in turn, implies that contract $A$ is taken only

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33 In Appendix B, we also provide mathematical proofs for our results.
by the S-type and so it becomes loss-making. As a result, at Stage 3, any application for that contract will be rejected. Anticipating that, the S-type will also choose $A'$, at Stage 2, and hence $A'$ becomes also loss-making (since to the right of $ICF_S$, $PZP_H$ does not exist, and $PZP_L$ lies to the right of $u^4_R$). Therefore, there is no profitable deviation to the right of A. ii) Consider now an F who deviates by offering a contract, say $A''$, in the area between $ICF_S$ and $u^4_R$ to the left of (above) A. Given contract A is still offered, contract $A''$ will reasonably attract only the S-type and so is loss-making. Thus, any application for contract $A''$ will be rejected at stage 3. Actually, anticipating the rejection of that application at Stage 3, no S-type would make it at Stage 2. Therefore, the pooling equilibrium at A is the unique equilibrium where funds are provided.\(^\text{34}\) Q.E.D.

The pooling equilibrium at A reflects a trade-off between information revelation and effort incentives. The securities issued by the R- and S-type are priced as a pool. Although, because of perfect competition, debt and equity are fairly priced collectively, at individual level they are mispriced. Not surprisingly, it is precisely this mispricing that provides the more prone to shirking type with the subsidy necessary to induce him to exert effort. More specifically, in the pooling equilibrium of Proposition 4 the R-type subsidises the S-type through the mispricing of the more valuable for him security (equity). Hence, given that the single-crossing condition is satisfied, the R-type has an incentive to deviate by choosing a contract involving more debt and less equity than the equilibrium contract. By doing so, he can credibly signal his type, reduce the cross-subsidisation and increase his expected return.

However, his attempt will be fruitless. If the R-type chooses such a contract, the equilibrium contract becomes loss-making for the financiers and so any application for that will be rejected. As a result, the S-type will always mimic the R-type preventing him from revealing his type and obtaining funds in better terms. What is more, the deviant contract gives less subsidy to the S-type and so destroys his effort incentives. The S-type shirks and the collective expected return falls significantly. A financier who offers a contract involving less equity than the equilibrium contract can break even only if he asks for a considerably greater face value of debt (higher interest rate on debt). But neither type prefers such a deviant contract to the equilibrium contract. Hence, no financier has an incentive to offer a contract involving less equity than the equilibrium contract and so the R-type stays in equilibrium and provides the S-type with just enough subsidy in order to induce him to work.

Loosely speaking, the R-type accepts to issue some equity and induce the S-type to exert effort because the increase in his net expected return (due to the lower interest rate he pays on debt) more than offsets the cost of the incremental subsidy (the adverse selection cost of issuing equity). That is, the R-type is better off in the pooling equilibrium of Proposition 4 where both debt and equity are issued and both types exert effort than in a pooling equilibrium where only debt is issued and so the S-type shirks.\(^\text{35}\)

\(^{34}\) It should be noted that uniqueness follows from the application of the “intuitive criterion”. All contracts along (the relevant part of) $PZP^H$ correspond to pooling perfect Bayesian equilibria under arbitrary out-of-equilibrium beliefs. However, contract A is the only one that survives the “intuitive criterion”\(^^\text{(Cho and Kreps, 1987).}\)

\(^{35}\) Notice that in the pooling equilibrium of Proposition 4 the R-type is worse off compared to the case where types are observable and he obtains funds at fair terms. However, social welfare exceeds that under full information about types (see also the discussion in Subsection 1.4.2.2 below).
On the other hand, the role of debt and equity as communication devices implies that no financier can make a profit by offering a contract involving more equity (subsidy) and less debt than the equilibrium contract. Given that the equilibrium contract is still offered, the deviant contract will not be taken by any E at Stage 2. If an E chooses this contract, the financier will infer that he is an S-type. As a result, the deviant contract is loss-making and any application for that will be rejected at Stage 3. Anticipating that, no E will apply for it at Stage 2.\textsuperscript{36}

That is, the existence of the socially efficient pooling equilibrium relies on two factors: i) the endogenous (discrete)\textsuperscript{37} choice of the effort level and ii) the three-stage game structure that allows for an (interior) pooling perfect-Bayesian equilibrium even if cross-subsidisation across types takes place and the single-crossing condition is met.\textsuperscript{38} Due to the presence of the third stage agents behave less myopically than in a two-stage screening game and so the non-existence problem is resolved.

If it exists, the pooling equilibrium of Proposition 4 has two interesting implications: First, it provides an explanation for the issue of combinations of debt and equity even if issue of equity implies an adverse selection cost. Firms issue some equity even if under pure adverse selection they would have issued just debt. Second, in contrast with the pure adverse selection case, the cross-subsidisation is socially beneficial. It converts a negative into a positive NPV project and improves social welfare.

4.2.1 Implications for the Issue of Securities

To fix ideas, let us compare the adverse selection cum moral hazard case with the pure adverse selection and pure moral hazard cases. As we have already seen, under pure adverse selection, the securities issued are only used to convey socially costless information about the type of the project. Therefore, firms issue combinations of debt and equity only if both securities are fairly priced not only collectively but also individually. Pooling equilibria involving cross-subsidisation can exist only if the less valuable for the subsidiser security is issued (corner solution). In this case, there is no channel through which the cross-subsidy can have positive effects for the subsidiser. As a result, the subsidiser maximises his return by minimising the subsidy he provides the other type.

On the other hand, in the presence of effort moral hazard, if the subsidiser cannot reveal his type, it may be in his interest to incur the adverse selection cost of issuing some of the more valuable for him security. By doing so, he provides the more prone to shirking type with the subsidy necessary to induce him to work and so the

\textsuperscript{36} A similar argument applies if the subsidiser is the S-type (see Appendix A for a graphical illustration). The only difference is that the cross-subsidisation now takes place through the mispricing of the more valuable for the S-type security (debt). That is, in this latter case, the equilibrium contract involves more debt and less equity than the S-type would wish. However, $D^*$ is the minimum level of debt issued consistent with the R-type exerting effort.

\textsuperscript{37} I conjecture that, under certain restrictions on the probability and cost functions, this pooling equilibrium exists even if the effort level is a continuous variable.

\textsuperscript{38} In a two-stage signalling game, such a pooling equilibrium cannot exist. Behaving myopically, the R-type tries to reveal his type by issuing more debt and less equity. However, the S-type always mimicks and, more importantly, his effort incentives are destroyed. Therefore, there can exist either pooling equilibria where only debt is issued (corner solution) and the R-type works whereas the S-type shirks or pooling equilibria where both types shirk and so no funds are provided. In either case, the resulting pooling equilibria are Pareto-inferior to that of Proposition 4.
collective expected return rises. If the resulting increase in his expected return exceeds this adverse selection cost, the subsidiser’s welfare improves. For example, in Proposition 4 the benefit (due to the lower interest rate he pays on debt) for the R-type from accepting to issue some equity and inducing the S-type to exert effort exceeds the adverse selection cost associated with the equity issue.

In the pure moral hazard case, the Fs observe the type of each individual E. As a result, each type is offered contracts along the corresponding zero-profit line, provided it exists. In the context of our simple model, the mode of financing is irrelevant. All combinations of debt and equity along the existing zero-profit line are offered and are equally preferred by the corresponding type.

4.2.2 Implications for Investment and Social Welfare

Under the conditions in Proposition 4, if types were observable only the R-type would receive financing. If the S-type receives funds at fair terms he shirks and so his project NPV is negative. Moreover, financiers have no incentive to transfer resources from the R-type to the S-type to induce the latter to exert effort. Thus, no rational financier will be willing to offer him the required for the investment funds and so the S-type project is not undertaken. That is, under full information about types a potentially positive NPV investment opportunity is forgone. Furthermore, because when the S-type works his project NPV exceeds the cost of effort, the social welfare also worsens.

These results are in sharp contrast with the pure adverse selection case. In Myers and Majluf (1984) adverse selection leads firms to forgo positive NPV projects whereas in de Meza and Webb (1987) it encourages firms to undertake negative NPV projects. Hence, in either case social welfare is lower than under full information about types. The key to this difference is that in the presence of (effort) moral hazard the cross-subsidisation taking place in a pooling equilibrium relaxes this additional constraint and so it can be beneficial. On the contrary, given risk neutrality, under pure adverse selection there is no channel through which the cross-subsidy can have positive effects but it may have negative consequences.

5. The Role of Warrants

So far, the available financial instruments have been debt and equity. The discussion of the previous section illustrated the roles of these two financial contracts as separation devices and means of incentivising the more prone to shirking type. In this

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39 Notice that, although the pooling equilibrium of Proposition 4 involves cross-subsidisation across types of Es, it does not involve cross-subsidisation across debt and equity. Once the equilibrium is determined, the value of these two contracts can be calculated independently and so debt and equity could be traded separately in a secondary market. In fact, the same equilibrium obtains even if instead of one F offering both debt and equity, the Fs specialise in one of the two contracts and debt and equity markets are perfectly competitive (see Appendix C for a proof).

40 This result is due to the assumption that in case of failure the project yields zero regardless of its type. If instead we assume that in case of failure the return is strictly positive then debt becomes the optimal contract (Innes (1990)). All the main results go through under the latter assumption. However, the zero-return assumption simplifies considerably the analysis without losing any insight.

41 Notice that the pooling equilibrium of Proposition 4 may exist even if the NPV of the S-type project is negative regardless of the effort level. Obviously, in such a case, adverse selection results in overinvestment and a fall in social welfare. Moreover, the pooling equilibrium can exist if there exist more than two types. In this case, it is possible that adverse selection leads to overinvestment but an improvement in social welfare.
section, we introduce financing instruments with option features. More specifically, the Es can borrow the required amount I by issuing a debt-warrant combination.

The warrant gives its holder the right to purchase a prespecified proportion of the firm’s equity, $\eta$, at an agreed price K (exercise price). The proceeds from the exercise of the option, $K_i$, are distributed as dividends to the shareholders. Therefore, a warrant holder will exercise if

$$\eta_i(X_i - D_i + K_i) \geq K_i, \quad K_i \geq 0, \quad i = R, S$$

(10)

This can be rewritten as

$$K_i \leq \frac{\eta_i}{1-\eta_i}(X_i - D_i), \quad i = R, S$$

(10’)

So, given risk neutrality and limited liability, the Es seek to maximise:

$$U_i(X_i, \eta_i, D_i, B_i, K_i) = \pi(B_i)\min \{ (1-\eta_i)(X_i - D_i + K_i), \max \{ (X_i - D_i), 0 \} \} + B_i$$

(11)

where $U_i$ is the expected utility of an E of type i when choosing the contract $\Xi_i = (\eta_i, D_i, K_i)$. Similarly, given limited liability, the expected profit of an F offering the contract $\Xi_i = (\eta_i, D_i, K_i)$ is given by:

$$P_F = \pi(B_i)\max \{ \eta_i(X_i - D_i + K_i) - K_i + D_i, \min(X_i, D_i) \} - I$$

(12)

To make the analysis interesting, we assume that the exercise price is set such that, in case of success, the option is exercised regardless of the type of the project. That is, the exercise price is given by:

$$K_i = \frac{\eta_i}{1-\eta_i}(X_S - D_i) \quad \text{where } \psi_i \in [0,1], \quad i = R, S$$

(13)

Without loss of generality, below we assume $\psi_R = \psi_S = \psi \in [0,1]$.\footnote{A combination of debt and equity is a special case of a debt-warrant combination that obtains for $\psi_R = \psi_S = 1$.} Basically, this assumption reduces the choice variables (signals) from three $(\eta, D, K)$ to two $(\eta, D)$. The choice of $\eta$ and $D$ completely determines $K$. By doing so, we considerably simplify the analysis without losing any insight.\footnote{No more than two choice variables are necessary for our purposes. Clearly, all the results go through if we increase their number to three by allowing for $\psi_R \neq \psi_S$.}

Using (13) and the assumption about $\psi$, the utility and profit functions simplify respectively to:

$$U_i(X_i, \eta_i, D_i, B_i, K_i) = \pi(B_i)[(1-\eta_i)(X_i - D_i) + \eta_i(1-\psi)(X_S - D_i)] + B_i$$

(14)

$$P_F = \pi(B_i)[\eta_i((X_i - D_i) - (1-\psi)(X_S - D_i)) + D_i] - I$$

(15)
5.1 Effort Incentive Constraints, Indifference Curves and Revelation Constraints

A given contract \((\eta, D)\) will induce the high effort level if

\[
(1 - \eta)(X_i - D + K) \geq c_i
\]  

(16)

So, the equations of the effort incentive frontiers \(ICF_S\) and \(ICF_R\) are given respectively by:

\[
(1 - \psi \eta)(X_S - D) = c_S
\]  

(17)

\[
(1 - \psi \eta)(X_R - D) - \eta(1 - \psi)(X_R - X_S) = c_R
\]  

(18)

The family of indifference curves of type \(i\) can be derived from (14). The indifference curves have the same slope as the corresponding effort incentive frontiers. As a result, no indifference curve of type \(i\) crosses \(ICF_i\) and therefore the indifference curves do not exhibit kinks in the \((\eta, D)\) space. For each type, one of the indifference curves coincides with the corresponding \(ICF\).

Finally, for any given pair of contracts \(\Xi_R = (\eta_R, D_R)\) and \(\Xi_S = (\eta_S, D_S)\) the revelation constraints are:

\[
U_R(\Xi_R) \geq U_R(\Xi_S)
\]  

(19)

\[
U_S(\Xi_S) \geq U_S(\Xi_R)
\]  

(20)

where \(U_i, i = R, S\) is given by Eq. (14).

Lemma 9: In the \((\eta, D)\) space:

a) \(ICF_R\) and \(ICF_S\) are downward sloping and strictly concave with slopes:

\[
\frac{d\eta}{dD}_{ICF_R} = - \frac{1 - \psi \eta}{\psi(X_S - D) + (X_R - X_S)} < 0
\]

\[
\frac{d\eta}{dD}_{ICF_S} = - \frac{1 - \psi \eta}{\psi(X_S - D)} < 0
\]

That is, at any \((\eta, D)\) pair, \(ICF_R\) is flatter than \(ICF_S\).

b) If \(c_R \geq c_S\) and \(X_R - c_R \geq X_S - c_S\), then for any \(\psi \in [0, \min[\overline{\psi}, 1]]\), \(ICF_R\) and \(ICF_S\) intersect at some \((1 \geq \eta \geq 0, D \geq 0)\). Otherwise, either \(IC_R \subset IC_S\) or \(IC_S \subset IC_R\).

where \(\overline{\psi} = \frac{(X_R - X_S)(1 - c_S/X_S)}{(X_R - c_R) - (X_S - c_s)}\)
c) The indifference curves of the R- and S-type have the same slope as the corresponding effort incentive frontiers.

**Proof:** See Appendix A.

Since $X_R > X_S$, one of the conditions ($c_R \geq c_S$) for the intersection of $ICF_R$ and $ICF_S$ to occur at some admissible value of the two choice variables ($\eta$ and $D$) in this case, is weaker than under a combination of debt and equity ($c_R/X_R \geq c_S/X_S$).

Intuitively, since $X_R > X_S$, at any given $(\eta, D)$ pair, a given fall in $\psi$ (increase in the exercise price) implies that the project’s return constitutes a smaller proportion of the total payment to the warrantholder if the warrant is issued by the S-type. That is, as the exercise price rises, the warrant value falls faster for the S-type and so the S-type is willing to increase faster the proportion of equity, $\eta$, offered to the financier than the R-type while still exerting effort. As a result, for $\psi$ sufficiently low ($\psi \leq \psi^*$), $ICF_S$ and $ICF_R$ intersect at some positive face value of debt, $D$, even if this is not possible when we restrict ourselves to debt and equity.

As far as the indifference curves are concerned, in the $(\eta, D)$ space, that of the R-type is flatter. Intuitively, regardless of the assumption about the distribution of returns, at any given $(\eta, D)$ pair, the warrant is more valuable for the R-type and debt for the S-type (even if, in absolute terms, both debt and the warrant issued by the S-type are more valuable). As a result, the R-type is willing to accept a greater increase in $D$ in exchange for a given reduction in $\eta$ than the S-type. That is, the single-crossing condition is satisfied.

### 5.2 Zero-profit Lines

The expected profit of an F offering a contract $(\eta, D)$ is given by (15). Given Assumption 1, the zero-profit line corresponding to the i-type $(ZP_i)$ exists only if the i-type exerts effort when he receives funds at fair terms. In other words, the existence of a zero-profit line $(ZP_i)$ requires that it belong to the corresponding set of effort incentive compatible contracts $(IC_i)$. Conditional on the choice of the high effort level there exist three zero-profit lines: that corresponding to the R-type $(ZP_R)$, to the S-type $(ZP_S)$, and the pooling zero-profit line $(PZP_H)$. The equations of the zero-profit lines $ZP_S$ and $ZP_R$ are respectively:

\[
\pi^S_c [\eta \psi (X_S - D) + D] = I \tag{21}
\]

\[
\pi^R_c \{\eta [\psi (X_S - D) + (X_R - X_S)] + D\} = I \tag{22}
\]

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44 Diagrammatically, in the $(\eta, D)$ space, as $\psi$ falls $ICF_S$ becomes steeper faster than $ICF_R$.

45 By Assumption 1, the NPV of both types of projects is strictly positive if the high effort level is chosen whereas it is strictly negative if shirking is chosen.
Lemma 10 summarises the key properties of the zero-profit lines and their relationship with the corresponding indifference curves and effort incentive frontiers.

**Lemma 10:** In the \((\eta, D)\) space,

a) All \(ZP_R^1, ZP_S^1, PZP^1\) are downward sloping and strictly concave with slopes:

\[
\left( \frac{d\eta}{dD} \right)_{ZP^1_R} = -\frac{1 - \psi\eta}{\psi(X^S - D) + (X^R - X^S)} < 0
\]

\[
\left( \frac{d\eta}{dD} \right)_{ZP^1_S} = -\frac{1 - \psi\eta}{\psi(X^S - D)} < 0
\]

\[
\left( \frac{d\eta}{dD} \right)_{PZP^1} = -\frac{(1 - \psi\eta)(\lambda_\pi^R + (1 - \lambda_\pi^S))}{\lambda_\pi^R((X^R - X^S) + \psi(X^S - D)) + (1 - \lambda_\pi^S\psi)(X^S - D)} < 0
\]

where

\[
\left| \frac{d\alpha}{dD} \right|_{ZP^1_R} \geq \left| \frac{d\alpha}{dD} \right|_{ZP^1_S} \geq \left| \frac{d\alpha}{dD} \right|_{PZP^1}
\]

b) If \(X^R - 1/\pi^R_c \geq X^S - 1/\pi^S_c\), then for any \(\psi \in [0, \min[\hat{\psi}, 1]]\) \(ZP^1_R\) and \(ZP^1_S\) intersect at some \((0 \leq \eta \leq 1, D \geq 0)\).

where \(\hat{\psi} = \frac{\pi^R_c(X^R - X^S)}{(\pi^S_c - \pi^R_c)X^S}\)

c) \(ICF, u_i, \) and \(ZP_i\) never cross each other, \(i = R, S\).

**Proof:** See Appendix A.

That is, the intersection of the zero-profit lines \(ZP^1_R\) and \(ZP^1_S\) can occur at some admissible value of the choice variables \((\eta, D)\) even under mean-reducing spreads. Recall that this is not possible if we restrict the contract space to debt and equity (see Lemma 7). Intuitively, since \(X^R > X^S\), at any given \((\eta, D)\) pair, a given fall in \(\psi\) (increase in the exercise price) implies that the project’s return constitutes a smaller proportion of the total payment to the warrantholder if the warrant is issued by the S-type. That is, as the exercise price rises, the net payoff of a financier offering funds to the S-type falls faster. As a result, the increase in the proportion of equity, \(\eta\), required in order for the financier to just break even is greater if the warrant is issued by the S-type.\(^{46}\) For \(\psi\) sufficiently low \((\psi \leq \hat{\psi})\), \(ZP^1_S\) and \(ZP^1_R\) intersect at some positive face value of debt, \(D\), even if this is not possible when we restrict ourselves to debt and equity. In other words, for a sufficiently high exercise price the warrant

\(^{46}\) Diagrammatically, in the \((\eta, D)\) space, as \(\psi\) falls \(ZP_S^1\) becomes steeper faster than \(ZP_R^1\).
issued by the R-type becomes more valuable than that of the S-type even if the S-type equity is more valuable.

In general, the intersection of the effort incentive frontiers and the zero-profit lines of the two types at some admissible value for both choice variables requires weaker conditions under debt coupled with a warrant than under a combination of debt and equity. This has important implications for the fair pricing of the securities issued under pure adverse selection and the restrictions on the parameter values required for the existence of the socially efficient pooling equilibrium under adverse selection and (effort) moral hazard. Below we consider each case separately.

5.3 The Pure Adverse Selection Case

In this case, as we have seen, no effort incentive constraint is binding and so all three zero-profit lines \( R_{ZP}, S_{ZP} \) and \( HP_{ZP} \) exist.\(^{47}\) Therefore, both separating and pooling equilibria can exist. Moreover, given that the single-crossing condition is satisfied and, by appropriately choosing \( \psi \), \( Z_{S} \) and \( Z_{R} \) can intersect at some admissible value of the choice variables, a “reasonable” pooling equilibrium can exist only if it does not involve cross-subsidisation across types.

Under pure adverse selection, the face value of debt and the proportion of equity (or exercise price) jointly serve as signals conveying socially costless information about the type of the project. Hence, in any pooling equilibrium where cross-subsidisation takes place, the subsidiser has an incentive to deviate by issuing more of the less valuable for him security. By doing so, he can credibly signal his type, reduce the cross-subsidisation and increase his expected return. As a result, no pooling equilibrium involving cross-subsidisation can sustain. That is, in any equilibrium (pooling or separating) the securities issued are fairly priced not only collectively but also individually. Moreover, by choosing \( \psi < \min(\tilde{\psi}, 1) \), we can achieve full separation under mean-preserving, mean-increasing and mean-reducing spreads. However, if the risky project dominates the safe one by first-order stochastic dominance, regardless of the value of \( \psi \), there exists a pooling equilibrium where both types issue only debt as well as a continuum of separating equilibria where the R-type issues only debt whereas the S-type issues a debt-warrant combination. More formally,

**Proposition 5:** If \( \pi_i X_i - I \geq \pi_i c_i \), \( i = R, S \) and \( X_R - I/\pi_C > X_S - I/\pi_C \), then for any \( \psi \in [0, \min(\tilde{\psi}, 1)] \) there always exist equilibria (pooling or separating) where both types of projects obtain funds and the securities issued are fairly priced regardless of the distributional assumption. In particular,

a) If the risky projects dominate the safe projects by first-order stochastic dominance, there exists a pooling equilibrium where both types issue only debt as well as a continuum of separating equilibria where the R-type issues only debt whereas the S-type issues a debt-warrant combination.

b) If the risky projects are mean-preserving, mean-increasing, or mean-reducing spreads of the safe ones, there exists a continuum of separating equilibria (as well

\(^{47}\) It can be easily shown that the results of Lemmas 4, 5, and 6 and Propositions 1 and 2 hold true regardless of the form of the contract.
as a pooling equilibrium) where both types issue a debt-warrant combination with \( \eta_R \leq \eta_S \) and \( D_R \geq D_S \).

**Proof:** Similar to Proposition 3 (see Figure 6).

Intuitively, full revelation requires that debt be more valuable for the S-type and the warrant for the R-type not only in relative but also in absolute terms. If these two conditions are met the S-type can credibly reveal his type by choosing a contract involving low face value of debt and a warrant with very high exercise price (a high proportion of equity is offered to the financier). The R-type has no incentive to mimic because the cost from the underpricing of such a warrant exceeds the gains from issuing a little overpriced debt. The first condition is satisfied under all four distributional assumptions. By appropriately choosing the warrant exercise price, the second condition can also be satisfied even under mean-reducing spreads.

If the risky projects dominate the safe ones by first-order stochastic dominance, debt of given face value issued by both types is equally valuable but the warrant issued by the R-type is more valuable. Hence, in order to avoid subsidising the S-type, in any equilibrium, the R-type issues only debt and debt issued is fairly priced. As a result, the S-type is indifferent between issuing just debt (pooling equilibrium) and any debt-warrant combination along \( ZP_S \) (separating equilibria). Therefore, there can exist a pooling equilibrium where only debt is issued as well as a continuum of separating equilibria where the R-type issues just debt whereas the S-type issues a debt-warrant combination along \( ZP_S \).

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\[ \text{Figure 6} \]

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48 The first (single-crossing) condition allows agents to send a credible signal while the second rules out pooling equilibria where only the less valuable for the subsidiser security is issued (corner solutions).
In summary, under pure adverse selection, a debt-warrant combination allows us to obtain equilibria (pooling or separating) where the securities issued are fairly priced even if it is not possible when we restrict ourselves to debt, equity and/or convertible debt. This result provides a rationale for the use of warrants.

5.4 The Adverse Selection cum Moral Hazard Case

In this subsection, we show that a debt-warrant combination allows for the existence of the socially efficient pooling equilibrium under weaker restrictions on parameter values than a debt-equity combination. For expositional simplicity, we only consider the case where, at any given identical effort level, the risky project dominates the safe one by first-order stochastic dominance (see Appendix B for a generalisation of Proposition 6). Also, if both types are offered funds at fair terms, the S-type shirks whereas the R-type exerts effort.

Proposition 6: Suppose the risky projects dominate the safe ones by first-order stochastic dominance \( X_R > X_S \), \( \pi_j^X = \pi_j^S = \pi_j \), \( j = C, 0 \) and \( \pi_c^R X_R - I > \pi_c^S c_R \), \( \pi_c^S X_S - I < \pi_c^S c_S \). Then for any \( \psi \leq \bar{\psi} \) and \( \lambda \geq \bar{\lambda} \), then there exists a unique pooling (funding) equilibrium where both types exert effort and obtain funds by issuing a debt-warrant combination (see Figure 7.b).

The equilibrium contract, \( A = (\eta^*, D^*) \), lies at the intersection of \( ICF_S \) and \( PZP_H \) with \( \eta^* \) and \( D^* \) given by:

\[
\eta^* = \frac{I - \lambda \pi_c^R (X_S - c_S)}{\lambda \pi_c^R (X_R - X_S)} (23)
\]

\[
D^* = X_S - c_S / (1 - \psi \eta^*) (24)
\]

Proof: Similar to Proposition 4 (see Figure 7.b).

To illustrate the role of warrants, we graphically compare the case where the firms can issue a debt-equity combination with the case they issue a debt-warrant combination (see Figures 7.a and 7.b). By Lemma 8, if firms can only issue debt and equity, under this distributional assumption, \( ICF_S \) lies entirely below \( ICF_R \). Also, because the R-type equity is more valuable, as his proportion in the population of entrepreneurs, \( \lambda \), decreases the pooling zero-profit line \( PZP_H \) becomes steeper and intersects \( ICF_S \) at points corresponding to a higher proportion of equity. A necessary condition for the

\[49\] Recall that if we restrict ourselves to debt and equity under mean-reducing spreads there exists a unique pooling equilibrium where only equity is issued and the S-type subsidises the R-type through the mispricing of equity at individual level.
existence of the efficient pooling equilibrium is that $PZP_H$ both intersects $ICF_S$ and lies below $ICF_R$ ($PZP_H$ is constructed conditional on both types exerting effort). If $\lambda$ falls below $\lambda_1$, $PZP_H$ lies entirely above $ICF_S$ and so it is not relevant (see Figure 7.a). As a result, the socially efficient pooling equilibrium collapses although the R-type would exert effort even if a higher proportion of equity was issued.

Because the warrant value falls with the exercise price faster for the S-type, as the warrant exercise price rises both $ICF_S$ and $PZP_H$ become steeper but $ICF_S$ becomes so at a higher rate. As a result, for a sufficiently high exercise price, $ICF_S$ and $PZP_H$ meet again and the existence of the socially efficient pooling equilibrium is restored (see Figure 7.b). That is, a debt-warrant combination allows for the existence of the efficient pooling equilibrium even if it collapses when firms can issue only debt and equity.

Intuitively, in this case, if firms can only issue debt and equity, at any given debt level, the proportion of equity issued consistent with exerting effort is strictly lower for the S-type. That is, the pooling equilibrium where both types exert effort may collapse although the R-type would have exerted effort even if a higher proportion of equity was issued (more subsidy was given to the S-type). Because the warrant value falls with the exercise price (proportionately) faster for the S-type, the S-type is willing to increase faster the proportion of equity offered to the financier than the R-type while still exerting effort. Thus, because in absolute terms the warrant issued by the S-type is less valuable than his equity, the warrant payoff function between $X_R$ and $X_S$ can be steeper than the equity payoff function without violating the S-type effort incentive constraint. This implies that the difference between the value of the warrants issued by the R- and S-type exceeds the corresponding difference of equity values consistent with both types working. This larger difference allows for the provision of the subsidy necessary to induce the S-type to work when the proportion of the R-type is so low that the socially efficient pooling equilibrium breaks if a debt-equity combination is used.
The mechanism at work here relies on the fact that the warrant exercise price can be chosen independently of (and be greater than) the face value of debt. By choosing a sufficiently high exercise price, we can create a sufficiently convex claim which allows us to exploit the difference between the returns of the two types of projects and satisfy the S-type effort incentive constraint under weaker conditions than if warrants were not available. In other words, through the appropriate choice of their exercise price, warrants allow for the implementation of the socially efficient outcome even if this is not possible when we restrict ourselves to debt, equity and/or convertible debt. If the roles of the two types reverse and so the required cross-subsidisation takes place through the mispricing of debt, a debt-warrant combination does not improve on a debt-equity combination.

6. Optimal Financial Contracts under Adverse Selection and Moral Hazard

If both the type and the actions of the entrepreneurs were observable (and verifiable), both types would exert effort if they were offered funds at fair terms. As a result, the net social surplus (social welfare) would be maximised (first best). However, if the choice of the effort level is not observable and one of the two types (the S-type) shirks if he receives funds at fair terms, the implementation of the socially efficient outcome requires cross-subsidisation across types. In this section, we address the following question: Can competitive financial markets implement the socially efficient outcome under the same conditions as a benevolent central authority (social planner) who aims at maximising social welfare?

Competitive financiers have no incentive to transfer resources from the R-type to the S-type to induce the latter to exert effort. Therefore, if types are observable or can be credibly revealed, they offer funds only to the R-type and so competitive markets cannot maximise social welfare. In a competitive environment, the implementation of the first-best solution can be achieved only in a pooling equilibrium where the required cross-subsidisation takes place through the mispricing of the R-type’s more valuable security (equity). We begin by characterising the social planner’s solution (the optimal contract) under adverse selection and effort moral hazard.

6.1 The Social Planner’s Solution: The Optimal Contract

The social planner’s objective is to induce both types to exert effort whenever feasible. Hence, the social planner will offer the S-type the required subsidy even if he can distinguish the two types, provided the R-type effort incentive constraint is not violated. Since the returns of the two types in case of success are different, observable and verifiable, the social planner can ex post distinguish the two types and promise to offer them funds at fair terms. Moreover, he can commit to making direct lump-sum transfers, $\tau$, from the R-type to the S-type so that the S-type effort incentive

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50 In our model, convertible debt does not improve on a debt-equity combination. If convertible debt is used, the exercise price of the option coincides with the face value of debt. This implies that if debt is converted into equity, the payment to the shareholders consists only of the project’s return. As a result, the maximum proportion of equity offered to the financiers consistent with the S-type working is exactly the same as under a debt-equity combination. In contrast, if a debt-warrant combination is used, the total payment to the shareholders, if the option is exercised, consists of two components: i) the project’s return and ii) the difference between the warrant exercise price and the face value of debt (which can be positive). Hence, the maximum proportion of equity issued consistent with the S-type working can be greater than under a debt-equity combination.

51 If the roles of the two types reverse and so the required cross-subsidisation takes place through the mispricing of debt, a debt-warrant combination does not improve on a debt-equity combination.
The constraint and the social planner feasibility constraint are just binding, and the R-type effort incentive constraint is not violated. Mathematically,

\[
(X_S - 1/\pi_C^S - \tau_S) = c_S
\]  
(25)

\[
(X_R - 1/\pi_C^R - \tau_R) \geq c_R
\]  
(26)

\[
\lambda \pi_C^R \tau_R + (1 - \lambda)\pi_C^S \tau_S = 0
\]  
(27)

Solving (25) and (26) for \( \tau_S \) and \( \tau_R \) respectively and substituting into (27), we obtain:

\[
\lambda \geq \frac{I - \pi_C^S (X_S - c_S)}{\pi_C^R (X_R - c_R) - \pi_C^S (X_S - c_S)} \equiv \lambda^{SP} = \lambda_2
\]  
(28)

Where \( \lambda^{SP} \) is the minimum proportion of the R-type (subsidiser) in the population of entrepreneurs consistent with both types exerting effort. In fact, it is the only restriction on the parameter values the social planner faces in his attempt to implement the socially efficient outcome. That is, the optimal contract involves the resolution of the adverse selection problem and lump-sum transfers.

6.2 Implementing the Optimal Contract with Debt, Equity and Warrants

Now that we have characterised the optimal contract, we examine its implementation as a competitive equilibrium using financial instruments observed in the real world. By Proposition 6, we know that, if the risky project dominates the safe one by first-order stochastic dominance, for any \( \lambda \geq \lambda_2 = \lambda^{SP} \) there exists a pooling equilibrium where both types exert effort and receive funds by issuing a debt-warrant combination. That is, the only restriction on parameter values required for the existence of the socially efficient pooling equilibrium is that the social planner also faces. Therefore, under this distributional assumption, debt coupled with a warrant can implement the optimal contract as a competitive equilibrium.

This really strong result relies on two factors: First, the fact that warrants allow for the intersection of the two effort incentive frontiers at some admissible value of the two choice variables, the proportion of equity, \( \eta \), and the face value of debt. This, in turn, implies that the socially efficient pooling equilibrium exists until the proportion of the R-type becomes so low that it is impossible to satisfy both effort incentive constraints. This is exactly the constraint the social planner faces. Second, the specific distributional assumption which ensures that the socially efficient pooling equilibrium Pareto-dominates any other equilibrium even if both effort incentive constraints are just binding. In other words, the R-type’s benefit from inducing the S-type to exert effort through the mispricing of warrants more than offsets the incremental subsidy (relative to the all-debt equilibrium where the S-type shirks) even if the total subsidy is so high that the R-type effort incentive constraint is just binding.

Under any other distributional assumption and/or a debt-equity combination, the existence of the socially efficient pooling equilibrium requires additional restrictions on the parameter values (see Propositions B1 and B2). If the two effort incentive
frontiers, $ICF_S$ and $ICF_R$, intersect at some admissible value of the choice variables (see Lemmas 1 and 9) $\lambda \geq \lambda_* = \lambda^{SP}$ is still a necessary condition for the existence of the socially efficient pooling equilibrium. However, for some $\lambda \geq \lambda_* = \lambda^{SP}$ this equilibrium collapses because the cost for the R-type of providing the S-type with a higher subsidy exceeds the benefit from inducing him to exert effort. The resulting pooling equilibrium involves the issue of just debt (corner solution) and the S-type shirking.

Notice, however, that because the social planner does not face the latter constraint, whenever $ICF_S$ and $ICF_R$, intersect at some admissible value of the choice variables, he can implement the optimal contract using a debt-equity or a debt-warrant combination.

7. Concluding Remarks

In this paper, we have analysed and discussed the roles of debt, equity and warrants under adverse selection and (effort) moral hazard. Several interesting results were obtained. First, we explained the issue of combinations of debt and equity as the outcome of the interaction between adverse selection and moral hazard. Firms accept to incur the adverse selection cost of issuing equity because this cost is more than offset by the benefit from relaxing the moral hazard problem. Second, we showed that, in the presence of moral hazard, adverse selection may result in the conversion of a negative into a positive NPV project and an improvement in social welfare. Third, we provided two rationales for the use of warrants. Under pure adverse selection, warrants can serve as separation devices in cases where other standard securities cannot. Under adverse selection cum moral hazard, warrants allow for the implementation of the socially efficient outcome even if this is not possible when we restrict ourselves to debt, equity and/or convertible debt. Finally, we showed that, under certain conditions, a debt-warrant combination can implement the optimal contract as a competitive equilibrium.

Our focus on a two-type model allowed us to illustrate the effects of the interaction between adverse selection and moral hazard and the mechanism that necessitates the use of warrants in the simplest possible way. Under certain conditions, most results should obtain if we extend the model to allow for more than two types. For example, under pure adverse selection, a debt-warrant combination allows for the existence of equilibria (separating, partial separating or pooling) where the securities issued are fairly priced even if there exist three or more types.

Also, the pooling equilibrium of Proposition 4 (Proposition 6) can obtain if a third type is added. What is more, it may exist even if the NPV of the third type project is negative regardless of whether he exerts effort or not. However, in such a case, the welfare properties of the pooling equilibrium are different. Adverse selection results in overinvestment and possibly in a fall in social welfare. This latter result does not depend on the number of types, it may obtain even with two types if the project of one of them has negative NPV regardless of the effort level.

Another natural extension of the model is to allow for more than two effort levels (possibly a continuum) and check the robustness of the results in the adverse selection cum moral hazard case. We conjecture that under certain distributional assumptions and restrictions on the cost and probability functions, a pooling equilibrium similar to those described in Propositions 4 and 6 should obtain. However, the combination of
financial contracts required for its existence as well as the implementation of the optimal contract in this case are interesting open questions.

References


Appendix A

Proof of Lemma 1

a) By totally differentiating (4), we obtain:

\[-(X_i - D)d\alpha - (1 - \alpha)dD = 0 \Rightarrow \left(\frac{d\alpha}{dD}\right)_{ICF_i} = -\frac{1 - \alpha}{X_i - D} < 0\]

Taking into account that $ICF_i$ implicitly defines $\alpha$ as a function of $D$, we obtain:

\[\left(\frac{d^2\alpha}{dD^2}\right)_{ICF_i} = -\frac{1 - \alpha}{(X_i - D)^2} + \frac{d\alpha}{dD} \frac{dD}{X_i - D} = \frac{2(1 - \alpha)}{(X_i - D)^2} < 0\]

Hence, $ICF_i$ is downward sloping and strictly concave. Also, since $X_R > X_S$, $ICF_S$ is steeper than $ICF_R$.

b) The effort incentive frontiers of the R- and S-type are respectively:

\[(1 - \alpha)(X_R - D) = c_R \quad (A1)\]

\[(1 - \alpha)(X_S - D) = c_S \quad (A2)\]

Using (A1) and (A2) and solving for $\alpha$ and $D$ we obtain:

\[\bar{\alpha} = \frac{(X_R - c_R) - (X_S - c_S)}{X_R - X_S}, \quad \bar{D} = \frac{X_S c_R - X_R c_S}{c_R - c_S} \quad (A3)\]

So, $\bar{\alpha} \geq 0 \iff X_R - c_R \geq X_S - c_S$

$\bar{\alpha} \leq 1 \iff c_R \geq c_S$

Also, $\bar{D} \geq 0 \iff c_R / X_R \geq c_S / X_S \implies c_R > c_S$

Therefore, $1 \geq \bar{\alpha} \geq 0$ and $\bar{D} \geq 0 \iff X_R - c_R \geq X_S - c_S$ and $c_R / X_R \geq c_S / X_S$.

If $X_R - c_R < X_S - c_S$, (the intersection of $ICF_S$ with the horizontal axis lies to the right of that of $ICF_R$), then because $ICF_S$ is steeper than $ICF_R$, at any $0 \leq \alpha \leq 1$, $ICF_S$ lies entirely above $ICF_R$ in the $(\alpha, D)$ space. That is, $IC_R \subset IC_S$.

If $c_R / X_R < c_S / X_S$, (the intersection of $ICF_S$ with the vertical axis lies below that of $ICF_R$), then because $ICF_S$ is steeper than $ICF_R$, at any $0 \leq \alpha \leq 1$, $ICF_S$ lies entirely below $ICF_R$ in the $(\alpha, D)$ space. That is, $IC_S \subset IC_R$.
c) $ICF_i$ meets the vertical axis at $\alpha_i = 1 - c_i / X_i$ and the horizontal axis at $D_i = X_i - c_i$. By Assumption 1, $X_i > c_i$ and $1 > c_i / X_i$. Also, by Part (a) of this Lemma, $ICF_i$ is downward sloping and strictly concave. Therefore, $IC_i$ cannot be empty (See Figure 1). Q.E.D.

Proof of Lemma 2

a) For any $0 \leq \alpha \leq 1$, $0 \leq D \leq R$, Eq. (1) becomes:

$$U_i = \pi(B_i)(1 - \alpha)(X_i - D) + B_i, \quad i = R, S$$

(A4)

Differentiating (A4), we obtain:

$$\left(\frac{d\alpha}{dD}\right)_{u=\pi} = -\frac{1 - \alpha}{X_i - D} \leq 0$$

$$u_i = u$$ implicitly defines $\alpha$ as a function of $D$ and so:

$$\left(\frac{d^2\alpha}{dD^2}\right)_{u=\pi} = -\frac{2(1 - \alpha)}{(X_i - D)^2} \leq 0$$

Hence, the indifference curves of both the R- and the S-type are downward sloping and concave.

b) Since $X_R > X_S$, at any $(\alpha, D)$ pair, $u_R$ is flatter than $u_S$ and hence they cross only once. Q.E.D.

Proof of Lemma 3

a) The equations for $ZP_i$ and $PZP_H$ are respectively:

$$\pi_i^c[\alpha(X_i - D) + D] = I, \quad i = R, S$$

(A5)

$$\lambda \pi_i^s[\alpha(X_R - D) + D] + (1 - \lambda) \pi_c^s[\alpha(X_S - D) + D] = I$$

(A6)

Differentiating (A5) and (A6) we obtain the slopes of $ZP_i$ and $PZP_H$ respectively. Since $X_R > X_S$ and $0 \leq \lambda \leq 1$, it is obvious that at any given $(\alpha, D)$ pair,

$$\left|\left(\frac{d\alpha}{dD}\right)_{ZP_i}\right| \geq \left|\left(\frac{d\alpha}{dD}\right)_{PZP_H}\right| \geq \left|\left(\frac{d\alpha}{dD}\right)_{ZP_s}\right|$$

b) By Lemmas 1, 2, and 3
\[
\left( \frac{d\alpha}{dD} \right)_{ICF_i} = \left( \frac{d\alpha}{dD} \right)_{ZP_i} = \frac{1 - \alpha}{X_i - D} < 0 , \quad i = R, S
\]  

Hence, \( ICF_i, \ u_i, \ ZP_i \ (i = R, S) \) never intersect.  

**Proof of Lemma 7**

Using (A5) and solving for \( \alpha \) and \( D \), we obtain the values of \( \alpha \) and \( D \) where \( ZP_R \) and \( ZP_S \) intersect in the \((\alpha, D)\) space.

\[
\hat{\alpha} = \frac{I(1/\pi^R_C - 1/\pi^S_C)}{X_R - X_S} , \quad \hat{D} = \frac{\pi^R_C X_R - \pi^S_C X_S}{(X_R - X_S) \pi^R_C \pi^S_C / I - (\pi^S_C - \pi^R_C)}
\]  

(A8)

Notice that \( X_R > X_S \) and \( \pi^S_C \geq \pi^S_S \) imply \( \hat{\alpha} \geq 0 \). Also, \( \hat{\alpha} \leq 1 \Leftrightarrow (X_R - X_S) \pi^R_C \pi^S_C / I \geq \pi^S_C - \pi^R_C \). Given \( X_R > X_S \), this condition may be violated only under mean-reducing spreads.

That is, in the range of parameters where \( \hat{\alpha} \) takes on an admissible value, the denominator of the equation for \( \hat{D} \) is positive. Hence, \( D \geq 0 \Leftrightarrow \pi^R_C X_R \geq \pi^S_C X_S \).

More analytically,

a) If the risky project dominates the safe one by first-order stochastic dominance \( (\pi^R_C = \pi^S_C = \pi^C, \ X_R > X_S) \), then \( \hat{\alpha} = 0, \ \hat{D} = 1/\pi^C > 0 \). Also, \( ZP_R \) is flatter than \( ZP_S \). Hence, for \( \alpha > 0 \) \( ZP_R \) lies below \( ZP_S \) in the \((\alpha, D)\) space.

b) If the risky project is a mean-preserving spread of the safe one \( (\pi^R_C X_R = \pi^S_C X_S) \), then \( \hat{\alpha} = 1/\pi^R_C X_R, \ \hat{D} = 0 \). Hence, since \( ZP_R \) is flatter than \( ZP_S \), for \( \hat{D} > 0 \) \( ZP_S \) lies below \( ZP_R \) in the \((\alpha, D)\) space.

c) If the risky project is a mean-increasing spread of the safe one \( (\pi^R_C X_R > \pi^S_C X_S) \), then \( \hat{\alpha} > 0, \ \hat{D} > 0 \).

d) If the risky project is a mean-reducing spread of the safe one \( (\pi^R_C X_R < \pi^S_C X_S) \), then \( \hat{\alpha} > 0 \) but \( \hat{D} < 0 \). Hence, since \( ZP_R \) is flatter than \( ZP_S \), for \( D \geq 0 \) \( ZP_S \) lies below \( ZP_R \) in the \((\alpha, D)\) space. That is, \( ZP_S \) and \( ZP_R \) do not intersect at any admissible value of the two choice variables.  

**Q.E.D.**

**Proof of Lemma 8**

a) In this case, \( \pi^R_j = \pi^S_j, \ j = C, 0 \). Hence, \( c_R = \frac{C}{\pi^R_C - \pi^R_0} = \frac{C}{\pi^S_C - \pi^S_0} = c_S \). Using (A3) we obtain: \( \overline{\alpha} = 1, \ \overline{D} = -\infty \). Also, since \( ICF_S \) is steeper than \( ICF_R \), \( IC_S \subset IC_R \).

b) Here, \( \pi^S_j X_R = \pi^S_j X_S, \ j = C, 0 \). Hence, \( c_S X_R = c_R X_S \). Then, (A3) implies \( 0 < \overline{\alpha} < 1, \ \overline{D} = 0 \). Also, since \( ICF_S \) is steeper than \( ICF_R \), \( IC_S \subset IC_R \).  

**Q.E.D.**
Proof of Lemma 9

The equations of the effort incentive frontiers $ICF_S$ and $ICF_R$ are given respectively by:

$$ (1 - \psi \eta)(X_S - D) = c_S $$ (A9)

$$ (1 - \psi \eta)(X_R - D) - \eta(1 - \psi)(X_R - X_S) = c_R $$ (A10)

a) By totally differentiating (A9) and (A10), we obtain the slopes of $ICF_S$ and $ICF_R$ respectively (the equations are provided in the text). Since $X_R > X_S$, at any given $(\eta, D)$ pair, $ICF_S$ is steeper than $ICF_R$.

b) Solving (A9) and (A10) for $\eta$ and $D$, we obtain:

$$ \bar{\eta} = \frac{(X_R - c_R) - (X_S - c_S)}{X_R - X_S} $$

$$ \bar{D} = \frac{(X_S - c_S)(X_R - (1 - \psi)X_S) - \psi(X_R - c_R)X_S}{(1 - \psi)(X_R - X_S) + \psi(c_R - c_S)} $$

(A11)

So, $\bar{\eta} \geq 0 \iff X_R - c_R \geq X_S - c_S$

$\bar{\eta} \leq 1 \iff c_R \geq c_S$

Notice that $\bar{\eta}$ is independent of $\psi$. Also, for $\psi = 1$ the expression for $\bar{D}$ in (A11) becomes identical to that in (A3). Moreover, for any admissible value of $\bar{\eta}$, the denominator in the expression for $\bar{D}$ is positive. Hence,

$$ \bar{D} \geq 0 \iff (X_S - c_S)(X_R - (1 - \psi)X_S) \geq \psi(X_R - c_R)X_S \iff $$

$$ \psi \leq \frac{(X_R - X_S)(1 - c_S/X_S)}{(X_R - c_R) - (X_S - c_S)} \equiv \overline{\psi} $$

(A12)

Therefore, if $X_R - c_R \geq X_S - c_S$ and $c_R \geq c_S$, then for any $\psi \in [0, \min[\overline{\psi}, 1]]$, $1 \geq \bar{\eta} \geq 0$ and $\bar{D} \geq 0$.

If $X_R - c_R < X_S - c_S$, (the intersection of $ICF_S$ with the horizontal axis lies to the right of that of $ICF_R$), then because $ICF_S$ is steeper than $ICF_R$, at any $0 \leq \eta \leq 1$, $ICF_S$ lies entirely above $ICF_R$ in the $(\eta, D)$ space. That is, $IC_R \subset IC_S$.

If $c_R < c_S$, the intersection of $ICF_S$ and $ICF_R$ occurs at some $\eta > 1$ regardless of the value of $\psi$. Hence, because $ICF_S$ is steeper than $ICF_R$, at any $0 \leq \eta \leq 1$, $ICF_S$ lies entirely below $ICF_R$ in the $(\eta, D)$ space. That is, $IC_S \subset IC_R$. 
c) Setting the utility (Eq. (14) in the text) of an E of type \( i \) equal to a constant and differentiating, we obtain the slopes of the indifference curves which are identical to the corresponding slopes of the effort incentive frontiers. \( Q.E.D. \)

**Proof of Lemma 10**

The equations of the zero-profit lines \( ZP_S \) and \( ZP_R \) are respectively:

\[
\pi_C^S \left[ \eta \psi (X_S - D) + D \right] = I \tag{A13}
\]

\[
\pi_C^R \left\{ \eta [\psi (X_S - D) + (X_R - X_S)] + D \right\} = I \tag{A14}
\]

a) By totally differentiating (A13) and (A14), we obtain the slopes of \( ZP_S \) and \( ZP_R \) respectively (the equations are provided in the text). Since \( X_R > X_S \), at any given \((\eta, D)\) pair, \( ZP_S \) is steeper than \( ZP_R \). Also, since \( 0 \leq \lambda \leq 1 \),

\[
\left| \frac{d\alpha}{dD} \right|_{ZP_S} \geq \left| \frac{d\alpha}{dD} \right|_{ZP_R} \geq \left| \frac{d\alpha}{dD} \right|_{ZP_i}
\]

b) Solving (B21) and (B22) for \( \eta \) and \( D \), we obtain:

\[
\hat{\alpha} = \frac{I (1/\pi_R^R - 1/\pi_C^S)}{X_R - X_S}, \quad \hat{D} = \frac{\pi_C^R (X_R - X_S) - \psi (\pi_C^S - \pi_C^R) X_S}{(X_R - X_S) \pi_C^R \pi_C^S / I - \psi (\pi_C^S - \pi_C^R)} \tag{A15}
\]

Notice that \( X_R > X_S \) and \( \pi_C^S \geq \pi_C^R \) imply \( \hat{\alpha} \geq 0 \). Also, \( \hat{\alpha} \leq 1 \iff X_R - I/\pi_C^R \geq X_S - I/\pi_C^S \). Given \( X_R > X_S \), this condition may be violated only under mean-reducing spreads. Also, in the range of parameters where \( \hat{\alpha} \) takes on an admissible value, the denominator of the equation for \( \hat{D} \) is positive. Hence,

\[
\hat{D} \geq 0 \iff \psi \leq \frac{\pi_C^R (X_R - X_S)}{(\pi_C^S - \pi_C^R) X_S} \equiv \hat{\psi} \tag{A16}
\]

Therefore, if \( X_R - I/\pi_C^R \geq X_S - I/\pi_C^S \), for any \( \psi \in [0, \min[\hat{\psi}, 1]] \) \( ZP_S \) and \( ZP_R \) intersect at some admissible value of the two choice variables \( (0 \leq \eta \leq 1, D \geq 0) \) under all four assumptions about the ordering of the distributions of returns.

c) By Lemmas 9 and 10,

\[
\left( \frac{d\eta}{dD} \right)_{ICF_i} = \left( \frac{d\eta}{dD} \right)_{u_i, \pi } = \left( \frac{d\eta}{dD} \right)_{ZP_i}, \quad i = R, S \tag{A17}
\]

Hence, \( ICF_i, \ u_i, \ ZP_i \ (i = R, S) \) never intersect. \( Q.E.D. \)
Appendix B

Proposition B1 (Generalisation of Proposition 4): Suppose the following are true
\[ \pi^R_c X_R - I > \pi^R_c c_R, \quad \pi^S_c X_S - I < \pi^S_c c_S, \quad \text{and} \quad I/\pi^R_c X_R < 1 - c_S/X_S. \] Then there exists a unique pooling (funding) equilibrium where both types choose the high effort level and obtain funds by issuing both debt and equity if either

a) \( IC_S \subset IC_R, \quad \lambda > \bar{\lambda}_1 \) and \( \pi^S_c / \pi^S_0 \geq X_R / X_S \) (it is possible if the risky projects dominate the safe ones by first-order stochastic dominance or they are mean-increasing spreads)

b) \( X_R - c_R > X_S - c_S, \quad c_R / X_R > c_S / X_S \) (\( ICF_R \) and \( ICF_S \) intersect), \( \lambda > \bar{\lambda}_2 \) and \( \pi^S_c / \pi^S_0 \geq (X_R - c_R)/(X_S - c_S) \) (it’s possible only under mean-increasing spreads).

where \( \bar{\lambda}_1 = \frac{I - \pi^S_c (X_S - c_S)}{(\pi^R_c X_R - \pi^S_c X_S)(1 - c_S/X_S)}, \quad \bar{\lambda}_2 = \frac{I - \pi^S_c (X_S - c_S)}{\pi^S_c (X_R - c_R) - \pi^S_c (X_S - c_S)} \)

The equilibrium contract, \( A = (\alpha^*, D^*) \), lies at the intersection of \( ICF_S \) and \( PZP_H \) with \( \alpha^* \) and \( D^* \) given by:

\[
\alpha^* = \frac{I - \left[ \lambda \pi^R_c + (1 - \lambda) \pi^S_c \right] (X_S - c_S)}{\lambda \pi^R_c (X_R - X_S)} \quad (B1)
\]

\[
D^* = X_S - c_S / (1 - \alpha^*) \quad (B2)
\]

![Diagram](image-url)
Proof:

The pooling equilibria described in this proposition exist if the following two conditions are satisfied: i) $PZP_H$ belongs to the intersection of $IC_s$ and $IC_R$ for $\lambda \leq 1$ and ii) the R-type indifference curve through the equilibrium contract, $u^*_R$, does not intersect $PZP_L$.

a) In this case, since $IC_s \subset IC_R$ the first condition is satisfied if $PZP_H$ crosses $ICF_s$. Provided $ZP_R$ intersects $ICF_s \ (1/\pi^R C X_R < 1-c_s/X_s)$, by Figures 5 and B1, it is clear that $PZP_H$ crosses $ICF_s$ if the intersection point of $PZP_H$ with the vertical axis lies below that of $ICF_s$. That is, if

$$
1 - \frac{c_s}{X_s} > \frac{I}{\lambda \pi^R C X_R + (1-\lambda)\pi^S C X_S} \iff \lambda > \frac{I - \pi^S C (X_s - c_s)}{(\pi^R C X_R - \pi^S C X_S)(1-c_s/X_s)} \equiv \lambda_i
$$

$I - \pi^S C (X_s - c_s)$: Minimum subsidy required to induce the S-type to exert effort.

$\pi^R C X_R - \pi^S C X_S$: Expected return differential (given the high effort level is chosen).

$1 - c_s/X_s$: Maximum $\alpha \in IC_s$

Regarding the second condition, since $X_R > X_s$ and $0 \leq \lambda \leq 1$, at any given $(\alpha, D)$ pair, $u^*_R$ is flatter than $PZP_L$. Therefore, it suffices to show that the intersection point of $u^*_R$ with the horizontal axis lies to the left of that of $PZP_L$.

The intersection point of $PZP_L$ with the horizontal axis is given by:

$$
D = \frac{I}{\lambda \pi^R C + (1-\lambda)\pi^S C}
$$

Moreover, the expected utility of the R-type in equilibrium is given by:

$$
U^*_R = (1 - \alpha^*)\pi^R C (X_R - D^*) + b
$$

At $\alpha = 0$, the R-type’s expected utility is:

$$
(U^*_R)_{\alpha=0} = \pi^R C (X_R - D) + b
$$

Setting $U^*_R = (U^*_R)_{\alpha=0}$ and using the expressions for $\alpha^*$ and $D^*$, we obtain:

$$
D = \frac{I - (1-\lambda)\pi^S C (X_s - c_s)}{\lambda \pi^R C}
$$
Hence, the second condition is satisfied if:

\[
\frac{I}{\lambda \pi_c^R + (1-\lambda) \pi_0^S} \geq \frac{I - (1-\lambda) \pi_c^S (X_S - c_S)}{\lambda \pi_c^R}
\]  

(B7)

Let \( f(\lambda) = \frac{I - (1-\lambda) \pi_c^S (X_S - c_S)}{\lambda \pi_c^R} \)

and \( g(\lambda) = \frac{I}{\lambda \pi_c^R + (1-\lambda) \pi_0^S} \)

then \( f'(\lambda) = -\frac{1}{\lambda^2 \pi_c^R} [I - \pi_c^S (X_S - c_S)] < 0 \), \( f''(\lambda) = \frac{2}{\lambda^3 \pi_c^R} [I - \pi_c^S (X_S - c_S)] > 0 \)

Since, by assumption, \( I - \pi_c^S (X_S - c_S) > 0 \).

Also, \( g'(\lambda) = -\frac{(\pi_c^R - \pi_0^S) I}{[\lambda \pi_c^R + (1-\lambda) \pi_0^S]^2} < 0 \), \( g''(\lambda) = \frac{(\pi_c^R - \pi_0^S)^2 I}{[\lambda \pi_c^R + (1-\lambda) \pi_0^S]^3} > 0 \)

Assuming \( \pi_c^R > \pi_0^S \), both \( f(\lambda) \) and \( g(\lambda) \) are strictly decreasing and strictly convex.

Furthermore, \( f(\lambda) \leq g(\lambda) \iff \lambda \leq 1 \) and \( \lambda \geq \frac{I - \pi_c^S (X_S - c_S)}{(\pi_c^R - \pi_0^S) \pi_c^R (X_S - c_S)} \equiv \tilde{\lambda} \)

Since \( 0 \leq \lambda \leq 1 \), both \( f(\lambda) \) and \( g(\lambda) \) are continuous, strictly decreasing and strictly convex, \( f(\lambda) \leq g(\lambda) \) for \( \lambda \geq \tilde{\lambda} \) and \( f(\lambda) > g(\lambda) \) for \( \lambda < \tilde{\lambda} \), then \( f(\lambda) \leq g(\lambda) \) for all \( \lambda \in [\tilde{\lambda}, 1] \). Therefore, \( u^*_R \) does not cut \( PZP_{L} \) for any \( \lambda \in [\tilde{\lambda}, 1] \) if and only if:

\[
\tilde{\lambda}_1 \geq \tilde{\lambda} \iff \pi_c^S X_R \leq \pi_c^S X_S
\]  

(B8)

Notice that if the risky projects dominate the safe ones by first-order stochastic dominance \( (\pi_j^k = \pi_j^m) \), \( j = C, 0 \), this condition is automatically satisfied (by Assumption 1). However, under mean-increasing spreads it may be violated. In such a case, \( \tilde{\lambda}_1 < \tilde{\lambda} \) and hence the socially efficient pooling equilibrium exists only if \( \lambda \geq \tilde{\lambda} > \tilde{\lambda}_1 \).

b) In this case, since \( PZP_{hl} \) is flatter than \( ICF_S \) and steeper than \( ICF_R \), the first condition is satisfied if

\[
\bar{\alpha} \geq \alpha^* \iff \lambda \geq \frac{I - \pi_c^S (X_S - c_S)}{\pi_c^R (X_R - c_R) - \pi_c^S (X_S - c_S)} \equiv \tilde{\lambda}_2
\]  

(B9)
Repeating the steps in Part (a), one can show that, for any $\lambda \in [\bar{\lambda}_2, 1]$, the second condition is satisfied if and only if

$$\frac{\pi^C_c}{\pi^S_0} \geq \frac{X_R - c_R}{X_S - c_S}$$  \hspace{1cm} (B10)

*Q.E.D.*

**Proposition B2 (Generalisation of Proposition 6):** Suppose the following are true $\pi^R_c X_R - I > \pi^S_c X_S - I < \pi^S_c c_S$ and $X_R - c_R > X_S - c_S$, $c_R \geq c_S$. Then, for any $\psi \in [0, \min[\bar{\psi}, 1]]$, $\lambda \geq \bar{\lambda}_2$, and $\pi^S_c / \pi^S_0 \geq (X_R - c_R)/(X_S - c_S)$, then there exists a unique pooling (funding) equilibrium where both types exert effort and obtain funds by issuing a debt-warrant combination (it is possible if the risky projects dominate the safe ones by first-order stochastic dominance or they are mean-increasing spreads).

where $\bar{\lambda}_2 \equiv \frac{I - \pi^S_c (X_S - c_S)}{\pi^R_c (X_R - c_R) - \pi^S_c (X_S - c_S)}$

The equilibrium contract, $A = (\eta^*, D^*)$, lies at the intersection of $ICF_S$ and $PZP_H$ with $\eta^*$ and $D^*$ given by:

$$\eta^* = \frac{I - \left[\lambda \pi^R_c + (1 - \lambda) \pi^S_c \right] (X_S - c_S)}{\lambda \pi^R_c (X_R - X_S)}$$ \hspace{1cm} (B11)

$$D^* = X_S - c_S / (1 - \psi \eta^*)$$ \hspace{1cm} (B12)

*Proof:* Similar to Proposition B1.

Notice that if the risky projects dominate the safe ones by first-order stochastic dominance we have $X_R > X_S$, $\pi^R_j = \pi^S_j = \pi^C_j$, $j = C, 0$. This implies:

- $c_R = c_S = c$
- $X_R - c_R > X_S - c_S$
- $\pi^S_c / \pi^S_0 \geq (X_R - c_R)/(X_S - c_S) \iff \pi^S_c X_S - \pi^S_0 X_R \geq C$ (always true by Assumption 1).

Hence, all the conditions, except for $\lambda \geq \bar{\lambda}_2 = \lambda^{SP}$, required for the existence of the socially efficient pooling equilibrium are automatically satisfied. The remaining condition ($\lambda \geq \bar{\lambda}_2 = \lambda^{SP}$) is identical to that the social planner faces.
**Appendix C: Separate Bond and Equity Markets**

The analysis in the text assumed that the required amount of funds I is provided by the same financier who purchases both debt and equity. In this appendix, I show that all the results go through even if the buyer of debt and the buyer of equity are different (debt and equity markets are separate). It suffices to show that the zero-profit lines of an equity-buyer, a bond-buyer and a financier purchasing both debt and equity coincide. The following assumptions are made:

i) The project is indivisible.

ii) Es have no storage technology (cannot lend) and the consumption good is perishable.

iii) Bond and equity markets are perfectly competitive.

The first assumption implies that the Es borrow at least I. The second implies that no E will borrow more than I. Therefore, Es borrow just I. Given these three assumptions, we have:

\[ I_B + I_E = I \]  \hspace{1cm} (C1)

\[ P_{BF} = \left[ \lambda \pi_j^R + (1 - \lambda) \pi_k^S \right] D - I_B = 0, \quad j = C, 0, \quad k = C, 0, \]  \hspace{1cm} (C2)

\[ P_{EF} = \alpha \left[ \lambda \pi_j^R (X_R - D) + (1 - \lambda) \pi_k^S (X_S - D) \right] - I_E = 0 \]  \hspace{1cm} (C3)

where

- \( I_B \): Amount the E borrows from the bond-financier
- \( I_E \): Amount the E borrows from the equity-financier
- \( P_{BF} \): Expected profit of the bond-financier
- \( P_{EF} \): Expected profit of the equity-financier
- \( P_F \): Expected profit of a financier purchasing both debt and equity

Using (C1), (C2) and (C3) we obtain:

\[ \lambda \left[ \alpha \pi_j^R (X_R - D) + \pi_j^R D \right] + (1 - \lambda) \left[ \alpha \pi_k^S (X_S - D) + \pi_k^S D \right] - I \]

\[ = P_{EF} = P_{BF} = P_F = 0 \]  \hspace{1cm} (C4)

That is, the zero-profit lines of an equity-financier, a bond-financier and a financier purchasing both debt and equity coincide. Therefore, the pooling equilibrium obtains regardless of whether the same investor purchases both debt and equity and provides the required amount I or the debt-financier and the equity-financier are different (bond and equity markets are separate).

Similar results can be derived for the individual zero-profit lines. Also, all the results go through if debt and warrants are issued instead of debt and equity.