Improved formulae for consistent combination of geometric and orthometric heights and their rates

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Objective

- Revise the theoretical constraint for geometrical and physical heights **beyond the simplified model**

$$h - N - H = 0 \qquad\qquad \dot{h} - \dot{N} - \dot{H} = 0$$

- What will we gain ?

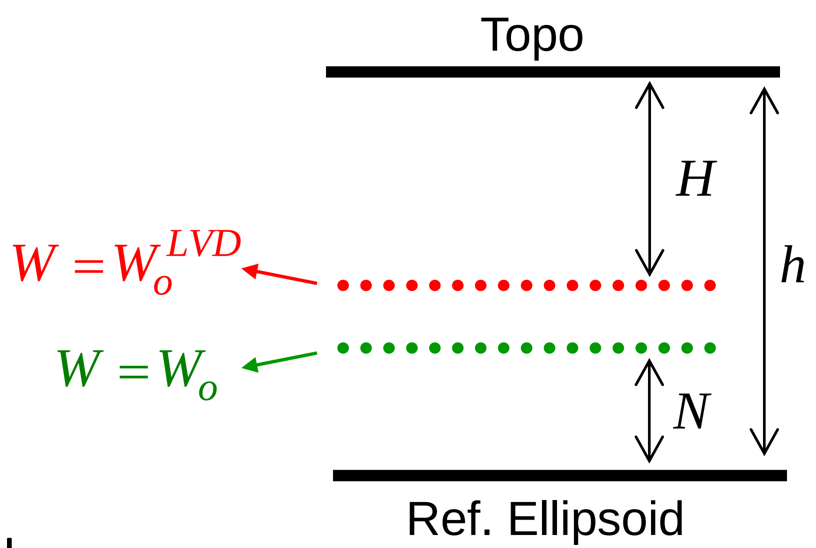
Standardized and geodetically meaningful de-trending
for the joint analysis of heterogeneous heights

Frame-consistent modeling of heterogeneous vertical velocities

Direct combination of quasi-geoids with orthometric heights !

Fundamental relationship

$$h - N - H = \frac{W_o - W_o^{LVD}}{g^{(W_o)}} = \frac{\delta W_o}{g^{(W_o)}}$$



Note:

- Expresses in linearized form the vertical separation between equipotential surfaces
- $g^{(W_o)}$ refers to the (almost constant) gravity on the geoid
- h and N should refer to the same GRS (ellipsoid + frame)
- DoV is ignored but the model error is negligible (< 1 mm)

Revised relationship

Instead of the (non-determinable) true orthometric heights, we shall employ the actual Helmert orthometric heights.

Using the formula: $$H = \frac{\bar{g}^{helm}}{\bar{g}} H^{helm}$$

we obtain the following condition:

$$h - N - H^{helm} = \frac{\bar{g}^{helm} - \bar{g}}{\bar{g}} H^{helm} + \frac{\delta W_o}{g^{(W_o)}}$$

Height-correlated residuals even with error-free data !

Revised relationship (cont'd)

$$h - N - H^{helm} = \boxed{\frac{\bar{g}^{helm} - \bar{g}}{\bar{g}}} H^{helm} + \boxed{\frac{\delta W_o}{g^{(W_o)}}}$$

Vertical scale factor (λ)

Zero-height vertical offset (μ)

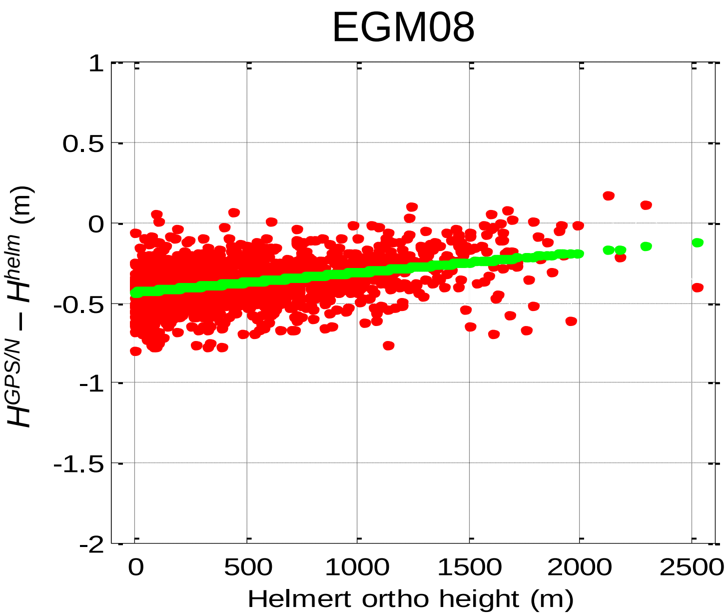
Looks like a *similarity transformation* for different VRFs

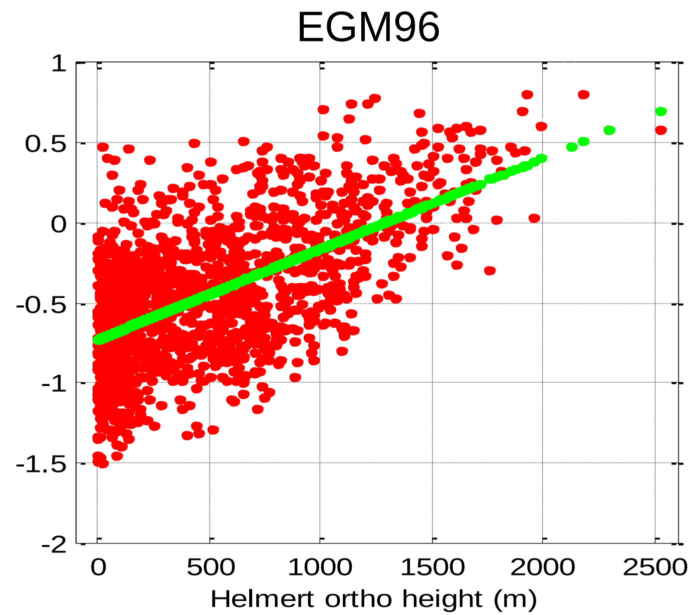
$$\boxed{H^{VRF2} - H^{VRF1} = \lambda H^{VRF1} + \mu}$$

Reflect the 'vertical datum disturbance' between the underlying height frames

Example (1542 Greek GPS/lev BMs)

$$H^{GPS/N} - H^{helm} = \underbrace{\lambda\, H^{helm} + \mu}_{\textit{Linear theoretical trend}} + e$$



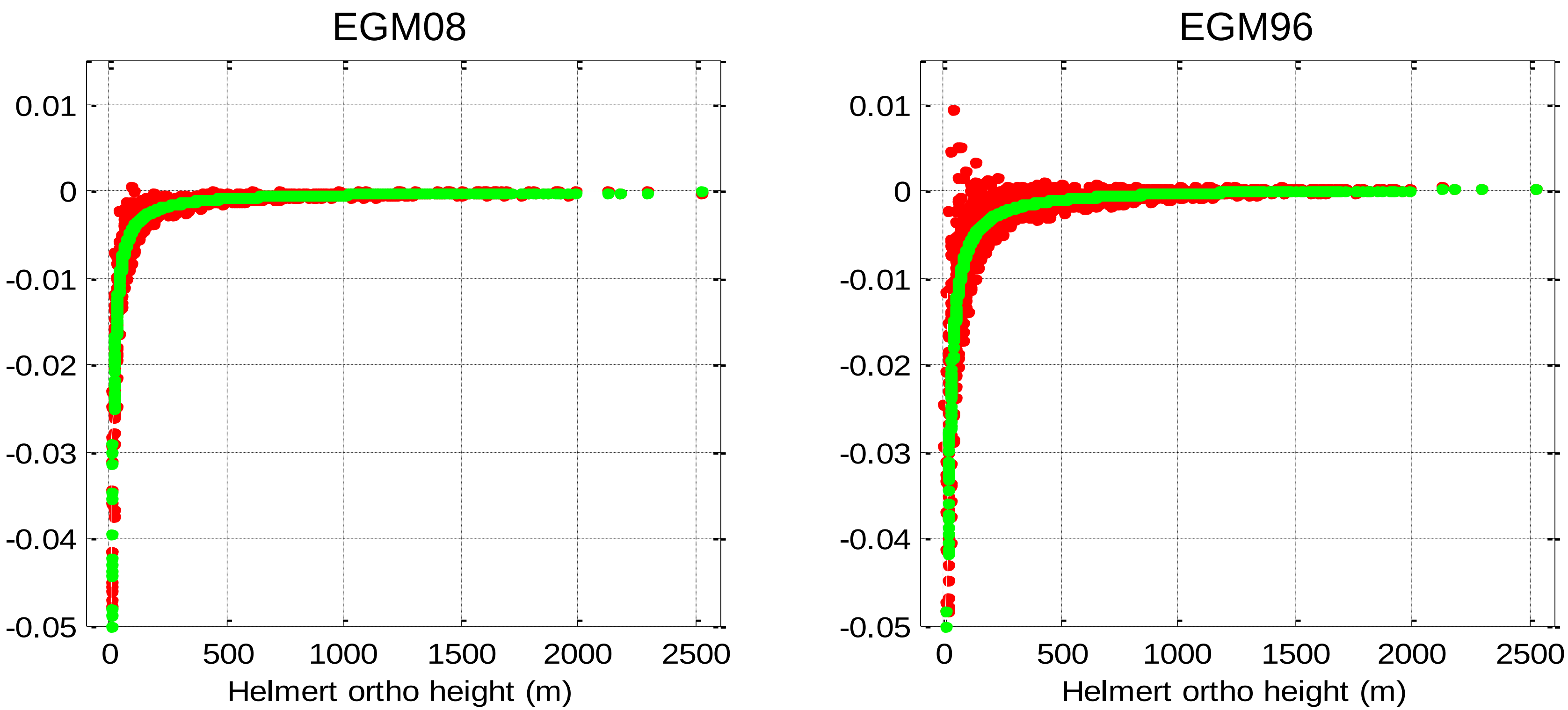


$$\hat{\mu} = -43.1 \text{ cm} \qquad\qquad \hat{\mu} = -72.6 \text{ cm}$$

$$\hat{\lambda} = 0.12\times 10^{-3} \qquad\qquad \hat{\lambda} = 0.57\times 10^{-3}$$

Example (1542 Greek GPS/lev BMs)

$$\frac{H^{GPS/N} - H^{helm}}{H^{helm}} = \underbrace{\lambda + \frac{\mu}{H^{helm}}}_{\textit{Nonlinear theoretical trend}} + \tilde{e}$$



Conclusion #1

Rigorous constraint between GPS/geoid and Helmert orthometric heights

$$H^{GPS/N} - H^{helm} = \lambda H^{helm} + \frac{\delta W_o}{g^{(W_o)}}$$

- *The parameters λ and δW_o reflect the systematic differences between the underlying VRFs*
- *Geodetically meaningful comparison of heterogeneous height data*
- *A constant conventional value $g^{(W_o)}$ is required for the LS inversion of the above model*

Use of quasi-geoid models

Based on the well-known formula: $N = \zeta + \frac{\bar{g}' - \bar{\gamma}}{\bar{\gamma}} H'$

and after few justifiable approximations (< 1 mm), we obtain the following condition:

$$h - \zeta - H^{helm} = \frac{\bar{g}^{helm} - \bar{\gamma}}{\bar{\gamma}} H^{helm} + \frac{\delta W_o}{\bar{\gamma}}$$

which is 'similar' to the formula

$$h - N - H^{helm} = \frac{\bar{g}^{helm} - \bar{g}}{\bar{g}} H^{helm} + \frac{\delta W_o}{g^{(W_o)}}$$

Conclusion #2

Rigorous constraints for heterogeneous heights

$$\begin{Bmatrix} h - N - H^{helm} \\ h - \zeta - H^{helm} \end{Bmatrix} = H^{helm} \begin{Bmatrix} \lambda \\ \lambda' \end{Bmatrix} + \begin{Bmatrix} 1/\, g^{(W_o)} \\ 1/\, \overline{\gamma} \end{Bmatrix} \delta W_o$$

Is there any value in using the second relationship for the joint analysis of ***quasi-geoid models with Helmert ortho heights*** *?*

Example (1542 Greek GPS/lev BMs)

Estimated VRF transformation parameters

	$\hat{\lambda}$	$\delta\hat{W}_o$	
$h - N^{EGM08} - H^{helm}$	$(12.45 \pm 0.75) \times 10^{-5}$	-8.55 ± 0.05 $m^2 s^{-2}$	**Model A**
$h - \zeta^{EGM08} - H^{helm}$	$(2.13 \pm 0.73) \times 10^{-5}$	-8.35 ± 0.05 $m^2 s^{-2}$	**Model B**

Statistics of adjusted residuals

	max	min	mean	σ
Residuals from **model A**	0.481	-0.473	0.000	**0.130**
Residuals from **model B**	0.465	-0.464	0.000	**0.127**

all values in m

Example (20 Swiss EUVN-DA BMs)

Estimated VRF transformation parameters

	$\hat{\lambda}$	$\delta\hat{W}_o$	
$h - N^{EGM08} - H^{EVRF07}$	$(-2.37 \pm 3.66) \times 10^{-5}$	-6.18 ± 0.41 $m^2\ s^{-2}$	**Model A**
$h - \zeta^{EGM08} - H^{EVRF07}$	$(-15.53 \pm 3.04) \times 10^{-5}$	-5.87 ± 0.34 $m^2\ s^{-2}$	**Model B**

Statistics of adjusted residuals

	max	min	mean	σ
Residuals from **model A**	0.160	-0.154	0.000	**0.090**
Residuals from **model B**	0.110	-0.156	0.000	**0.075**

all values in m

Joint modeling of heterogeneous vertical velocities

By taking the time derivative of the (static) constraint

$$h - \zeta - H^{helm} = \frac{\bar{g}^{helm} - \bar{\gamma}}{\bar{\gamma}} H^{helm} + \frac{\delta W_o}{\bar{\gamma}}$$

(and after some lengthy derivations) we obtain the following condition

$$\dot{h} - \dot{\zeta} - \dot{H}^{helm} = \frac{H^{helm}}{\bar{\gamma}} \dot{g} + \frac{\delta \dot{W}_o}{\bar{\gamma}} + \ldots \text{negligible terms}$$

Joint modeling of heterogeneous vertical velocities

Effect of total gravity variation on the Earth's surface

Numerical effect in mm/yr

H^{helm} \ $\dot{g}$	0.1 μGal/yr	1.0 μGal/yr	10 μGal/yr
100 m	**10^{-5}**	**10^{-4}**	**10^{-3}**
1000 m	**10^{-4}**	**10^{-3}**	**10^{-2}**

$$\dot{h} - \dot{\zeta} - \dot{H}^{helm} = \frac{H^{helm}}{\bar{\gamma}}\dot{g} + \frac{\delta \dot{W}_o}{\bar{\gamma}} + \ldots \text{ negligible terms}$$

Joint modeling of heterogeneous vertical velocities

Required datum-related term !

Its role is similar to that of the shift-rate parameters in TRF velocity transformation

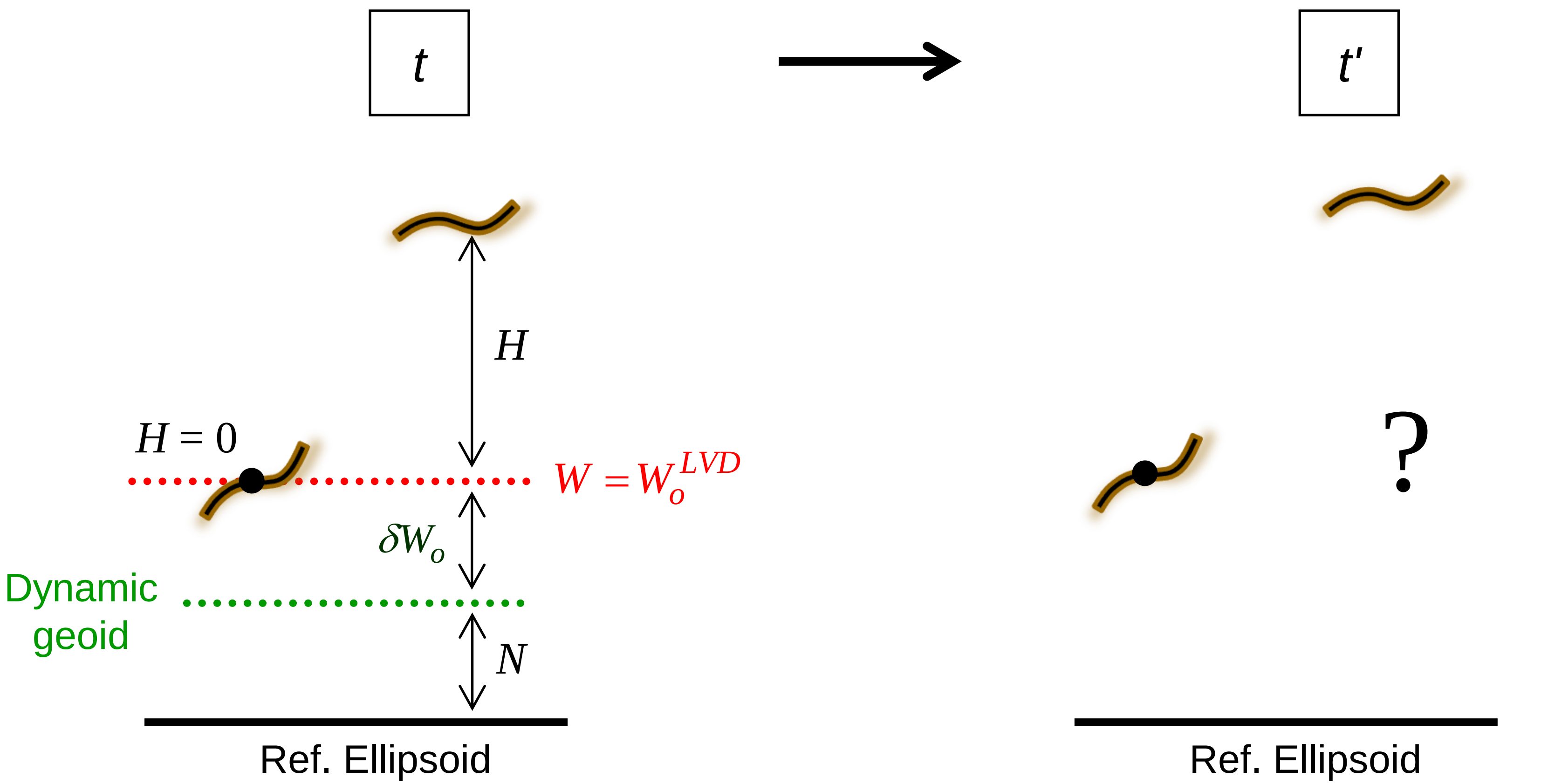
$$\mathbf{v}_x = \mathbf{v}'_x + \dot{\mathbf{t}}_x$$

It is zero provided that there is no-net-vertical-motion between the underlying VRFs !

$$\dot{h} - \dot{\zeta} - \dot{H}^{helm} = \frac{H^{helm}}{\bar{\gamma}}\dot{g} + \frac{\delta \dot{W}_o}{\bar{\gamma}} + \ldots \text{negligible terms}$$

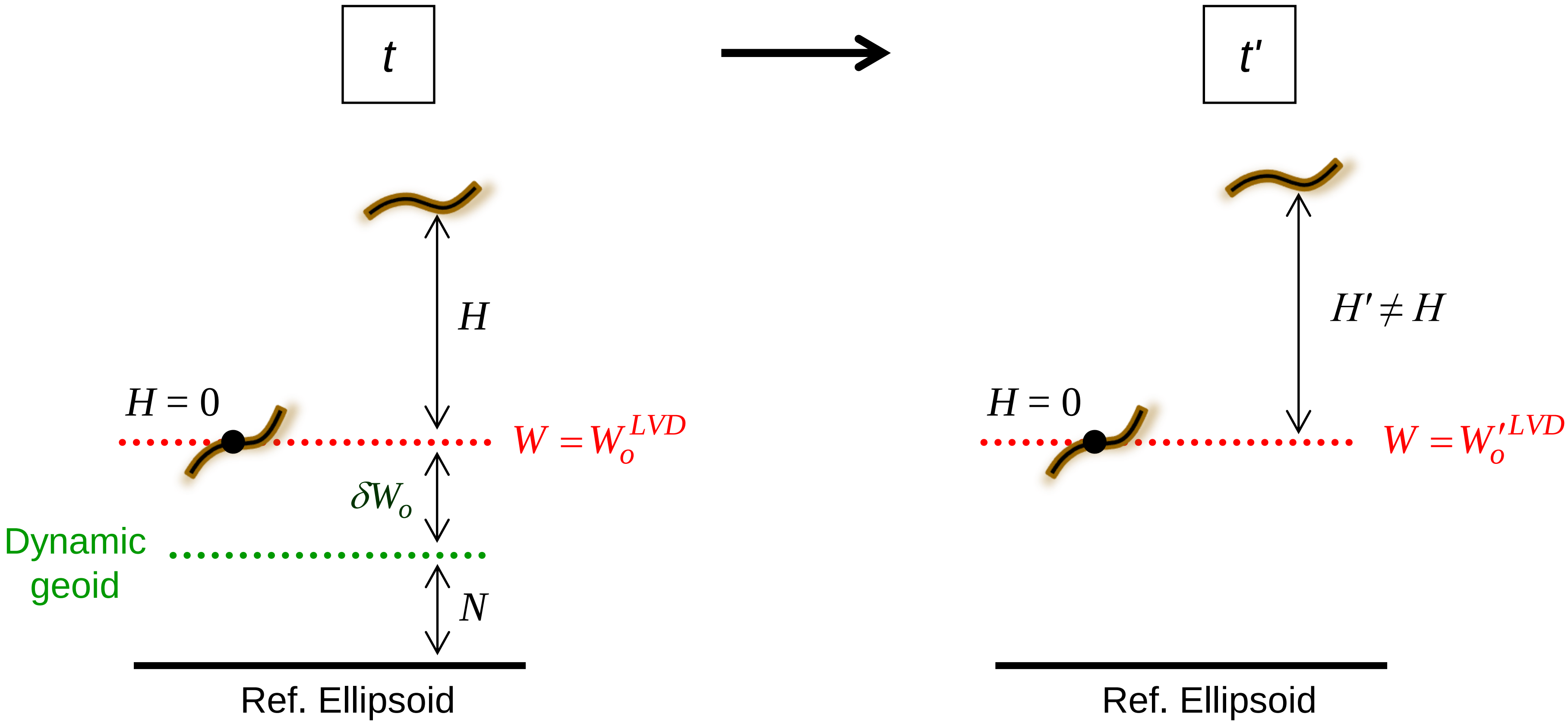
$$\dot{H}^{VRF2} - \dot{H}^{VRF1}$$

Example



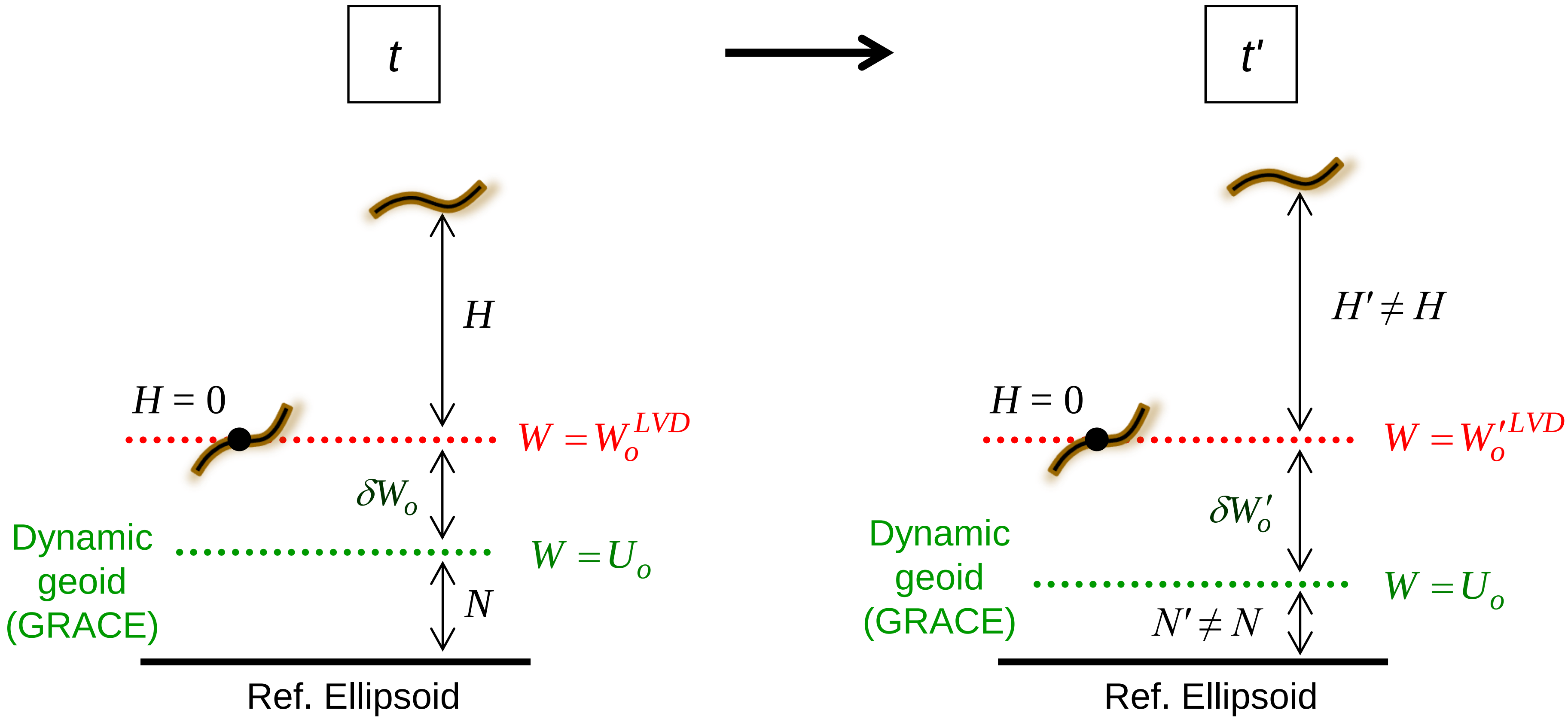
Vertical crustal motion + gravity field variation

Example



Vertical crustal motion + gravity field variation

Example



Vertical crustal motion + gravity field variation

The meaning of $\delta \dot{W}_o$

- It is associated with the **temporal evolution of the zero-height equipotential reference surfaces**

- It is affected by the realization of:
  - the dynamic vertical datum $\rightarrow$ $H^{helm}(t)$
  - the time-dependent geoid model $\rightarrow$ $N(t)$ or $\zeta(t)$

- It is a fundamental 'datum parameter' that needs to be a priori constrained when computing a dynamic VRF from the optimal combination of **multiple data sources**

A useful theoretical constraint

Based on the previous condition, we have

$$\frac{\dot{g}}{\dot{h}} = \frac{\dot{h} - \dot{\zeta} - \dot{H}^{helm}}{\dot{h}} \frac{\overline{\gamma}}{H^{helm}} - \left(\frac{\delta \dot{W}_o}{H^{helm}\, \dot{h}} \right)$$

which can be used for estimating the **gravity-to-height ratio** from heterogeneous vertical velocities

- Repeated gravity measurements are not required!
- Useful for validation of Earth models and comparison with geophysical predictions for various physical processes (e.g. PGR)

Conclusion – Future work

- A revision of the simplified model

$$h - N - H = 0 \qquad \dot{h} - \dot{N} - \dot{H} = 0$$

is necessary in the context of modern VRF theory and practice

- A general conventional re-formulation has been presented in this study

- Numerical tests with heterogeneous vertical velocities need to be performed (under our new formulation) over key areas, i.e. Canada, Fennoscandia

Thanks for your attention !