Estimation of the geopotential value Wo for the local vertical datum of continental Greece using EGM08 and GPS/leveling data

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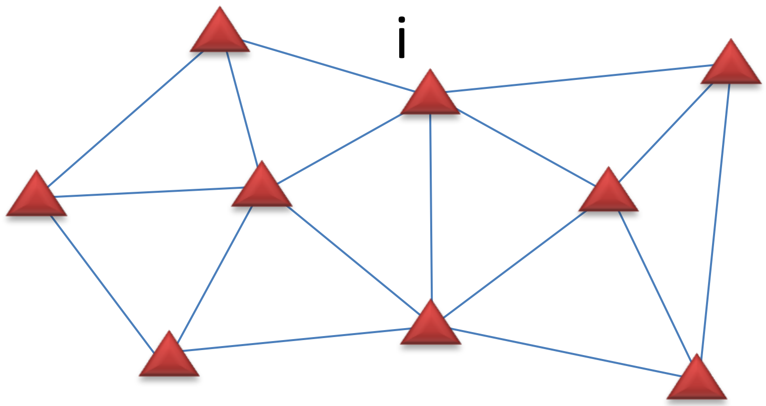
Estimate the zero-height geopotential value **Wo** for the local vertical datum of continental Greece

using a **methodology** based on **Helmert orthometric heights** and **geopotential models**

in order to allow the **connection** to other regional, continental and global **height systems**.

Terrestrial Network



Leveling BMs
$i = \{1, 2, \ldots, m\}$

Local Vertical Datum
$W = W_o$ (unknown)

H_i (orthometric heights – leveling)

ϕ_i, λ_i, h_i (spatial position – GNSS methods)

g_i, W_i (gravimetry and/or geopotential model)

→ W_o (estimated value)

Basic physical model (Helmert ortho heights):

$$H_i^{helm} = \frac{W_o^{\mathrm{LVD}} - W_i}{g_i + 0.0424 \cdot 10^{-5} H_i^{helm}} = \frac{W_o^{\mathrm{LVD}} - W_i}{\bar{g}_i^{helm}}$$

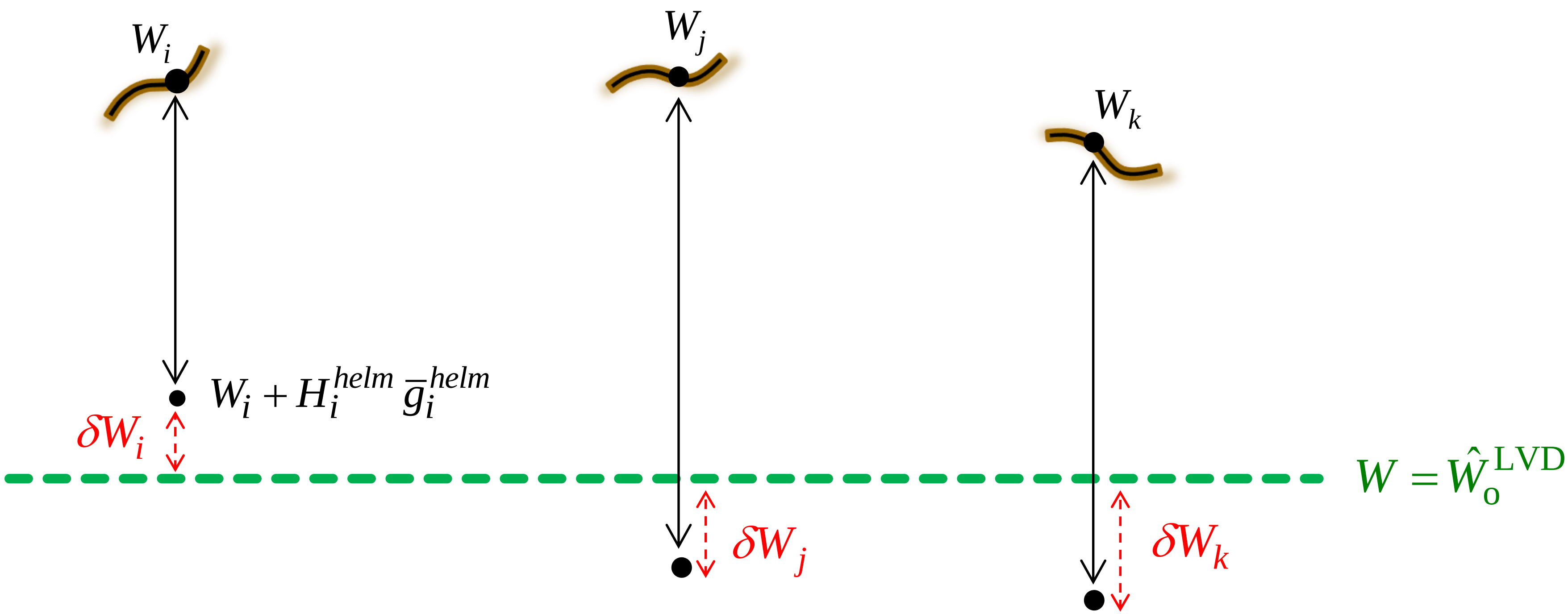
LS estimate of LVD's zero-height level

$$\hat{W}_o^{\mathrm{LVD}} = \frac{\sum_i p_i y_i}{\sum_i p_i}$$

$y_i \longrightarrow W_i + H_i^{helm}\,\bar{g}_i^{helm}$

$p_i \longrightarrow$ 'weights'

Method's rationale



Estimate the LVD zero-height level such that: $\sum_i p_i\, \delta W_i^2 = \min$

Method's advantages

- Does not rely on the use of geoid heights and thus it is not affected by 'geoid modeling errors'

- Robust with respect to the uncertainty of the surface gravity (g_i)

$$\sigma_{\hat{W}_o^{\mathrm{LVD}}} = \frac{\sqrt{\sum_i^m (H_i^{helm})^2}}{m} \sigma_g$$

<< 0.1 $m^2 s^{-2}$

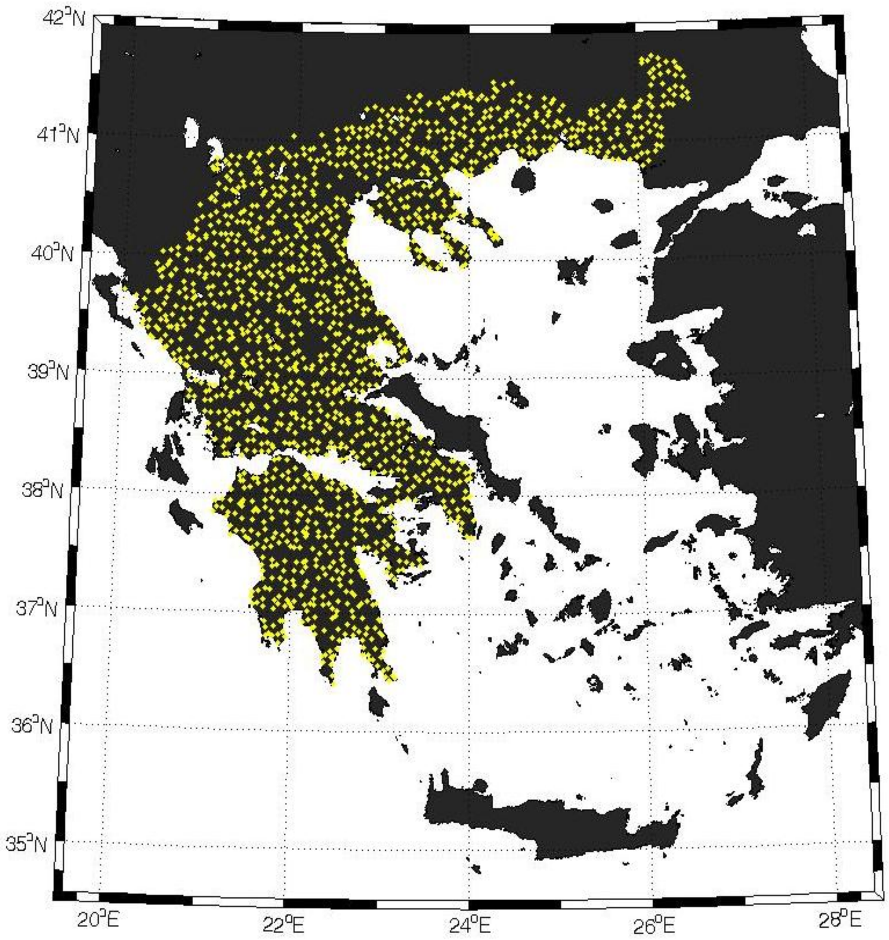
even for σ_g up to 10-20 mGal over mountainous areas

- The results can be compared with corresponding estimates from other techniques that may employ the same and/or different geopotential information

HELLENIC VERTICAL DATUM

- The Hellenic Vertical Datum was established by the Hellenic Geographic Military Service within the period 1963-1986
- In principle, physical heights in the HVD were modeled as Helmert orthometric heights
- An unknown (non-specified) W_o value is associated with the HVD
- The Helmert orthometric heights refer to the tide-gauge station located at the Piraeus port (Athens) – MSL measurements were performed over the period 1933-1978
- The true accuracy of the HVD's leveling network is largely unknown

AVAILABLE DATA



- 1542 GPS/lev BMs (*mainland*)
- X, Y, Z (ITRF00, t=2007.236)
  from HEPOS/GPS campaign
- Helmert ortho heights
  from Hellenic Geogr. Military Service
- Stations with identified blunders/outliers were removed
- GPS Accuracy (1σ): 1-4 cm (horiz)
  2-5 cm (vert)

- Mean-tide to tide-free conversion for orthometric heights using:

$$H_{TF} - H_{MT} = \left(29.6\sin^2\varphi - 9.9\right)\gamma \ [cm]$$

- Computation of surface gravity at each benchmark using:

$$g_{\mathrm{BM}} = \gamma_{\mathrm{BM}} - \frac{\partial T}{\partial r}$$ computed from EGM08

- Computation of surface geopotential values W_i from EGM08

All SHS computations were carried out in a tide-free system using Pavlis-Holmes software

Un-weighted LS estimate

$$\hat{W}_o^{\mathrm{LVD}} = 62636859.37 \pm 0.04 \ \mathrm{m^2/s^2}$$

Weighted LS estimate $(p_i = 1/H_i^{helm})$

$$\hat{W}_o^{\mathrm{LVD}} = 62636860.16 \pm 0.03 \ \mathrm{m^2/s^2}$$

Difference: $\delta\hat{W}_o^{\mathrm{LVD}} = 0.79 \ \mathrm{m^2/s^2} \approx 8 \ \mathrm{cm}$!

$\hat{W}_o^{\mathrm{LVD}} = 62636859.44 \ \mathrm{m^2/s^2}$ (from *Sima et al., EUREF 2009*)

Height threshold for used BMs	$\hat{W}_{o}^{\mathrm{LVD}}$ Un-weighted	$\hat{W}_{o}^{\mathrm{LVD}}$ Weighted $(p_i = 1/H_i^{helm})$	Difference
< 200 m, 514 pts	62636860.04	62636860.20	~ 1.6 cm
< 500 m, 866 pts	59.90	60.19	~ 3.0 cm
< 1000 m, 1308 pts	59.65	60.17	~ 5.3 cm
< 1500 m, 1487 pts	59.45	60.17	~ 7.3 cm
< 2000 m, 1535 pts	59.37	60.16	~ 8.1 cm

Evidence of a height-correlated bias in the data

	$\hat{W}_o^{LVD}$		Difference
	Un-weighted	Weighted $(p_i = 1/H_i^{helm})$	
< 200 m, 514 pts	62636860.04	62636860.20	~ 1.6 cm
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NUMERICAL RESULTS

Evidence of a height-correlated bias in the data

More robust estimates due to data weighting

	$\hat{W}_{o}^{\mathrm{LVD}}$		
	Un-weighted	Weighted $(p_i = 1/H_i^{helm})$	Difference
< 200 m, 514 pts	62636860.04	62636860.20	~ 1.6 cm
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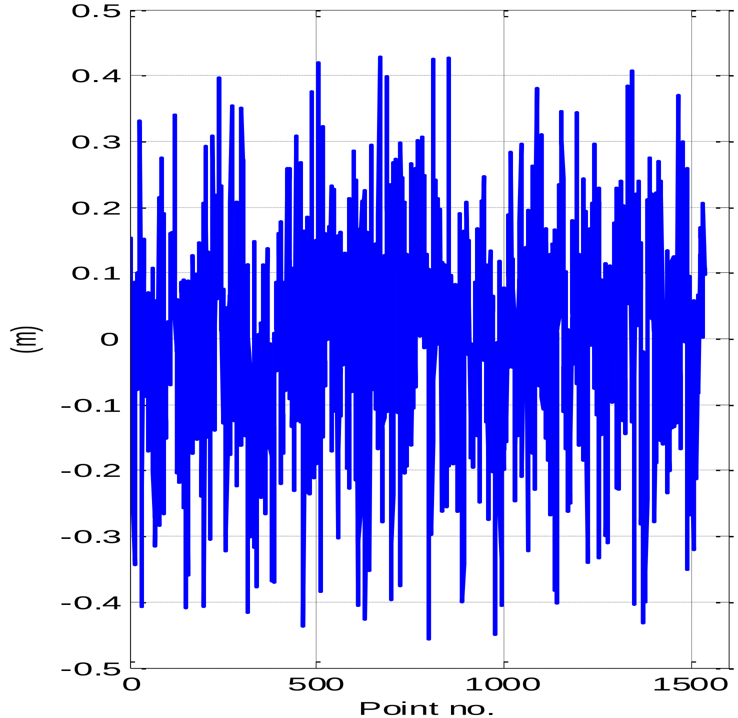
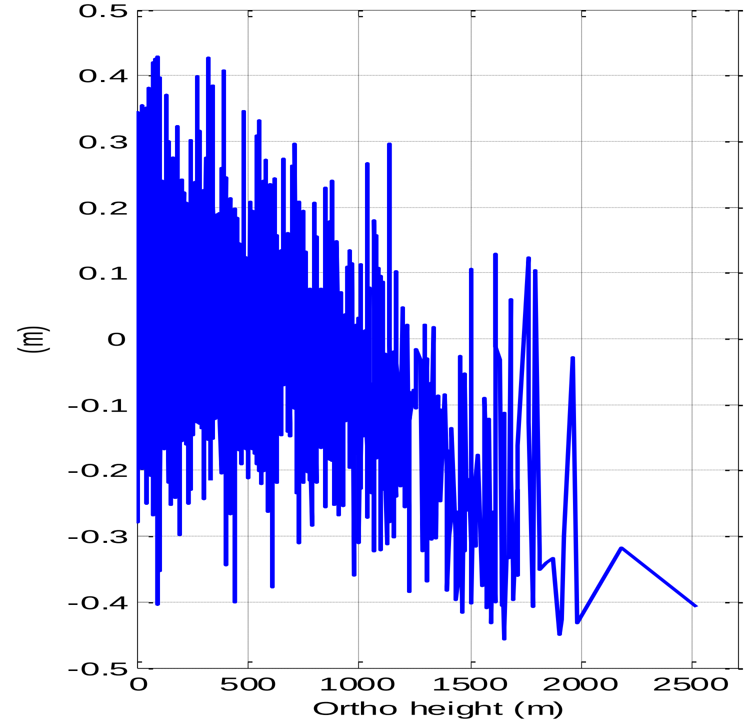
W_o-reduced Helmert ortho heights

(height residuals from unweighted LSA)

$$e_i = H_i^{helm} - \frac{\hat{W}_o^{LVD} - W_i}{\bar{g}_i^{helm}} \qquad \hat{W}_o^{LVD} = 62636859.37 \text{ m}^2/\text{s}^2$$





max = 0.429 m **min** = -0.456 m **mean** = 0.000 m **σ** = 0.150 m

Revised model

(considering height-correlated data errors)

$$H_i^{helm} = \frac{W_o^{LVD} - W_i}{g_i + 0.0424 H_i^{helm}} + \lambda H_i^{helm}$$

LSA schemes	$\hat{W}_o^{LVD}$	$\hat{\lambda}$
Un-weighted	62636860.30 ± 0.05	(-1.882 ± 0.073) × 10^{-4}
Weighted $p_i = (1/H_i^{helm})^{1/2}$	60.28 ± 0.04	(-1.832 ± 0.095) × 10^{-4}
Weighted $p_i = (1/H_i^{helm})$	60.23 ± 0.03	(-1.725 ± 0.221) × 10^{-4}
Weighted $p_i = (1/H_i^{helm})^2$	60.12 ± 0.01	(1.339 ± 3.660) × 10^{-4}

W_o-reduced Helmert ortho heights

(height residuals after LSA)

$$e_i = H_i^{helm} - \frac{\hat{W}_o^{\mathrm{LVD}} - W_i}{g_i + 0.0424 H_i^{helm}} - \hat{\lambda} H_i^{helm}$$

	Un-weighted	Weighted $p_i = (1/H_i^{helm})^{1/2}$	Weighted $p_i = (1/H_i^{helm})$	Weighted $p_i = (1/H_i^{helm})^2$
Max	0.415	0.412	0.405	0.340
Min	-0.481	-0.479	-0.474	-0.821
Mean	0.000	0.000	0.000	-0.144
Std	0.125	0.125	0.126	0.188

'Apparent bias' in $\hat{W}_o^{\mathrm{LVD}}$ due to EGM08 long-wavel. errors

$$\delta\hat{W}_o^{\mathrm{LVD}} = \sqrt{\sum_{n=2}^{n^*} \sigma_e^2(V_n)}$$

where

$$V = \frac{GM}{r} + \sum_{n=2}^{\infty} V_n \qquad \sigma_e^2(V_n) = \left(\frac{GM}{a}\right)^2 \sum_{m=0}^{n} (\sigma_{\bar{C}_{nm}}^2 + \sigma_{\bar{S}_{nm}}^2)$$

and n^* depends on the extent of the local test area ($n^* << 180/\Delta\sigma$)

For our study:

$\Delta\sigma = 5.5°\times 6°$ *Selected* $n^* \approx 15$ (~ 1300 km, half-wavelength)

'W_o *bias*' ≈ 3.5 cm

Based on the previous results, **our final estimate** for the zero-height level of the Hellenic Vertical Datum is:

$$\hat{W}_o^{LVD} = 62636860.30 \pm 0.40 \ \ m^2/s^2$$

- ✓ An estimate for the W_o of the Hellenic Vertical Datum was determined from Helmert orthometric heights and an ultra-high resolution geopotential model (EGM08)
- ✓ A series of LS adjustment tests using (i) empirical height-dependent weighting and (ii) an extended parametric model, was necessary to account for the systematic part of the data errors
- ➢ Future work involves the estimation of W_o using other techniques and datasets including geoid heights and GOCE based geopotential models

Thanks for your attention !