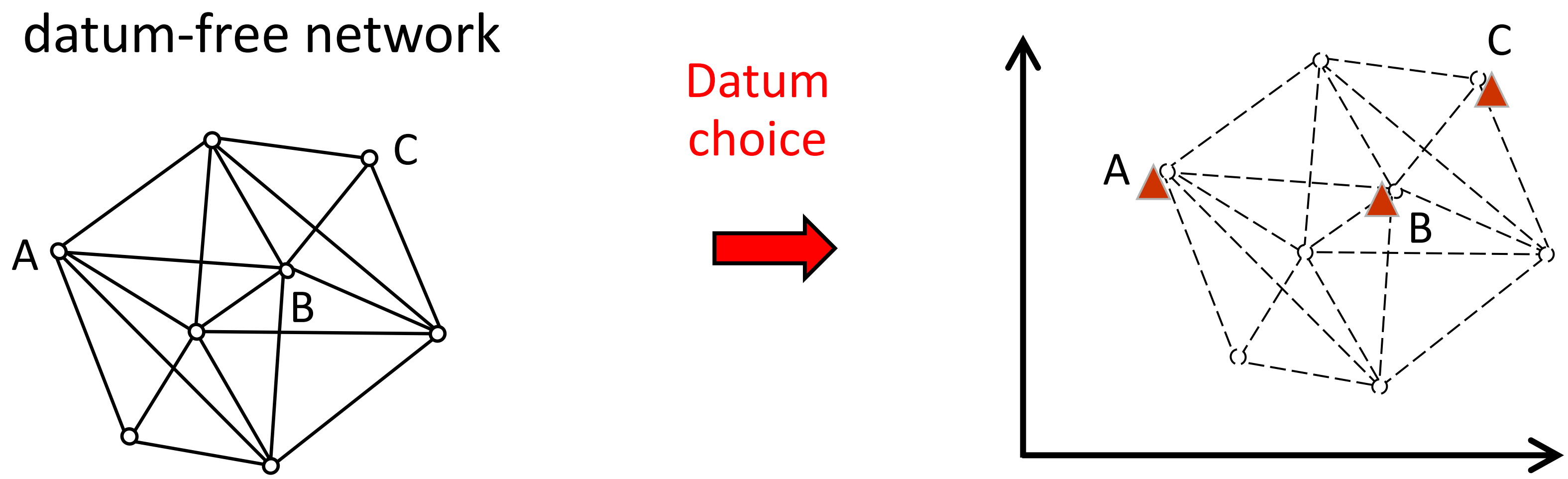
Weighted vs. un-weighted MCs for the datum definition in regional networks

M. Chatzinikos and C. Kotsakis

Department of Geodesy and Surveying
Aristotle University of Thessaloniki
Thessaloniki, Greece

IAG Commission I Symposium
"Reference Frames for Applications in Geosciences" (REFAG 2014)
Kirchberg, Luxemburg, October 13-17, 2014

Introduction



Minimal constraints on reference stations

$$\mathbf{E}(\mathbf{x}-\mathbf{x}^{\mathrm{ref}}) = \mathbf{0}$$ or, more generally $$\mathbf{E}\mathbf{P}(\mathbf{x}-\mathbf{x}^{\mathrm{ref}}) = \mathbf{0}$$

what we consider here: choice of the weight matrix P

Example

Classic form of NNT/NNR conditions	$\sum_i (\mathbf{x}_i - \mathbf{x}_i^{\text{ref}}) = \mathbf{0}$ $\sum_i \mathbf{x}_i^{\text{o}} \times (\mathbf{x}_i - \mathbf{x}_i^{\text{ref}}) = \mathbf{0}$
Weighted form of NNT/NNR conditions	$\sum_i p_i (\mathbf{x}_i - \mathbf{x}_i^{\text{ref}}) = \mathbf{0}$ $\sum_i \mathbf{x}_i^{\text{o}} \times p_i (\mathbf{x}_i - \mathbf{x}_i^{\text{ref}}) = \mathbf{0}$

Simplified scheme: diagonal weight matrix with a single scalar weight for each reference station

Frame optimality in classic (un-weighted) MC adjustment

- The realized frame of the adjusted network is optimized at the stations participating in the MCs

  (what about the other network stations?)

- The optimality of the realized frame considers only the data noise effect in the estimated coordinates

  (what about the “datum noise” effect?)

What do classic MCs optimize?

Rank-deficient NEQs: $\mathbf{N}\begin{bmatrix}\boldsymbol{\delta}\mathbf{x} \\ \boldsymbol{\delta}\mathbf{x}'\end{bmatrix} = \mathbf{u}$ — reference stations ($\boldsymbol{\delta}\mathbf{x}$)

MCs applied to reference stations: $\mathbf{E}(\mathbf{x}-\mathbf{x}^{\text{ref}}) = \mathbf{0}$

Covariance matrix of MC solution:

$$\boldsymbol{\Sigma} = \mathbf{N}^{-} = \left[\begin{array}{c|c}\boldsymbol{\Sigma}_{\hat{\mathbf{x}}} & \boldsymbol{\Sigma}_{\hat{\mathbf{x}}\hat{\mathbf{x}}'} \\ \hline \boldsymbol{\Sigma}_{\hat{\mathbf{x}}'\hat{\mathbf{x}}} & \boldsymbol{\Sigma}_{\hat{\mathbf{x}}'}\end{array}\right]$$

$\boldsymbol{\Sigma}_{\hat{\mathbf{x}}}$ → Minimum trace

$\mathbf{N}^{-}$ ↑ Data noise effect

Minimization of data noise effect only at the reference stations!

What can weighted MCs optimize?

$$\mathbf{N}\begin{bmatrix}\boldsymbol{\delta}\mathbf{x}\\ \boldsymbol{\delta}\mathbf{x}'\end{bmatrix} = \mathbf{u}$$

reference stations

$$\mathbf{E}\mathbf{P}(\mathbf{x}-\mathbf{x}^{\mathrm{ref}}) = \mathbf{0}$$

Minimization of data noise over *any station group*

$$\boldsymbol{\Sigma} = \mathbf{N}^{-} = \begin{bmatrix}\boldsymbol{\Sigma}_{\hat{\mathbf{x}}} & \boldsymbol{\Sigma}_{\hat{\mathbf{x}}\hat{\mathbf{x}}'}\\ \boldsymbol{\Sigma}_{\hat{\mathbf{x}}'\hat{\mathbf{x}}} & \boldsymbol{\Sigma}_{\hat{\mathbf{x}}'}\end{bmatrix}$$

minimum trace

Minimization of data+datum noise over *any station group*

$$\boldsymbol{\Sigma} = \mathbf{N}^{-} + \boldsymbol{\Sigma}^{\mathrm{ref}} = \begin{bmatrix}\boldsymbol{\Sigma}_{\hat{\mathbf{x}}} & \boldsymbol{\Sigma}_{\hat{\mathbf{x}}\hat{\mathbf{x}}'}\\ \boldsymbol{\Sigma}_{\hat{\mathbf{x}}'\hat{\mathbf{x}}} & \boldsymbol{\Sigma}_{\hat{\mathbf{x}}'}\end{bmatrix}$$

minimum trace

Data noise effect ($\mathbf{N}^{-}$)

Datum noise effect ($\boldsymbol{\Sigma}^{\mathrm{ref}}$)

Datum choice problem

Rank-deficient NEQs

$$\mathbf{N}\begin{bmatrix} \boldsymbol{\delta}\mathbf{x} \\ \boldsymbol{\delta}\mathbf{x}' \end{bmatrix} = \mathbf{u}$$

reference stations

Arbitrary MCs

$$\mathbf{H}(\mathbf{x}-\mathbf{x}^{\mathrm{ref}}) = \mathbf{0}$$

Optimization problem to be solved

$$\min_{\mathbf{H}} \ trace \ \mathbf{S}\left[\begin{array}{c|c} \boldsymbol{\Sigma}_{\hat{\mathbf{x}}} & \boldsymbol{\Sigma}_{\hat{\mathbf{x}}\hat{\mathbf{x}}'} \\ \hline \boldsymbol{\Sigma}_{\hat{\mathbf{x}}'\hat{\mathbf{x}}} & \boldsymbol{\Sigma}_{\hat{\mathbf{x}}'} \end{array}\right]\mathbf{S}^T$$

Total CV matrix of MC solution

where **S** is a "station selection" matrix

Problem solution

Frame/network optimality principle

$$\min_{\mathbf{H}} \; trace \; \mathbf{S} \begin{bmatrix} \boldsymbol{\Sigma}_{\hat{\mathbf{x}}} & \boldsymbol{\Sigma}_{\hat{\mathbf{x}}\hat{\mathbf{x}}'} \\ \boldsymbol{\Sigma}_{\hat{\mathbf{x}}'\hat{\mathbf{x}}} & \boldsymbol{\Sigma}_{\hat{\mathbf{x}}'} \end{bmatrix} \mathbf{S}^T$$

Optimal MC matrix (applied to reference stations)

$$\mathbf{H} = \mathbf{E}\left(\mathbf{Q}_{\mathbf{x}} + \boldsymbol{\Sigma}_{\mathbf{x}^{\mathrm{ref}}}\right)^{-1}$$

optimal weight matrix

where:

$$(\mathbf{N} + \mathbf{S}^T\mathbf{S}\tilde{\mathbf{E}}^T\tilde{\mathbf{E}}\mathbf{S}^T\mathbf{S})^{-1} = \begin{bmatrix} \mathbf{Q}_{\mathbf{x}} & \# \\ \# & \# \end{bmatrix}$$

inner-constraint matrix
for the entire network ($\mathbf{N}\tilde{\mathbf{E}}^T = \mathbf{0}$)

(*) *see Kotsakis (2013, JGeod)*

Numerical tests

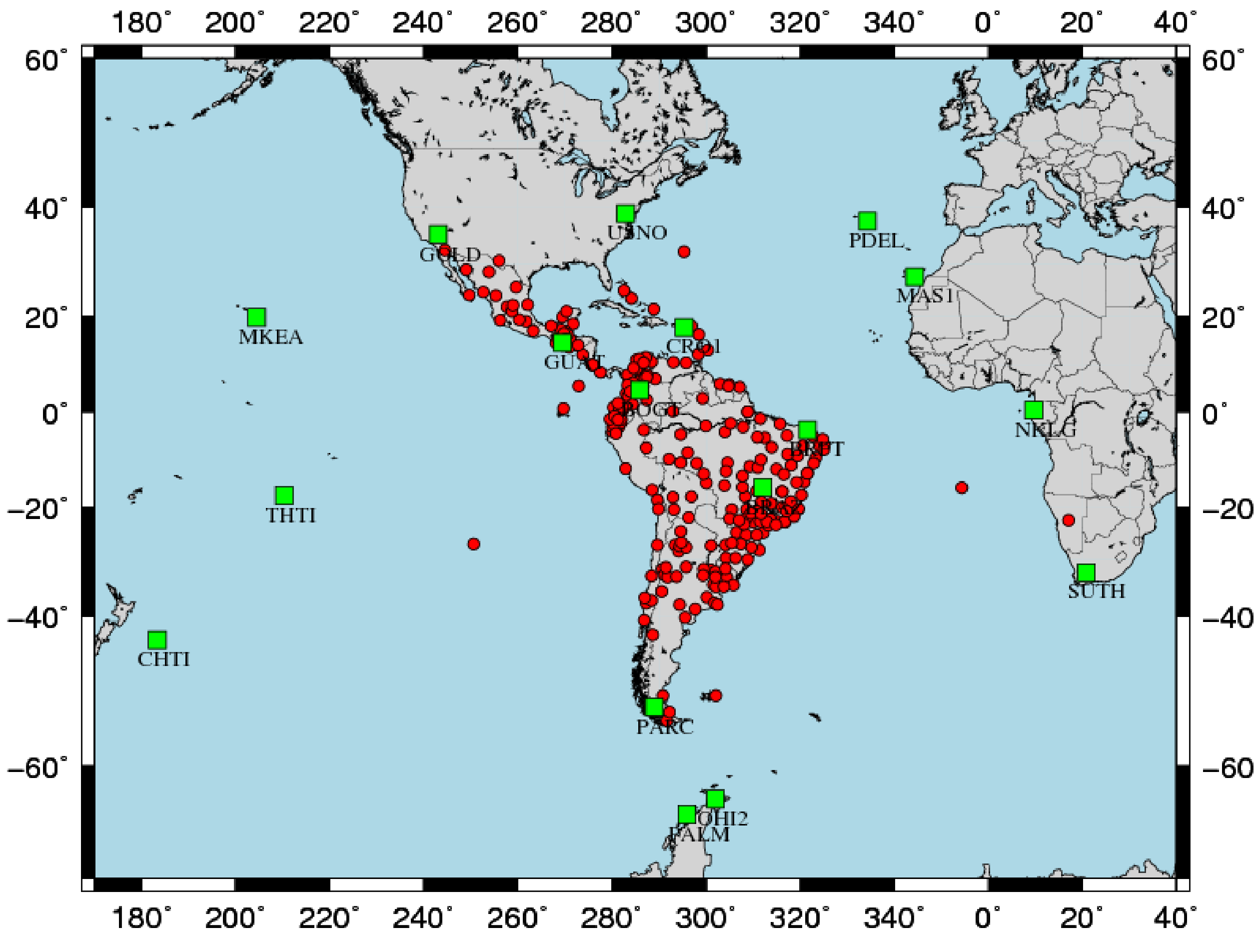
- Compare optimally weighted & un-weighted weekly MC solutions (NNT alignment to ITRF08) in various regional GNSS networks

- Evaluation scheme based on:
  - Average (per week) accuracy improvement for the estimated coordinates
  - RMS (per week) of the coordinate differences between the corresponding MC solutions
  - Coordinate time series behavior

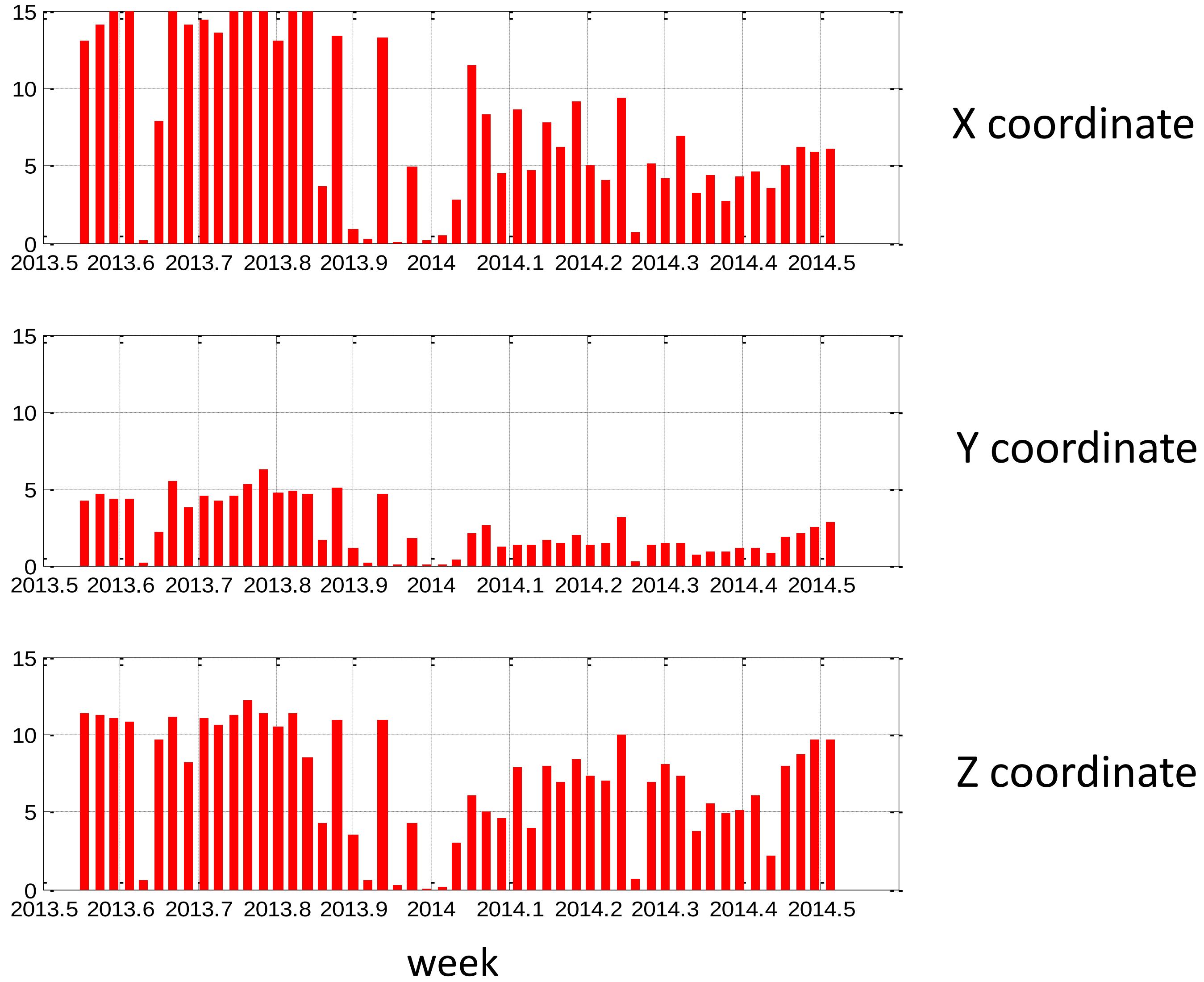
SIRGAS network

18 REF Stations, 256 NEW Stations

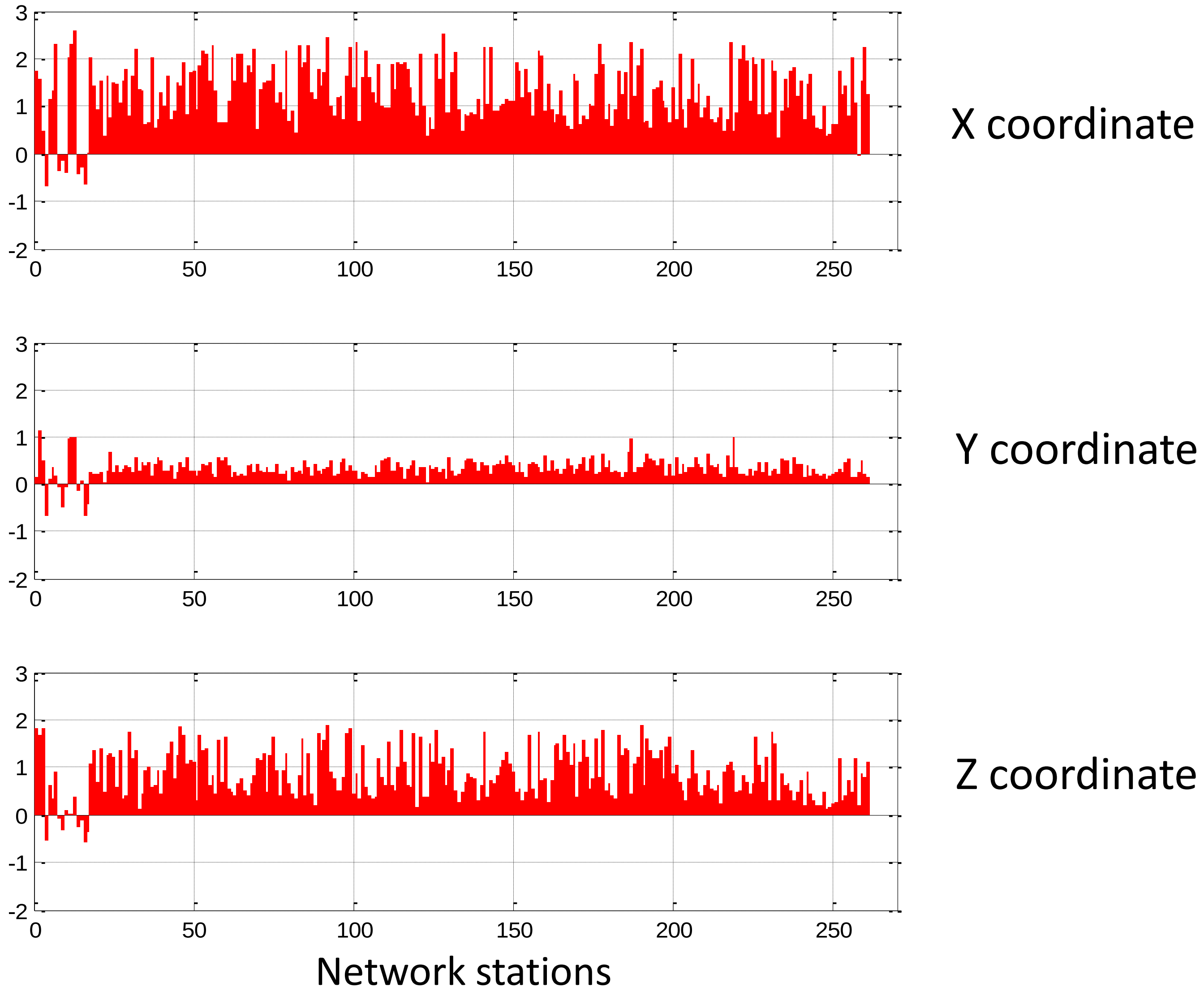
51 weekly solutions (2013-2014)



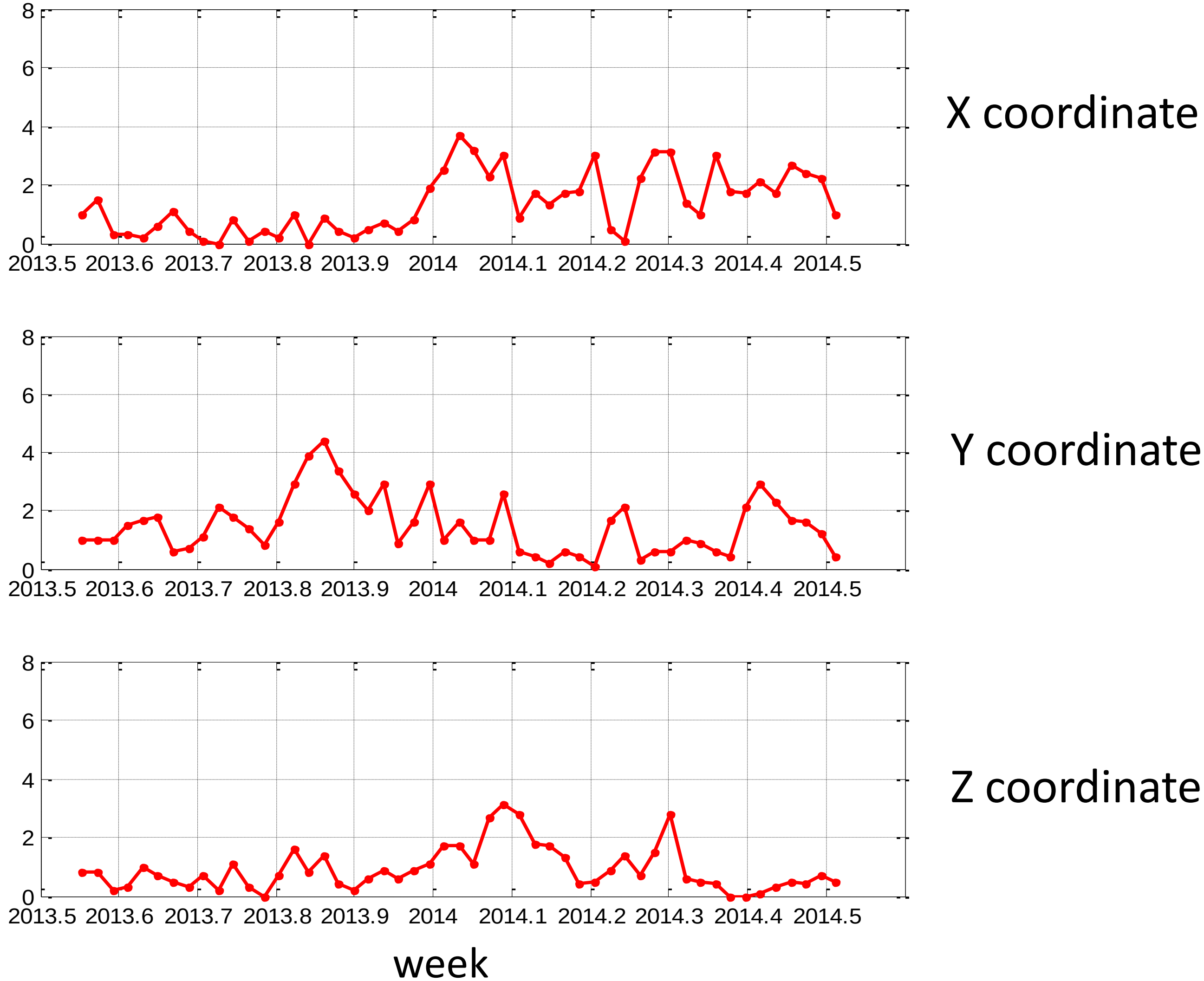
Avg accuracy improvement (%) for each weekly solution



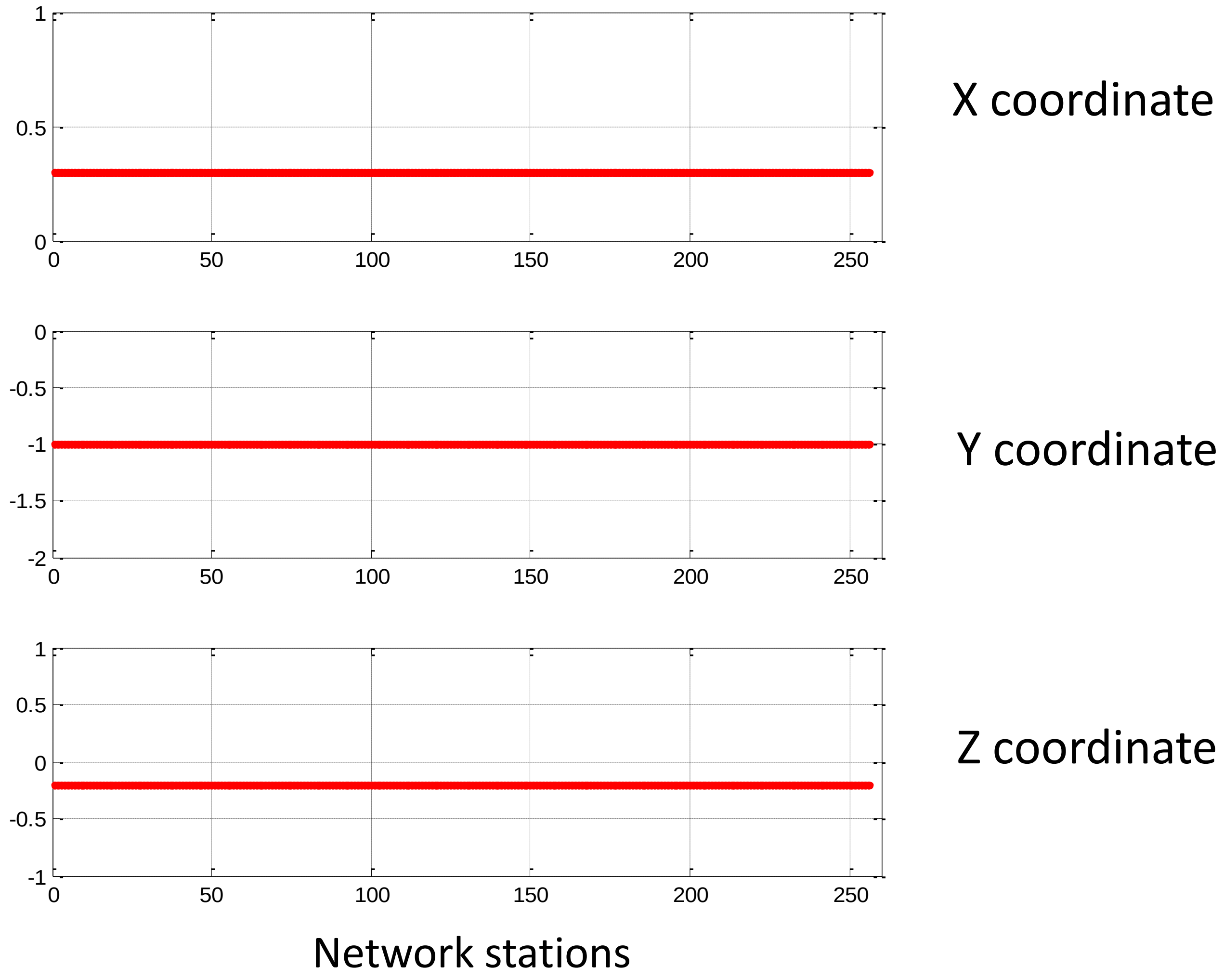
Coordinate sigma improvement (mm) for week 1752



Weekly RMS (mm) of the coordinate differences btw weighted and un-weighted MC solutions



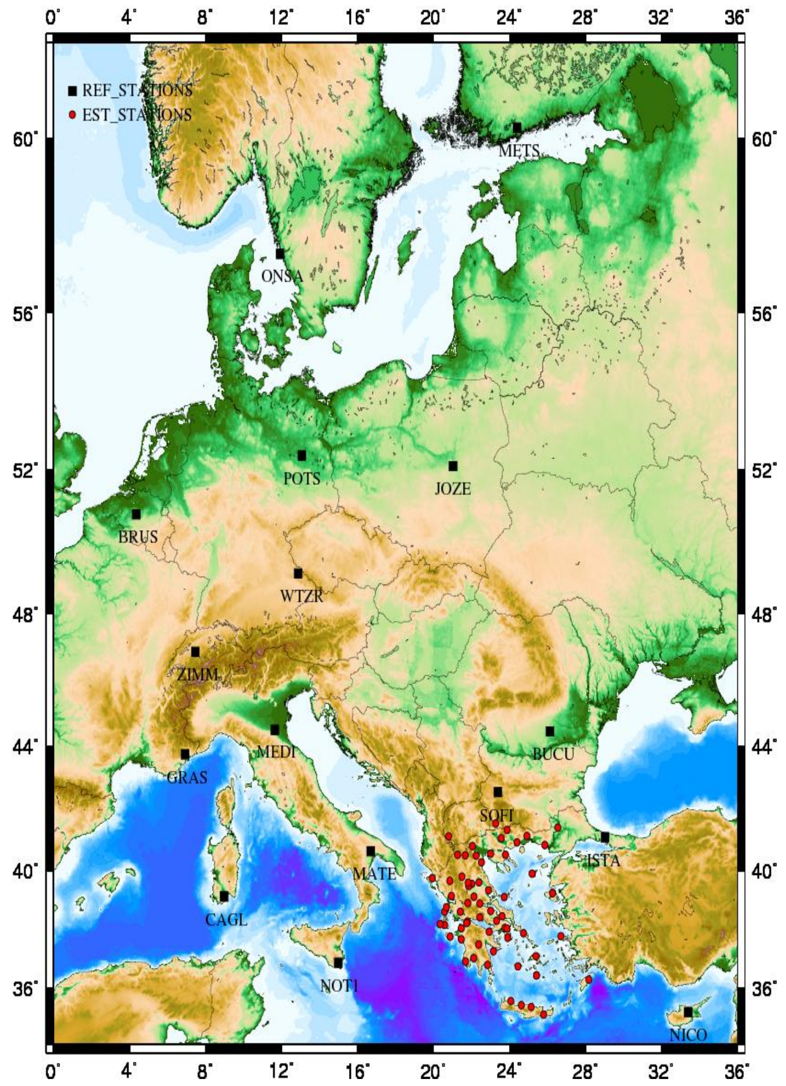
Coordinate differences (mm) between weighted and un-weighted MC solutions for GPS week 1752



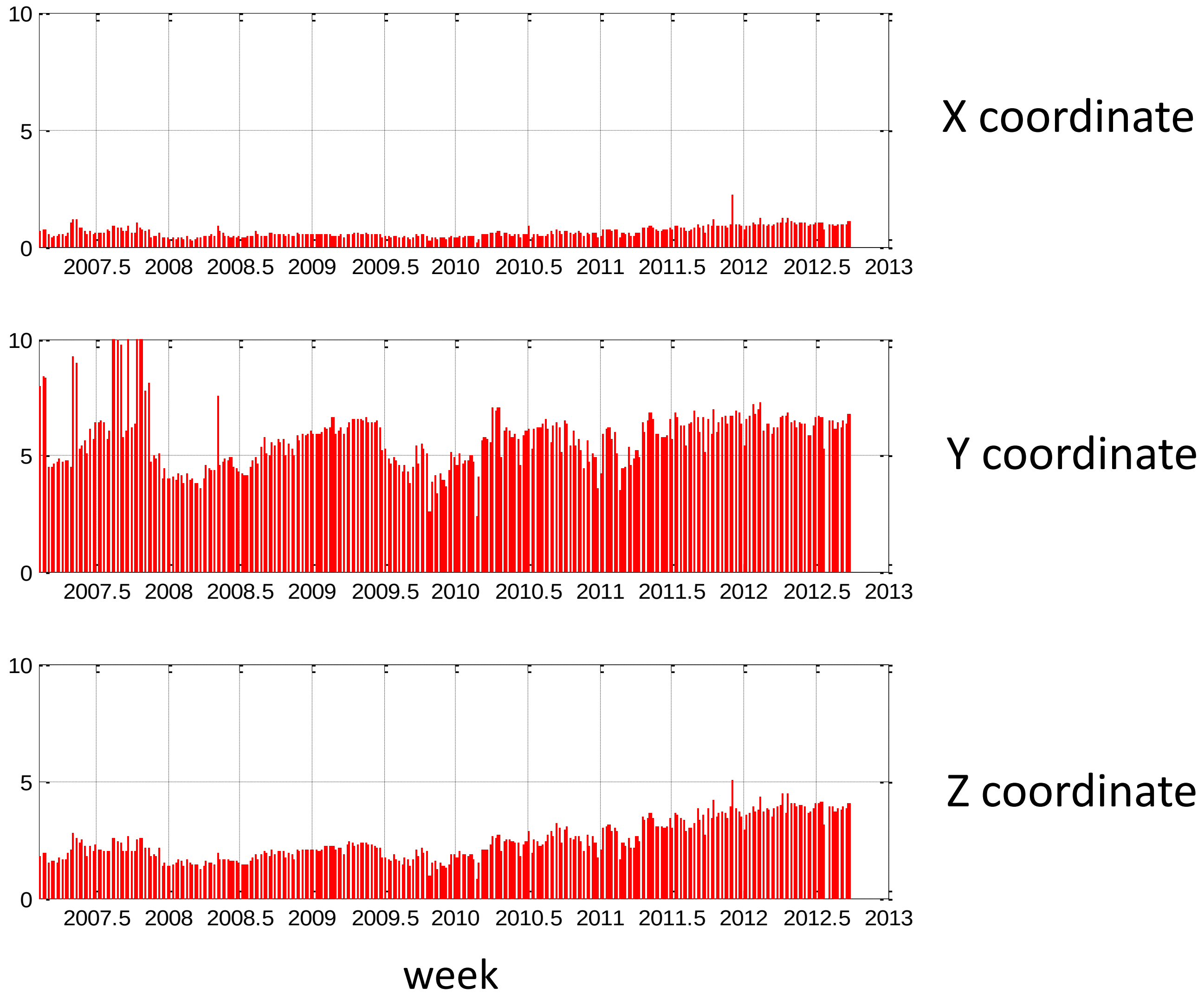
Hellenic network

16 REF Stations, 68 NEW Stations

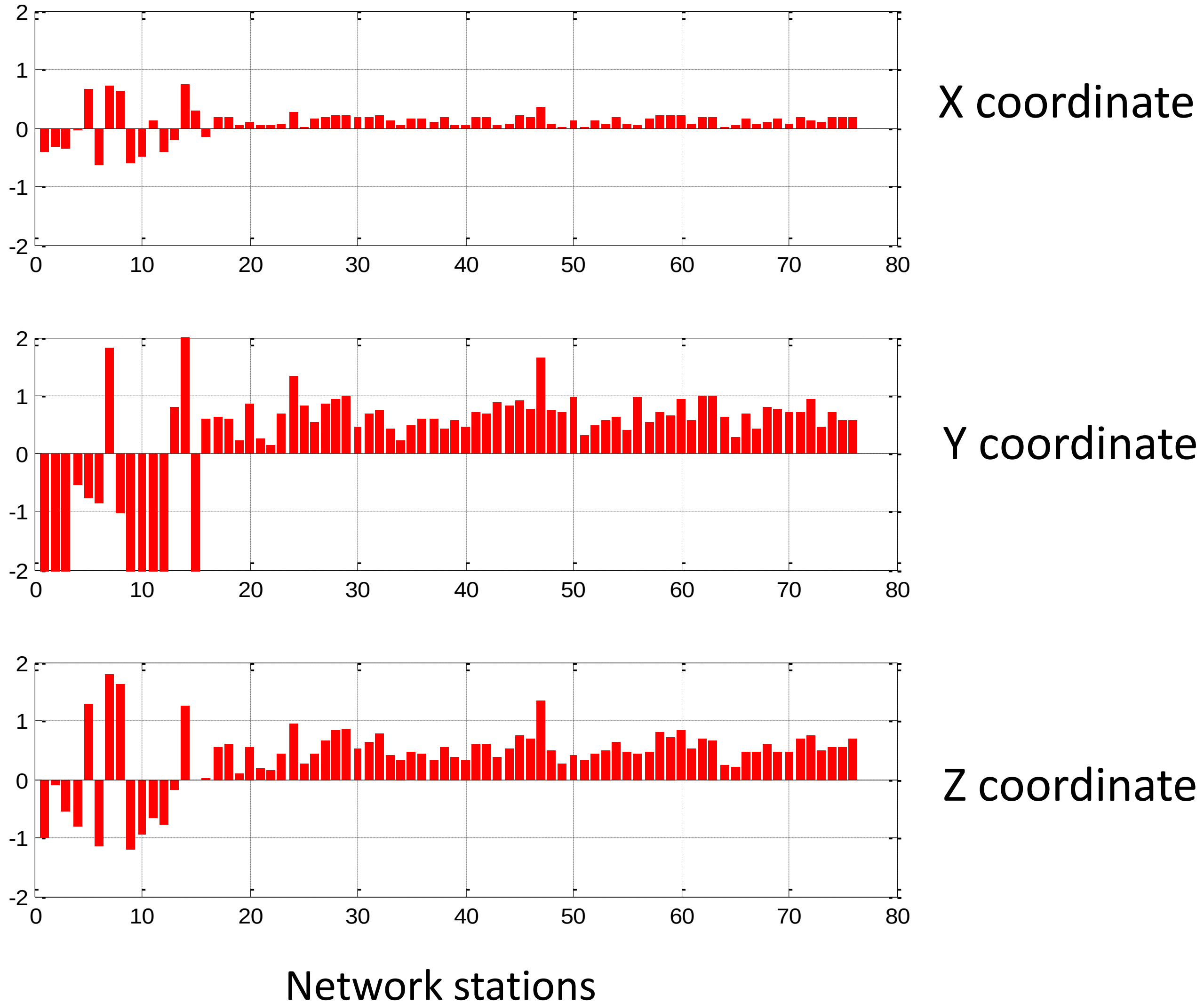
300 weekly solutions (2007-2013)



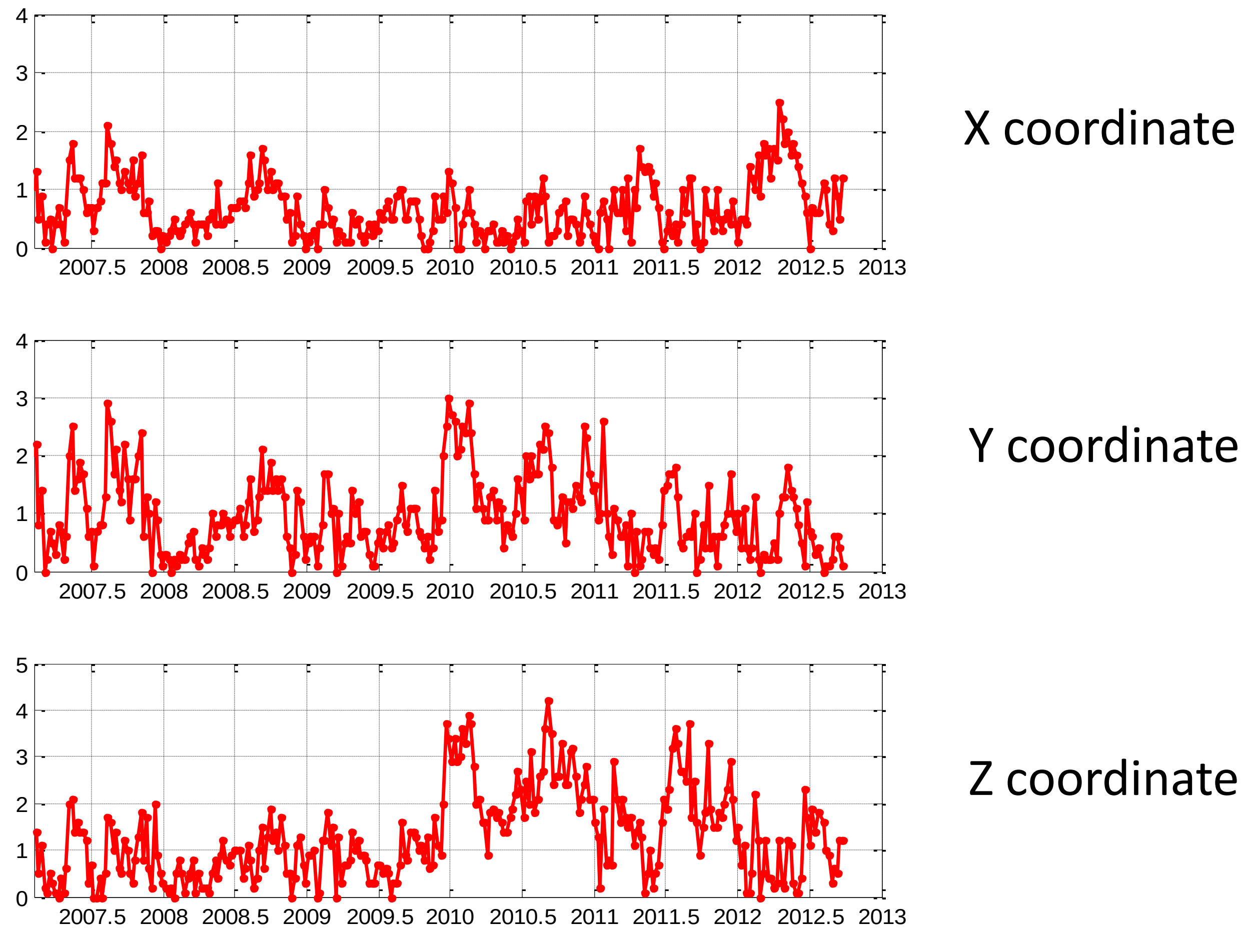
Avg accuracy improvement (%) for each weekly solution



Coordinate sigma improvement (mm) for week 1669

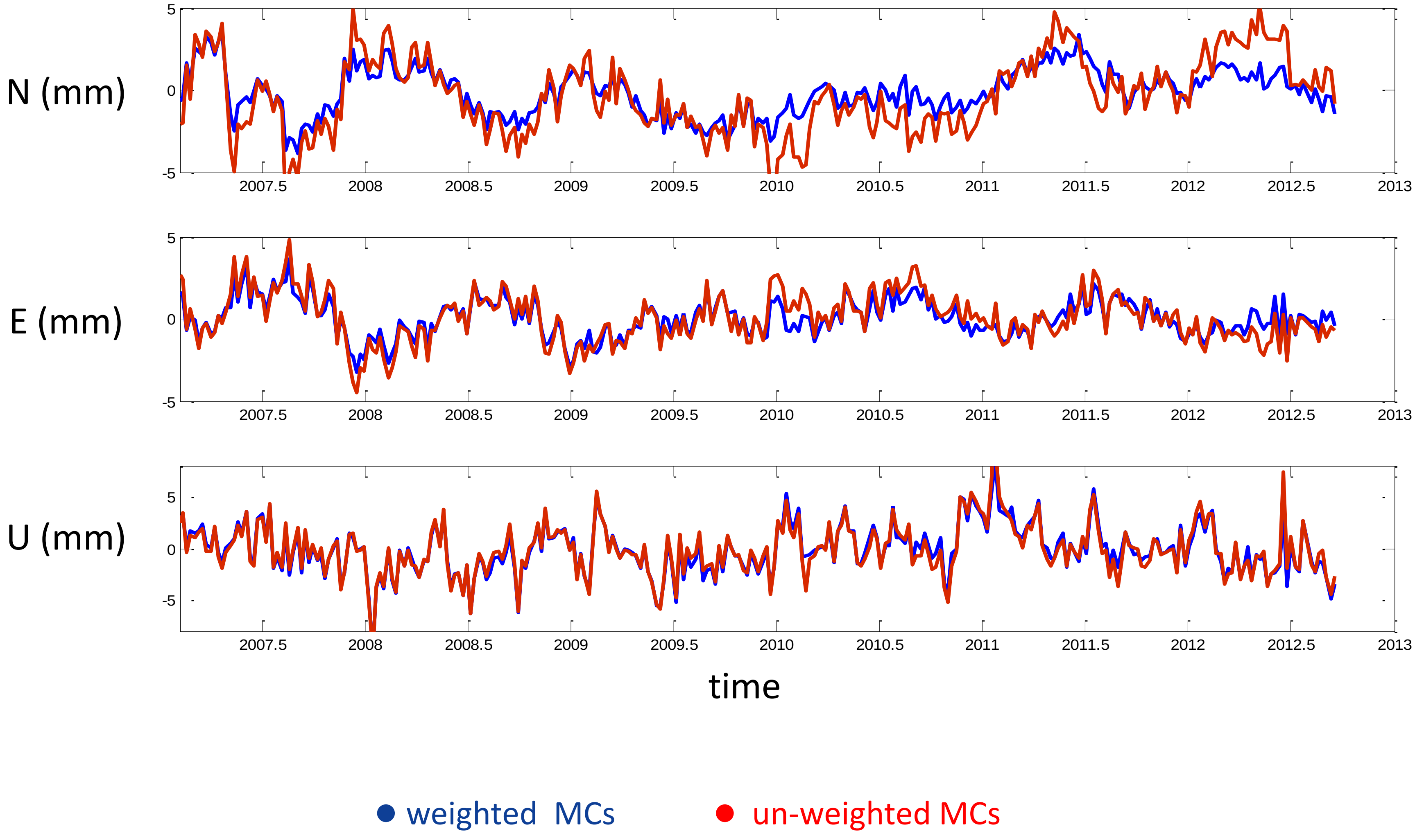


Weekly RMS (mm) of the coordinate differences btw weighted and un-weighted MC solutions

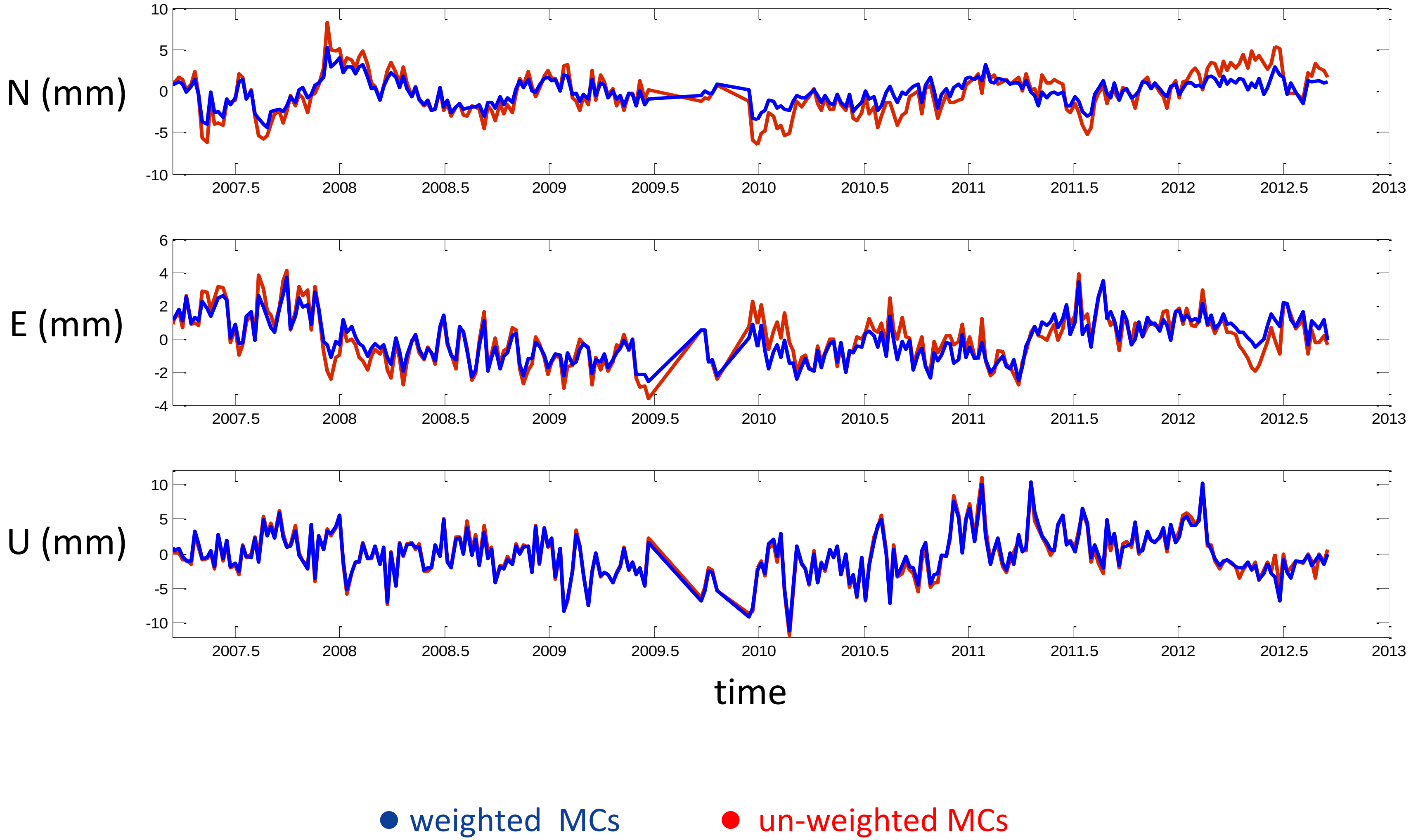


week

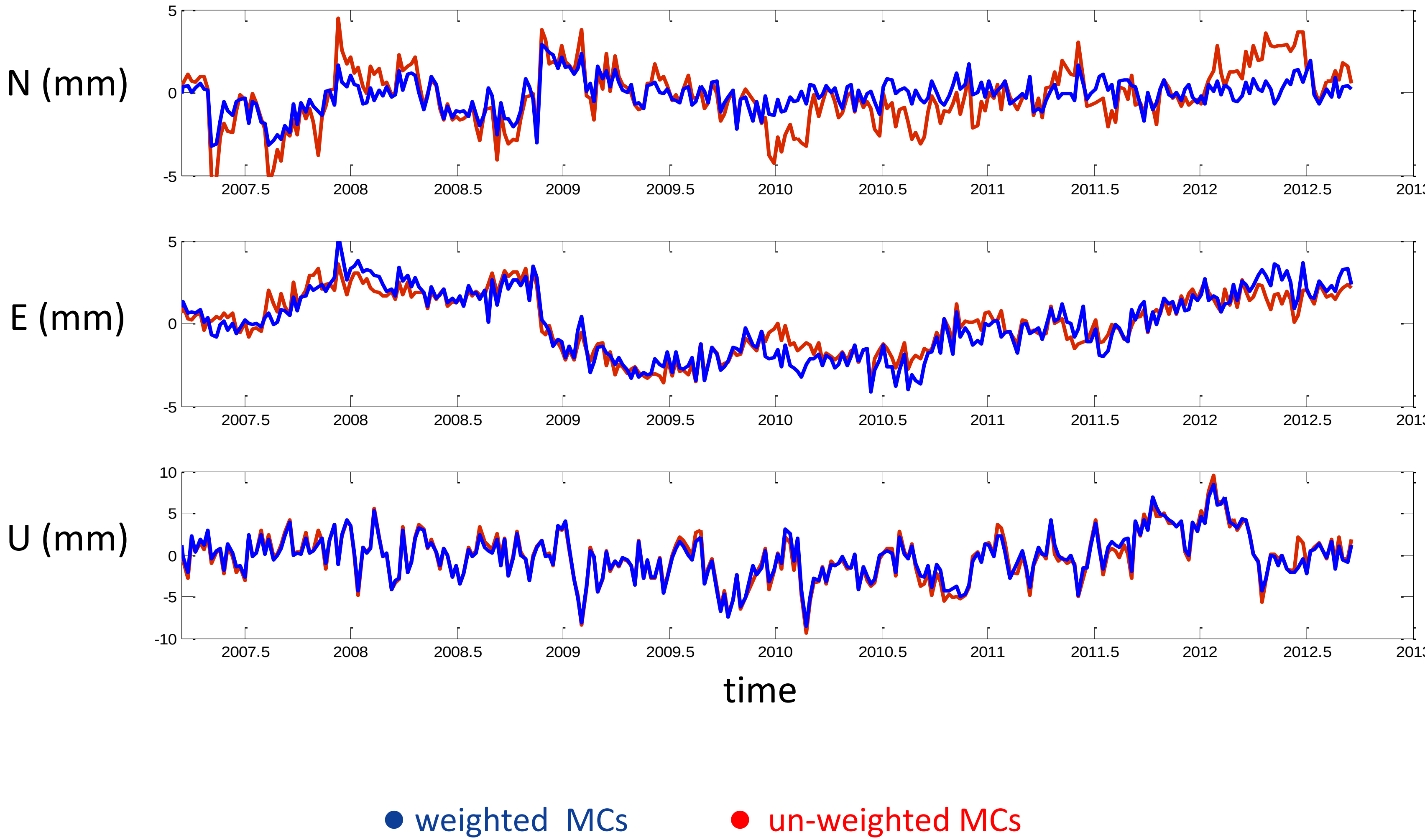
Coordinate time series for station AUT1 (linear trend removed)



Coordinate time series for station MET0
(linear trend removed)



Coordinate time series for station MATE (linear trend removed)



Conclusions

- Reference station weighting (within the MCs) can lead to different types of frame optimality

- Reference station weighting can be used to optimize the accuracy of a MC solution in terms of

  - the data and datum noise effects
  - the network stations over which these effects are considered

- More detailed numerical testing will be presented in a forthcoming paper