Comparison of frame alignment strategies in GNSS coordinate time series

M. Chatzinikos and C. Kotsakis

Department of Geodesy and Surveying
Aristotle University of Thessaloniki
Thessaloniki, Greece

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Objective:

The comparison of four different **ITRF-alignment strategies** for generating coordinate time series in geodetic networks:

- Constrained weekly adjustment directly to the target frame

  using unweighted MCs

  using weighted MCs

- Helmert transformation of weekly “free” solutions to the target frame

  using the “standard” approach

  using a “revised” approach

Unweighted vs. Weighted MCs

unconstrained weekly NEQs: $\mathbf{N}\begin{bmatrix}\boldsymbol{\delta}\mathbf{x}\\ \boldsymbol{\delta}\mathbf{z}\end{bmatrix} = \mathbf{u}$ *reference stations*

Datum conditions applied to the reference stations without any weighting:

$$\mathbf{E}\,(\mathbf{x}-\mathbf{x}^{\mathrm{trf}}) = \mathbf{0}$$

Optimal property: minimization of data noise effect only at reference stations!

Datum conditions applied to the reference stations with an optimal weight matrix:

$$\mathbf{E}\,\mathbf{P}(\mathbf{x}-\mathbf{x}^{\mathrm{trf}}) = \mathbf{0}$$

$$\mathbf{P} = \left(\boldsymbol{\Sigma}+\boldsymbol{\Sigma}_{\mathbf{x}^{\mathrm{trf}}}\right)^{-1}$$ **weight matrix**

Optimal property: minimization of data & datum noise effect at all network stations!

(see Kotsakis 2013, JGeod)

Standard vs. Revised Helmert Transformation

(applied on a free-net solution)

$$\hat{\boldsymbol{\theta}} = \left(\mathbf{E}(\boldsymbol{\Sigma}_{\mathbf{x}^{\mathrm{trf}}} + \boldsymbol{\Sigma}_{\mathbf{X}'})^{-1}\mathbf{E}^{\mathrm{T}}\right)^{-1}\mathbf{E}(\boldsymbol{\Sigma}_{\mathbf{x}^{\mathrm{trf}}} + \boldsymbol{\Sigma}_{\mathbf{X}'})^{-1}(\mathbf{x}^{\mathrm{trf}} - \mathbf{X}')$$

Standard approach:

Transformed coords are obtained by forward implementation of the HT model, after the initial estimation of the frame transformation parameters.

$$\hat{\mathbf{x}}^{\mathrm{st}} = \mathbf{X}' + \mathbf{E}^T\hat{\boldsymbol{\theta}}$$ *common stations*

$$\hat{\mathbf{z}}^{\mathrm{st}} = \mathbf{Z}' + \tilde{\mathbf{E}}^T\hat{\boldsymbol{\theta}}$$ *other stations*

Revised approach:

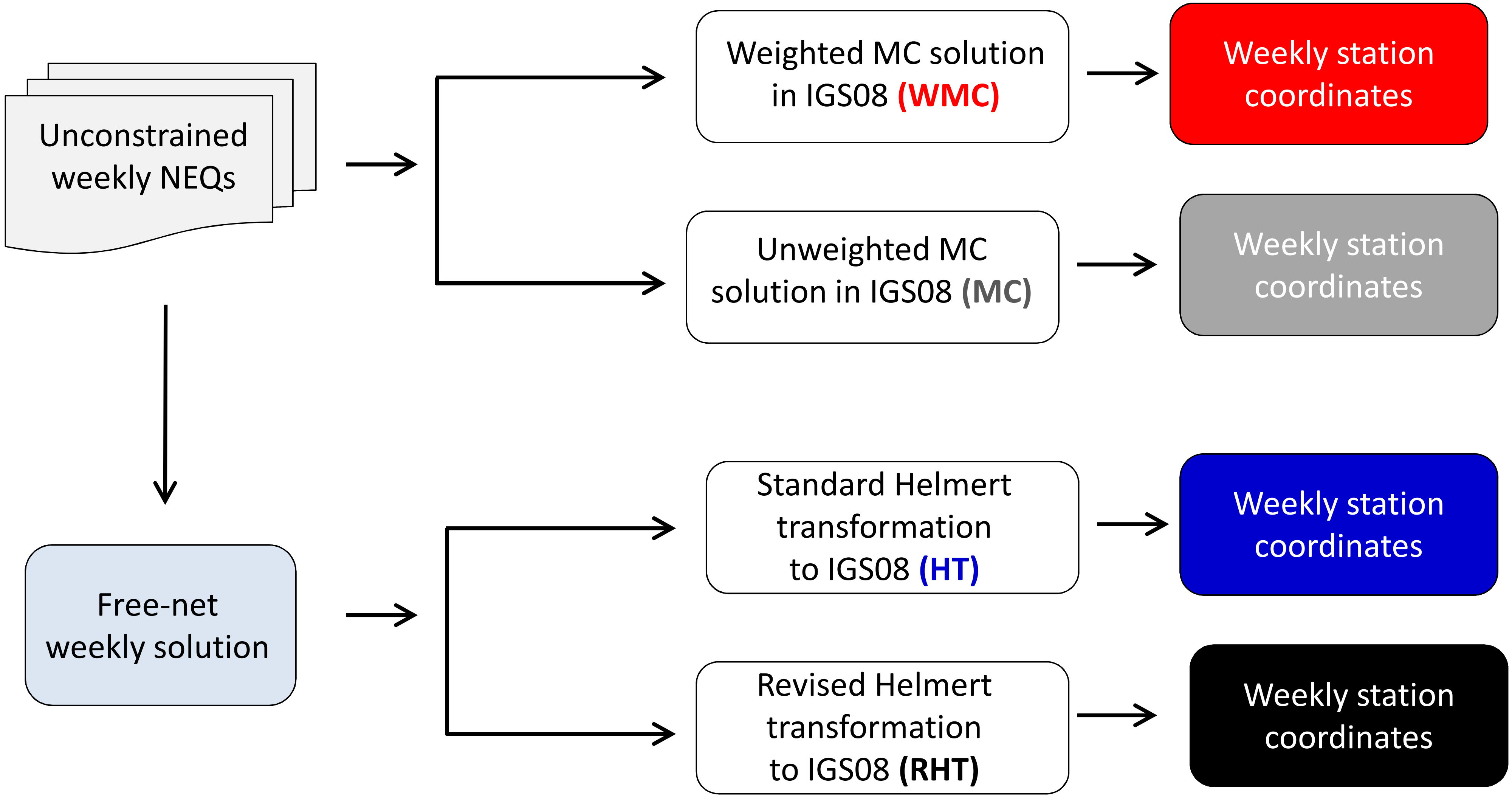
Transformed coords are simultaneously estimated with the frame transformation parameters in a single least-squares adjustment step.

correction term

$$\begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{z}} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{x}}^{\mathrm{st}} \\ \hat{\mathbf{z}}^{\mathrm{st}} \end{bmatrix} + \underbrace{\begin{bmatrix} \boldsymbol{\Sigma}_{\mathbf{X}'} \\ \boldsymbol{\Sigma}_{\mathbf{Z}'\mathbf{X}'} \end{bmatrix}(\boldsymbol{\Sigma}_{\mathbf{x}^{\mathrm{trf}}} + \boldsymbol{\Sigma}_{\mathbf{X}'})^{-1}(\mathbf{x}^{\mathrm{trf}} - \hat{\mathbf{x}}^{\mathrm{st}})}_{\text{correction term}}$$

(see Kotsakis et al. 2014, JGeod)

Processing Scheme



Comparison of weekly coordinate time series (for the period: 2007-2014) generated by:

- **Unweighted MCs** (NNT to IGS08)
- **Weighted MCs** (NNT to IGS08)
- **Standard 6/7-parameter HT** to IGS08
- **Revised 6/7-parameter HT** to IGS08

(*) all strategies used the same reference stations.

Our evaluation looks at the following:

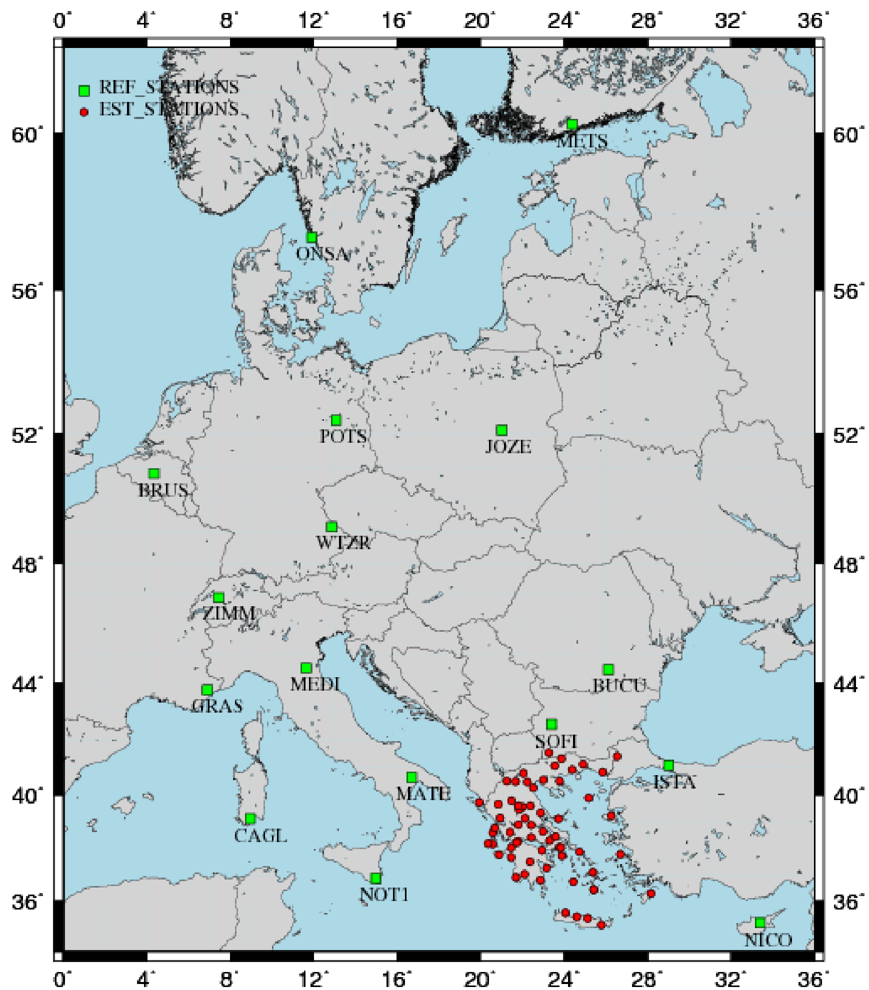
sensitivity to existing outliers at the ref stations

RMS of coordinate time series

differences of estimated station velocities

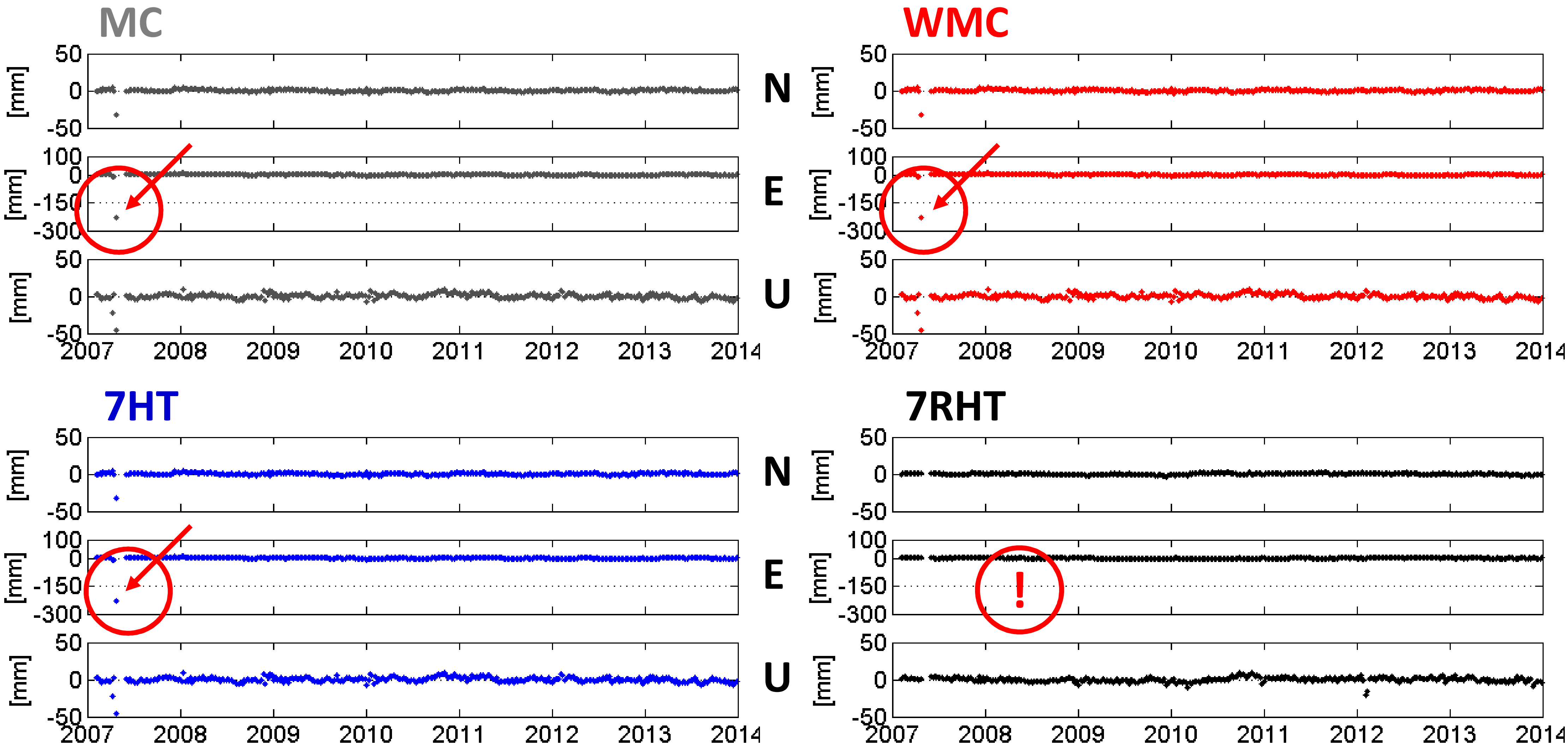
Test network

16 REF Stations, 68 OTHER Stations

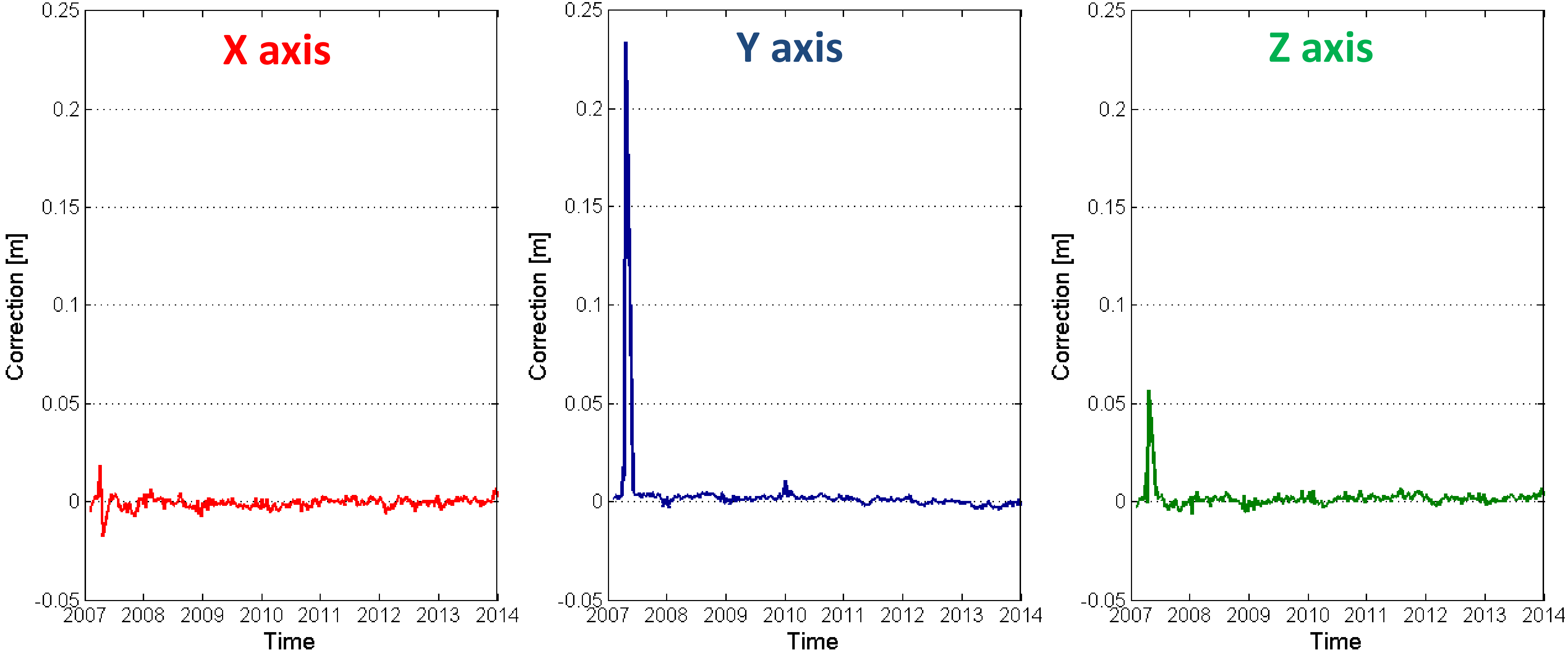


Sensitivity to existing outliers at the used reference stations

Residuals after linear trend removal (GRAS ref station)

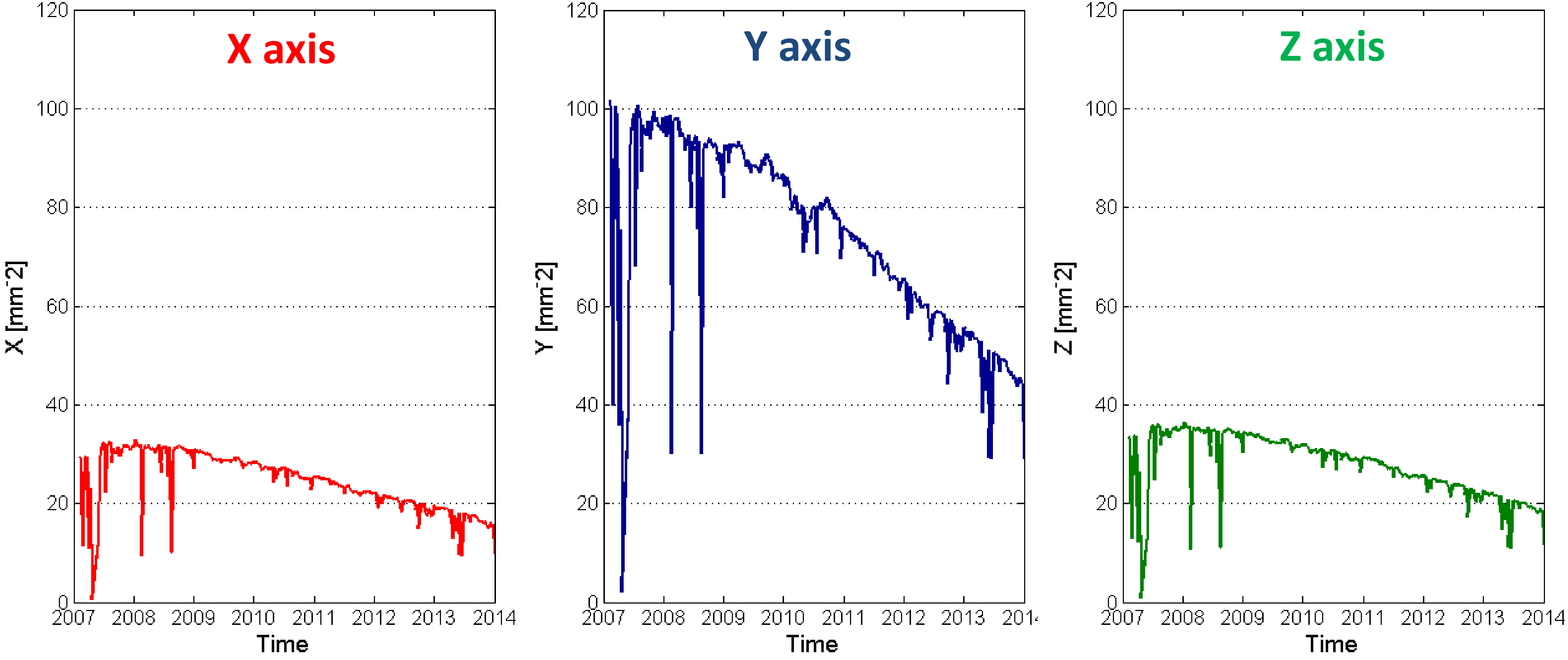


Time series of the **HT correction term** for GRAS ref station

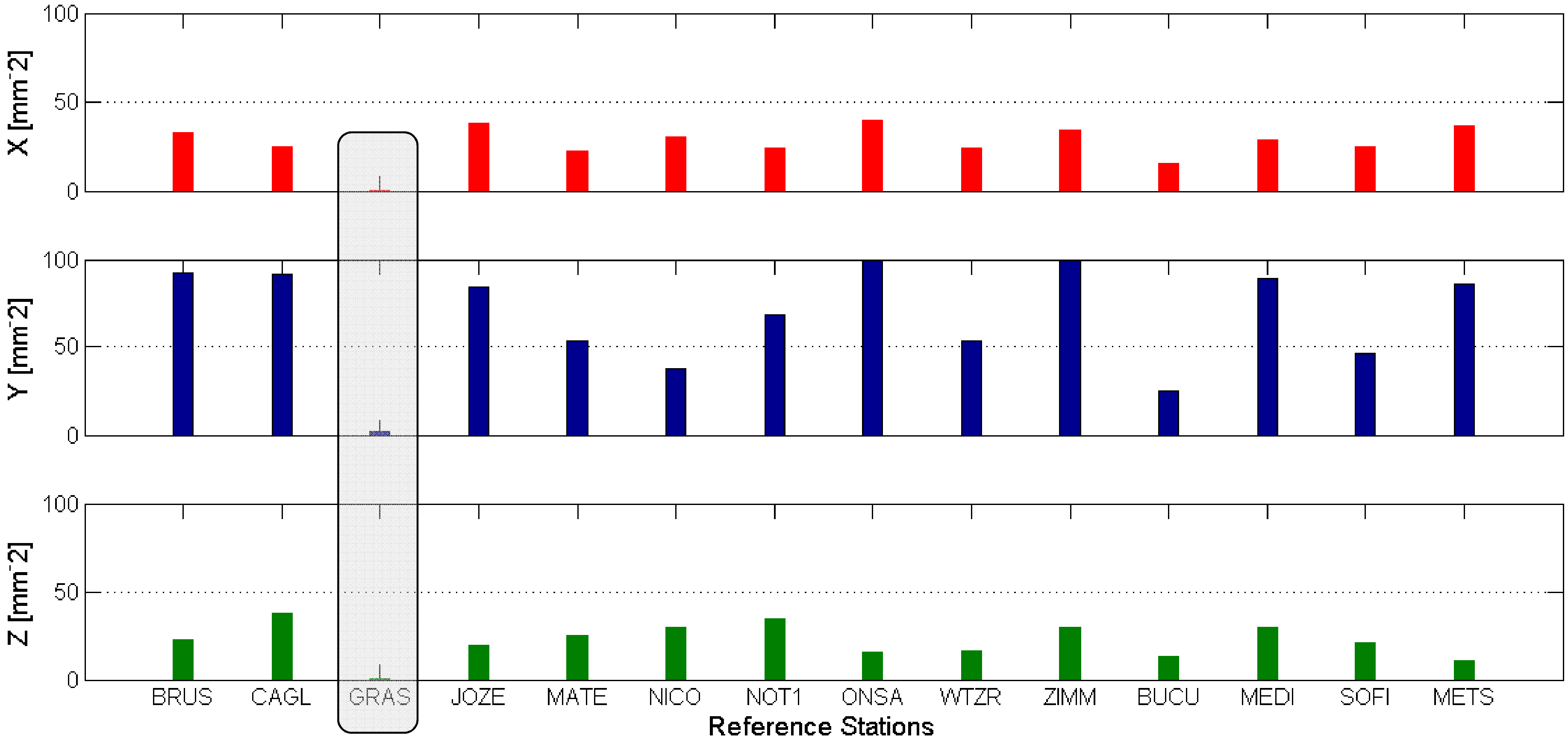


Time series of the **computed weights** for GRAS ref station

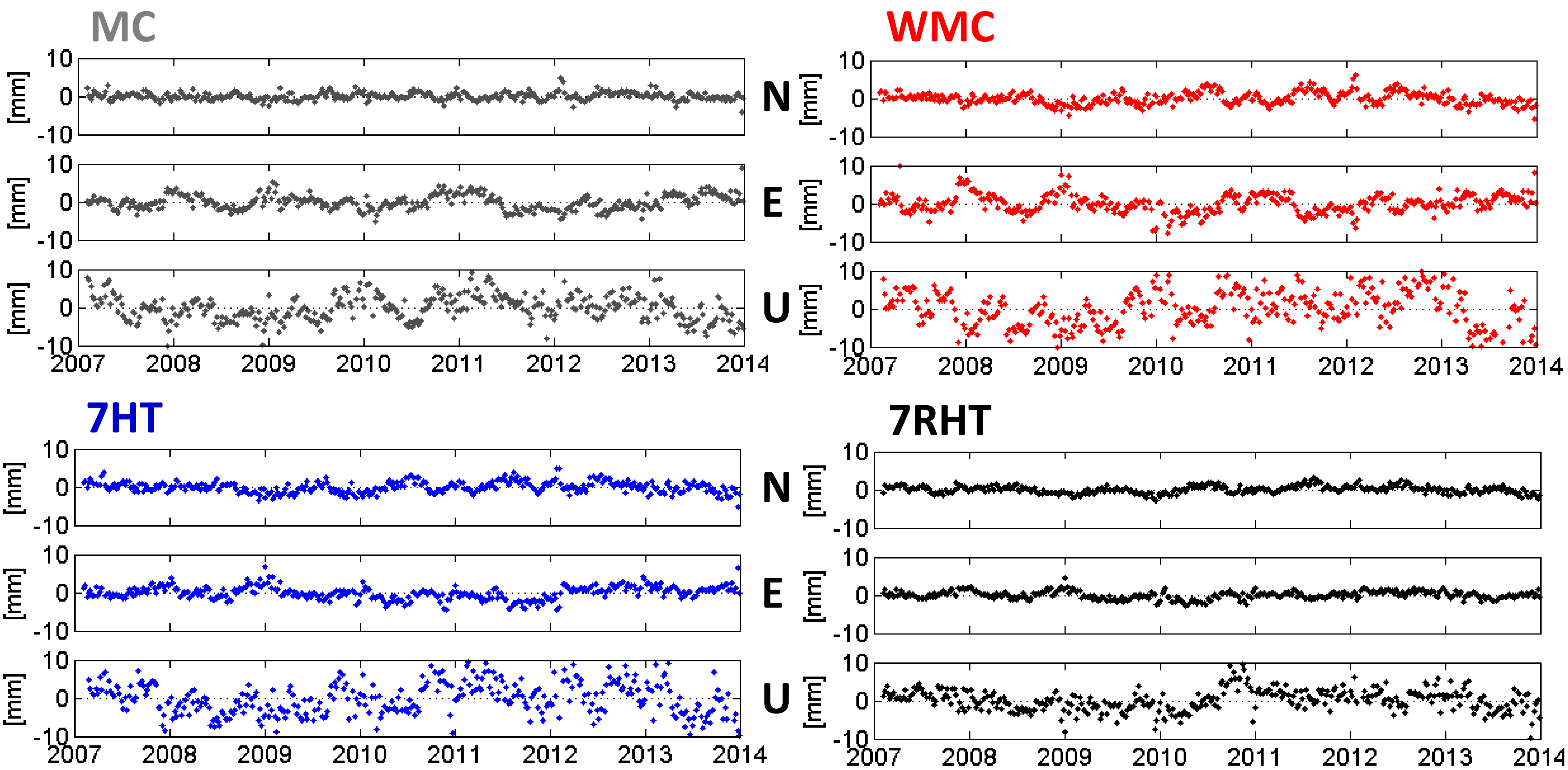
(as used in the weighted MCs)



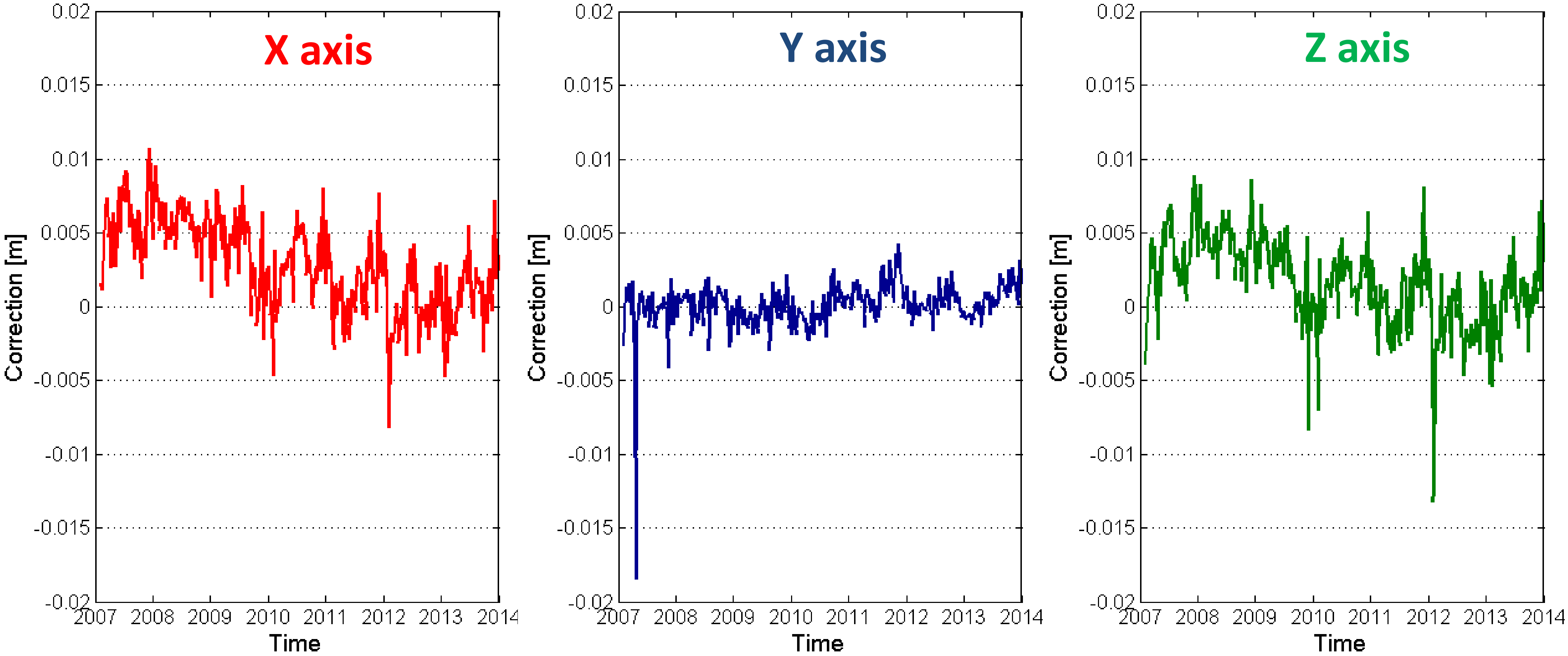
Diagonal elements of the optimal weight matrix **P** for the used reference stations (day 112/2007)



Residuals after linear trend removal (ZIMM ref station)

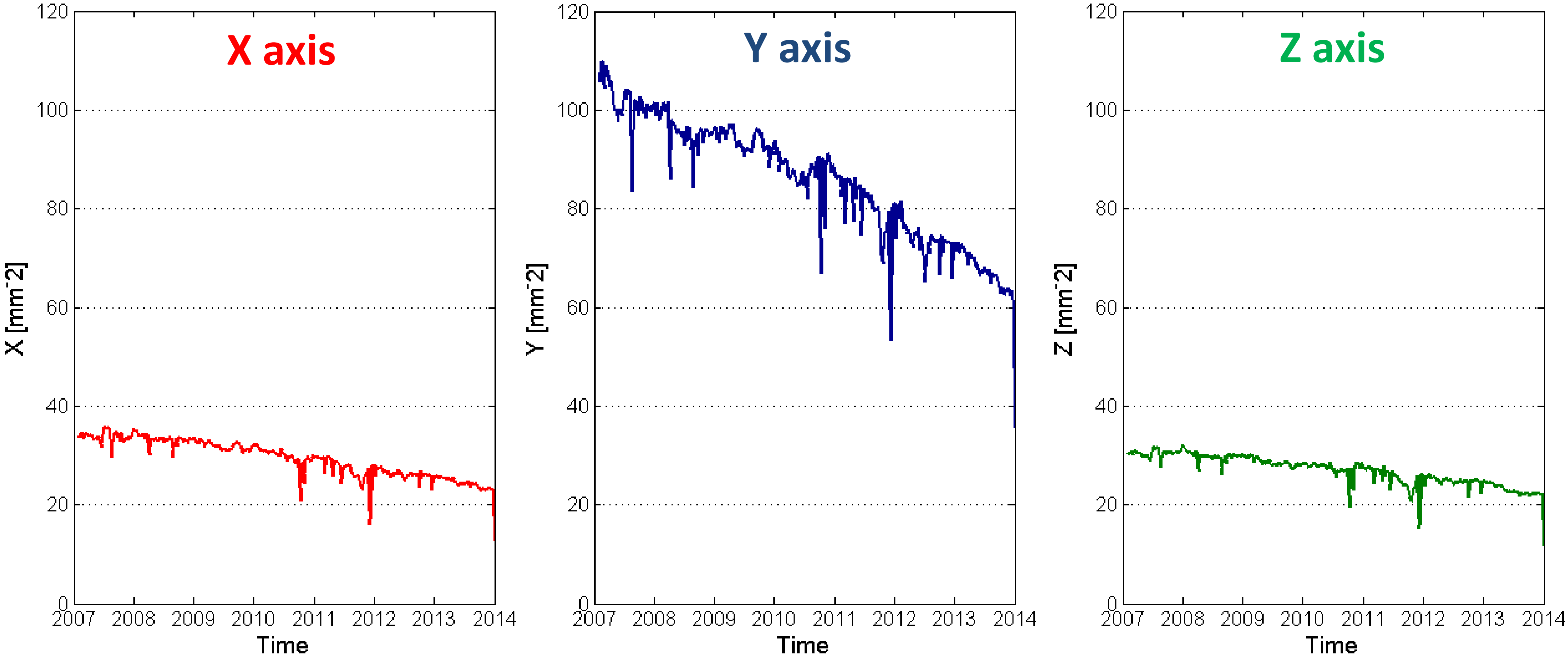


Time series of the **HT correction term** for ZIMM ref station

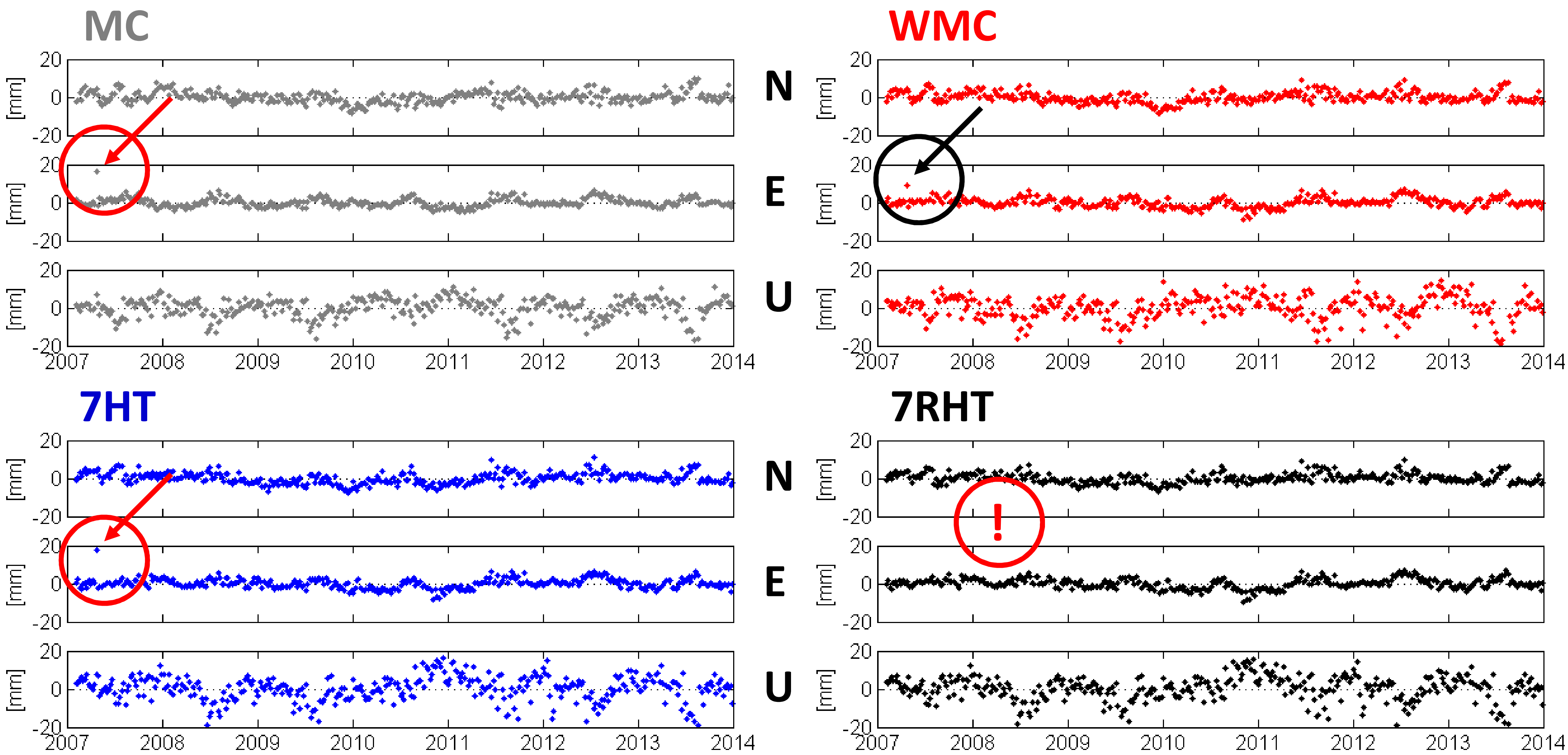


Time series of the **computed weights** for ZIMM ref station

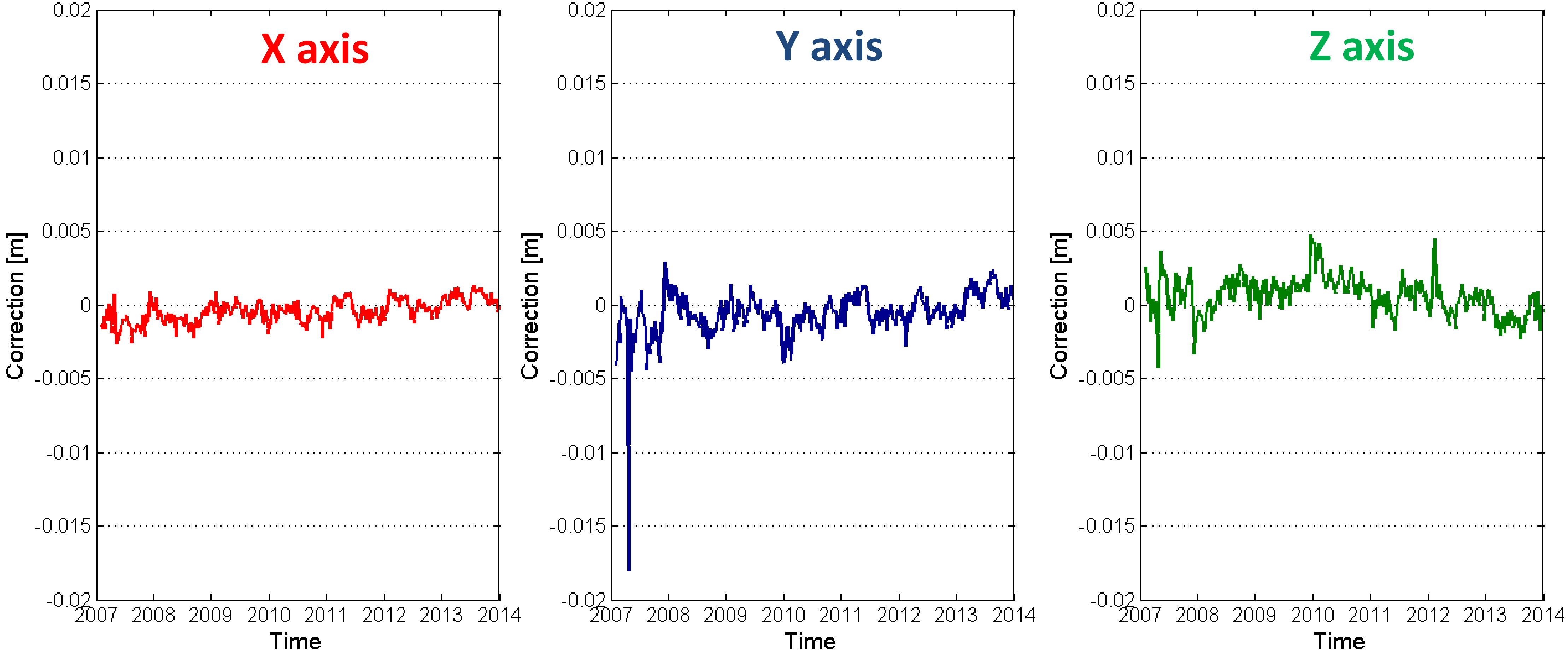
(as used in the weighted MCs)



Residuals after linear trend removal (NOA1 station)



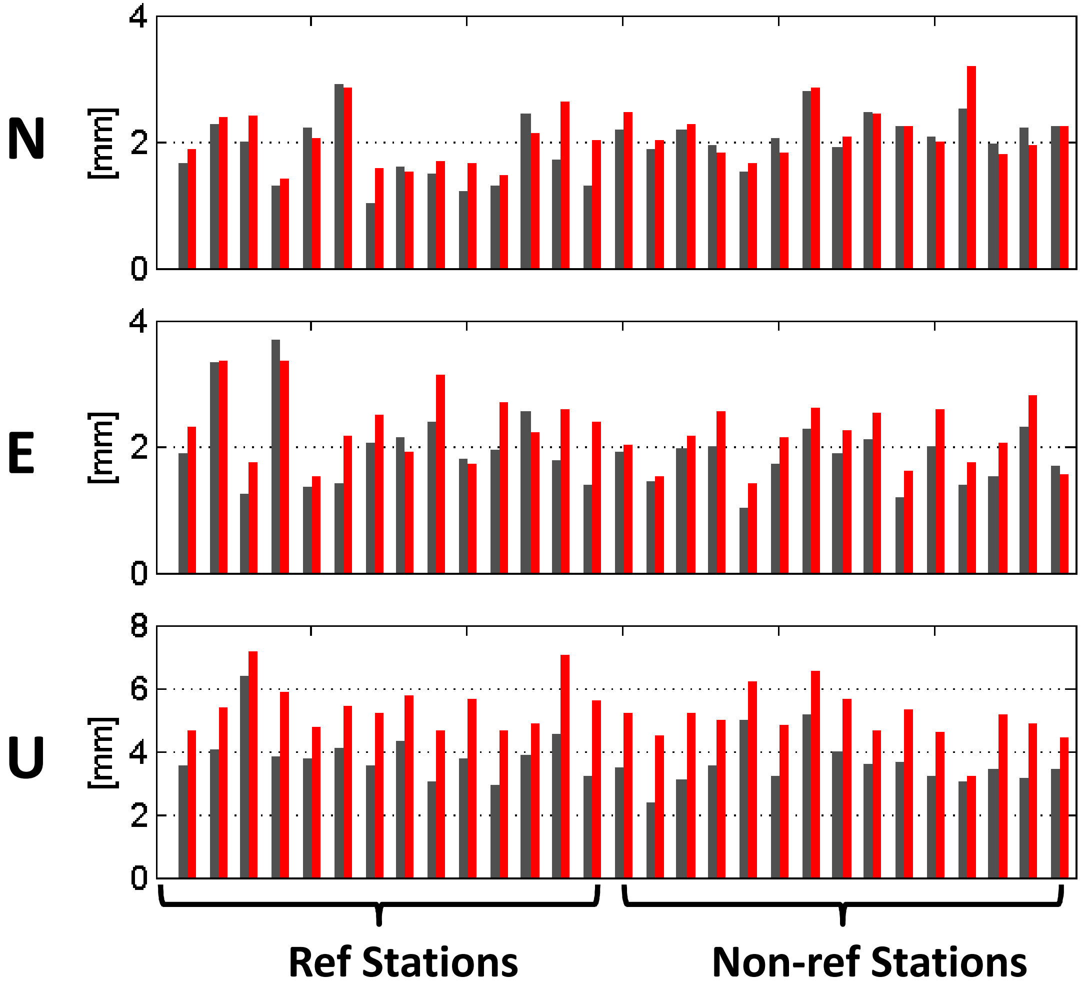
Time series of the **HT correction term** for NOA1 station



RMS behavior of the residual coordinate time series

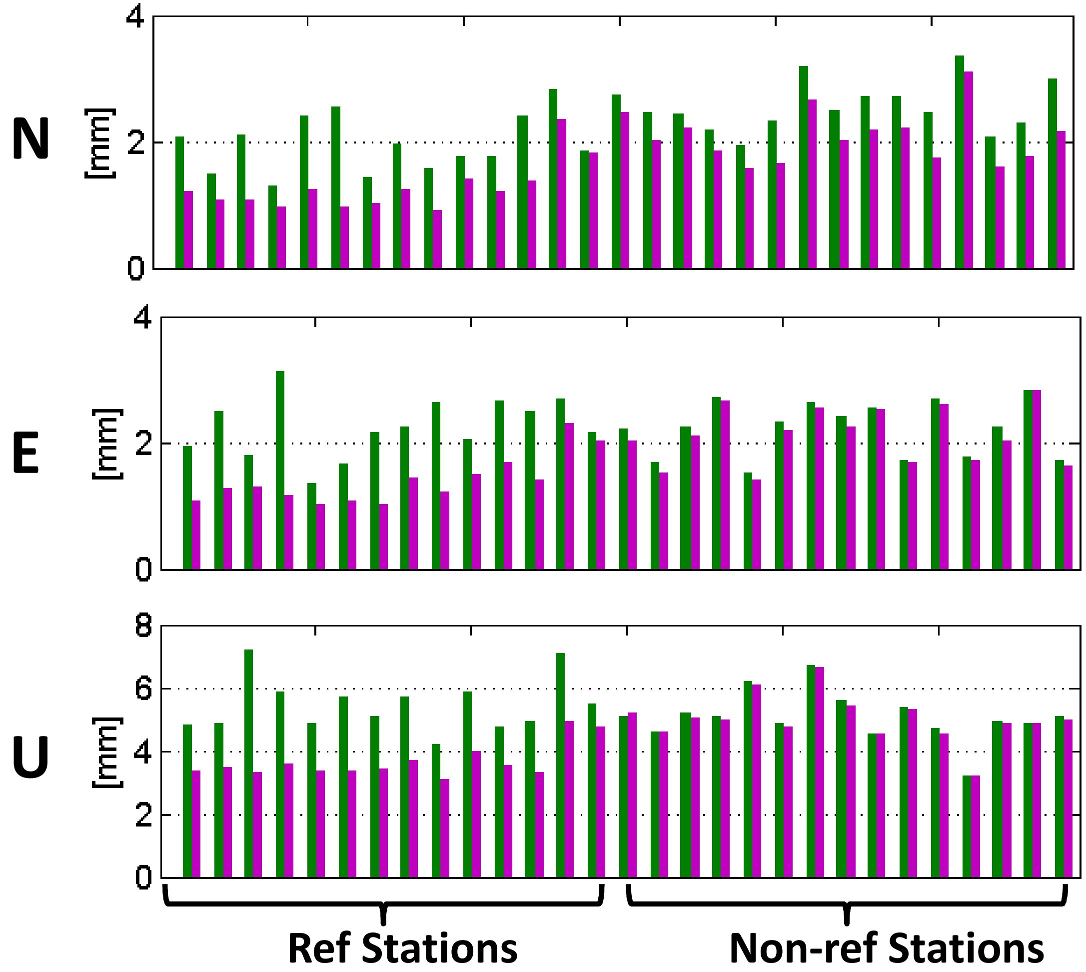
RMS of coordinate time series after linear trend removal

MCs vs. **WMCs**



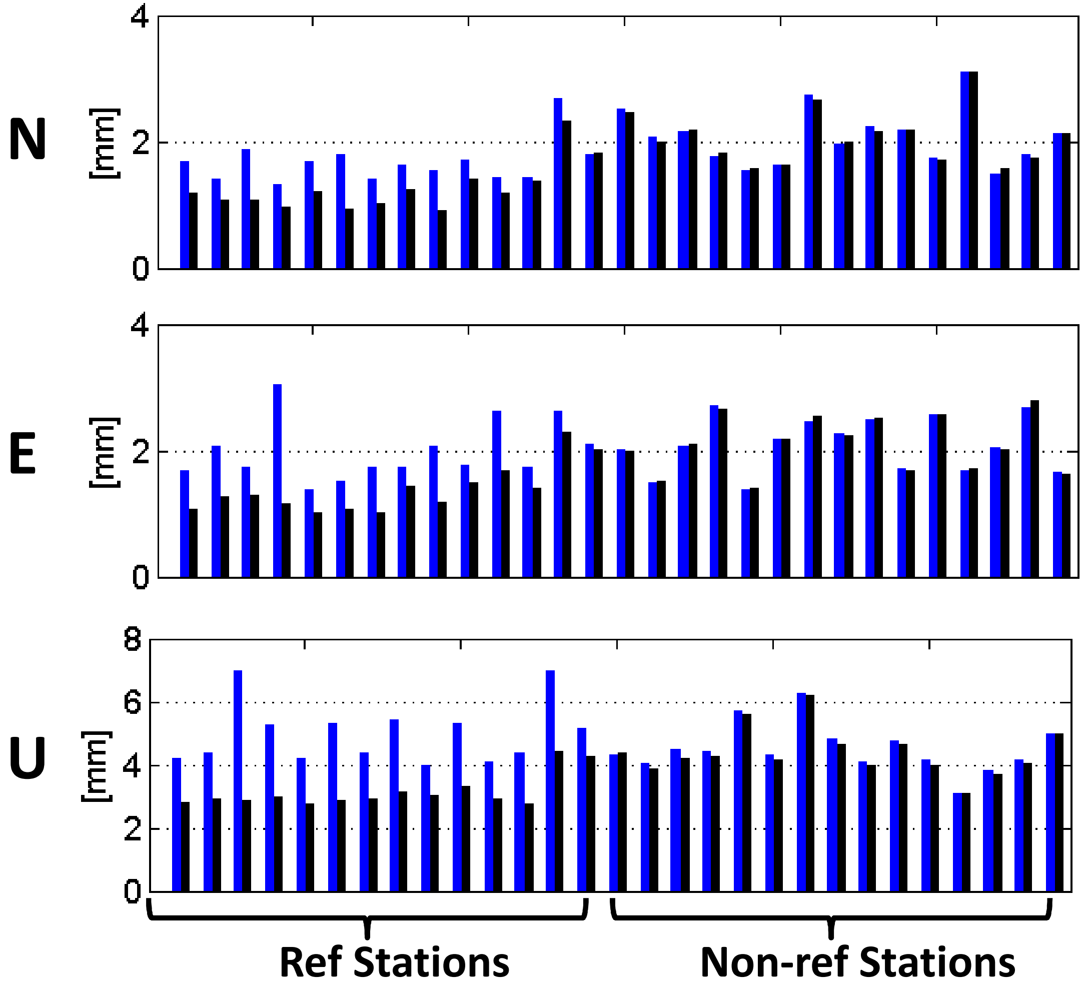
RMS of coordinate time series after linear trend removal

6-parameter **HT** vs. **RHT**



RMS of coordinate time series after linear trend removal

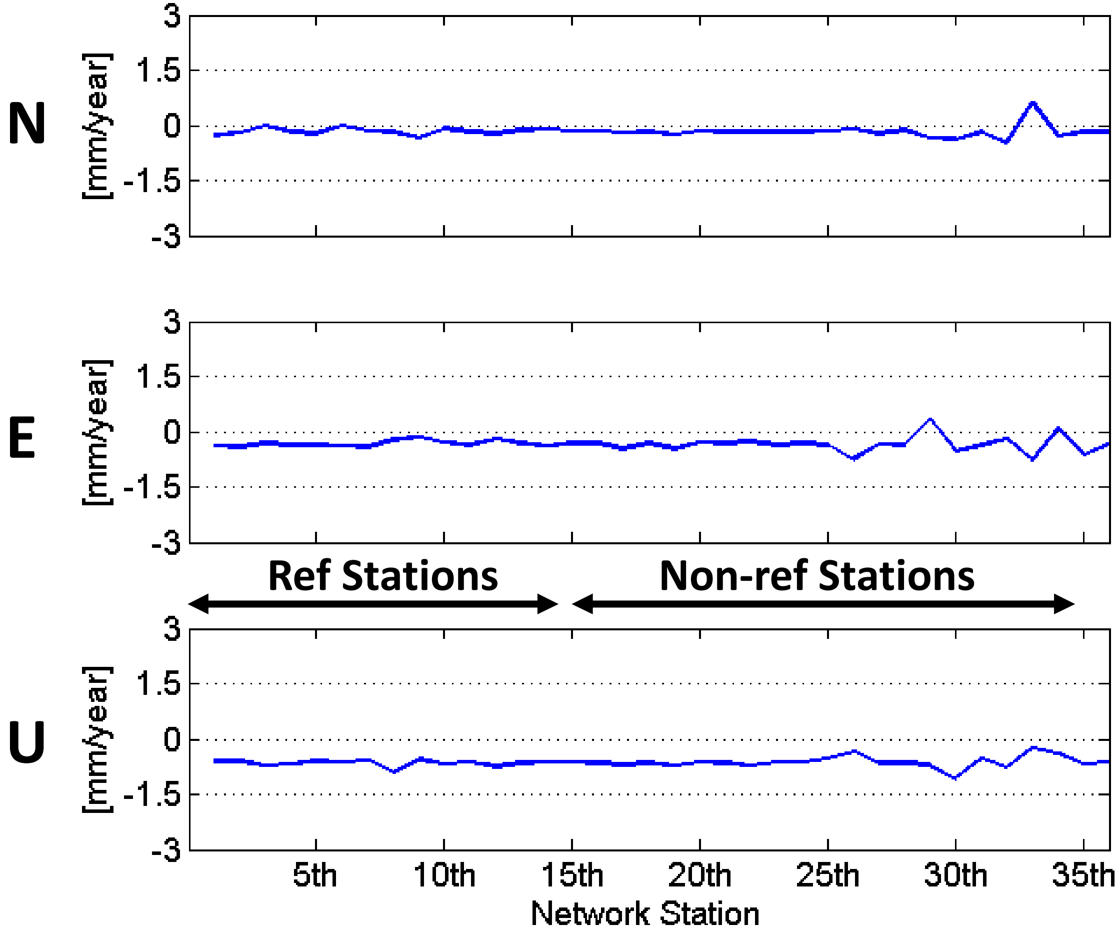
7-parameter **HT** vs. **RHT**



Differences of estimated station velocities among the frame-alignment schemes

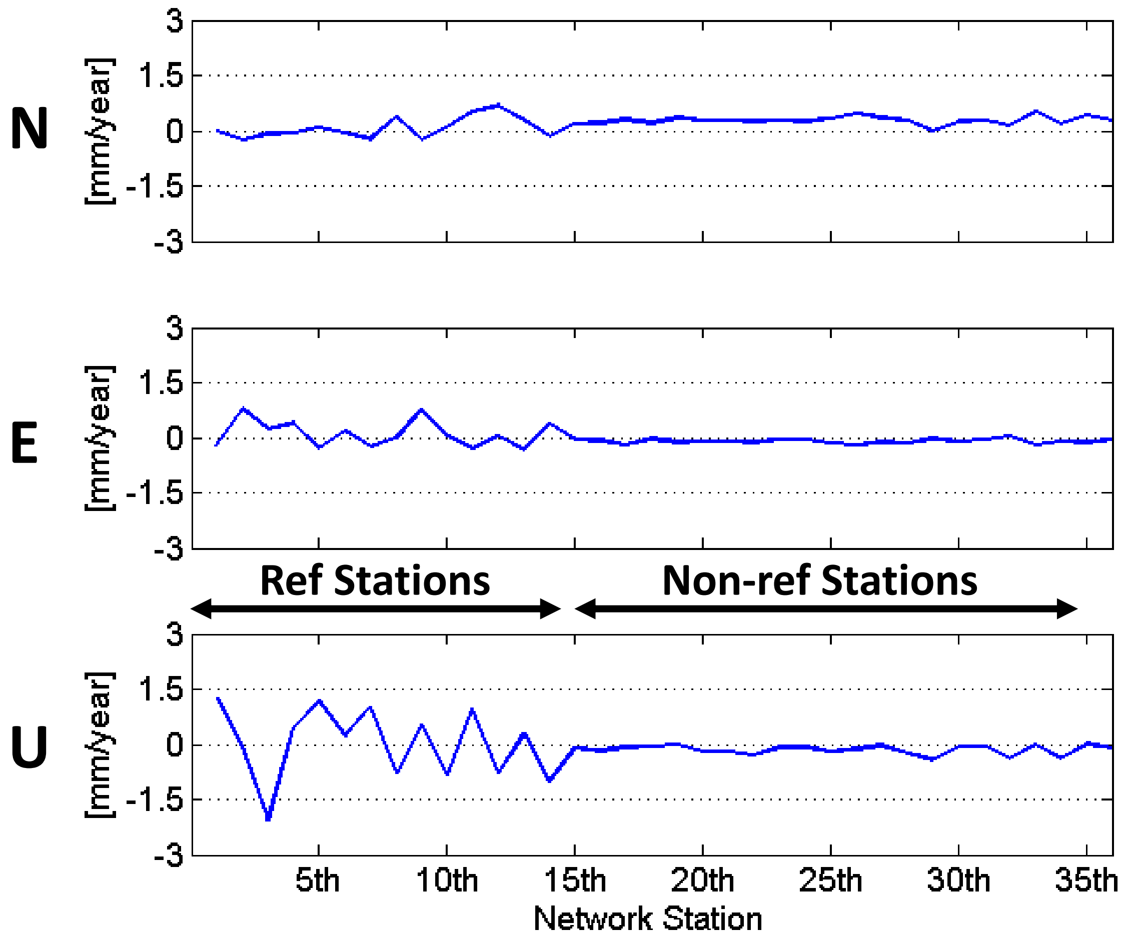
Differences of estimated station velocities

(MCs vs. WMCs)



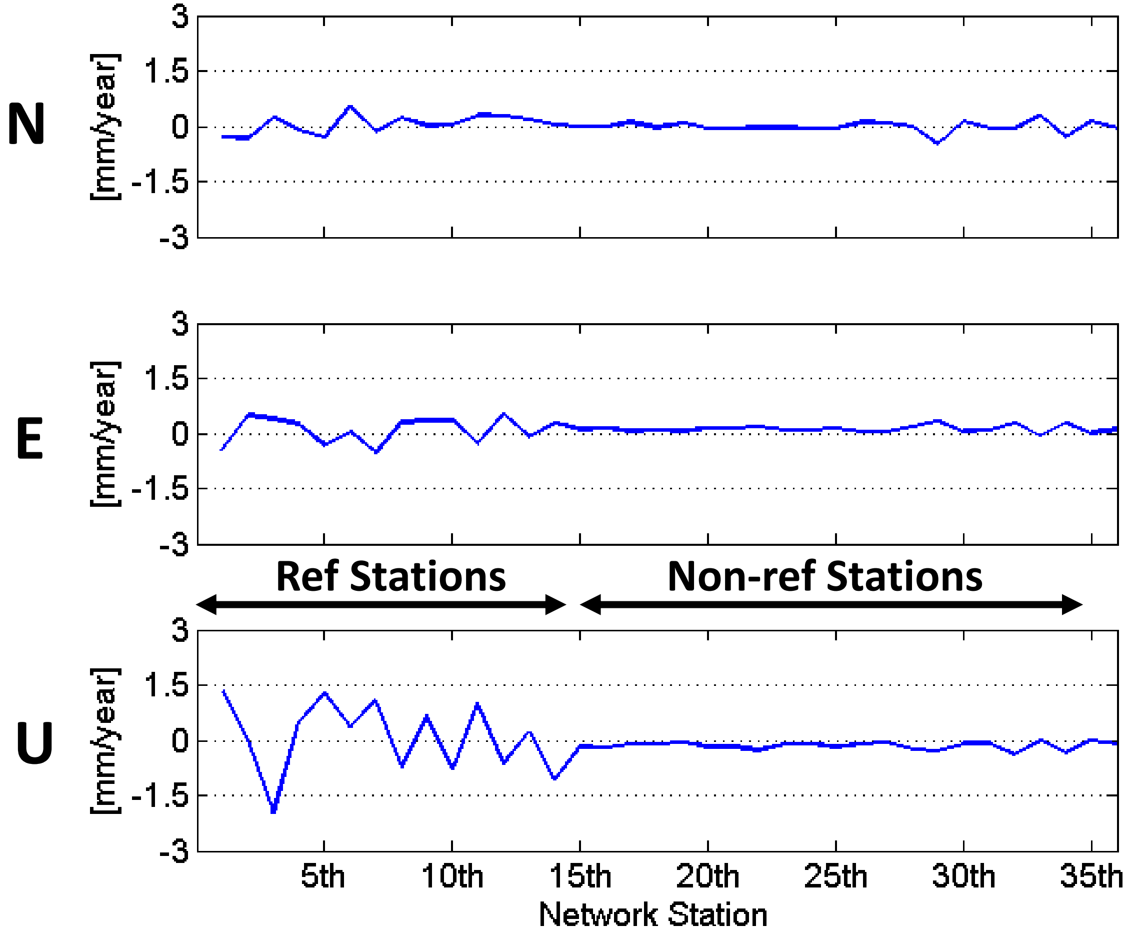
Differences of estimated station velocities

6-parameter **HT** vs. **RHT**



Differences of estimated station velocities

7-parameter **HT** vs. **RHT**



Conclusions

- The weighted MCs and the revised HT model seem to be **more robust** to existing outliers of the reference stations, than the unweighted MCs and the standard HT model, when generating coordinate time series in a geodetic network.

- In terms of the RMS for the derived coordinate time series (after trend removal):

  - standard vs. revised HT model: the former model gives higher RMS values at the reference stations by 1-3 mm.

  - standard vs. revised HT model: both give similar RMS values at the non-ref stations, except in the 6-parameter case for the NORTH component.

  - unweighted vs. weighted MCs: the former generally give smaller RMS values by 1-3 mm than the latter, especially in the UP component.

- **The consideration of the target frame noise** (i.e. taking into account the CV matrix of the reference stations coordinates in the weighted MCs and in the HT-based frame alignment) amplifies the RMS of the derived coordinate time series by 0.5 - 1 mm.

Conclusions

- In terms of the estimated velocities at the network stations:
    - the unweighted and weighted MCs give practically the same velocities in all stations (their differences are smaller than 1mm/year).
    - the standard and revised HT models give the same velocities at the non-ref stations.
    - the standard and revised HT models give velocity differences at the reference stations up to 2 mm/year.
- The differences of the estimated velocities among the different frame-alignment methods are more significant in the UP component.
- More work needs to be done in order to assess the performance of the four frame-alignment strategies for the analysis of (unmodeled) loading signals in geodetic coordinate time series.

Thank you for your attention!!!

email: mchatzin@topo.auth.gr

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