Assessment of weighted and un-weighted inner constraints on multi-session solutions for estimating station velocities in regional GNSS networks

C. Kotsakis and M. Chatzinikos

Department of Geodesy and Surveying
Aristotle University of Thessaloniki
Thessaloniki, Greece

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Introduction

- Datum definition in geodetic networks is often implemented by unweighted inner constraints (ICs) over selected reference stations.

- This study investigates the application of optimal weighting to the reference stations that are used for datum definition in GNSS network adjustment:
  - choice of optimal weight matrix
  - behavior of optimal weight matrix
  - effect of weight matrix on the results
  - tests on multi-year GNSS network solutions

The rationale of the study

Unconstrained NEQs

$$\mathbf{N}\begin{bmatrix}\mathbf{x}-\mathbf{x}^{o}\\ \cdots\cdots\cdots\\ \bar{\mathbf{x}}-\bar{\mathbf{x}}^{o}\end{bmatrix}=\mathbf{u}$$

$\mathbf{x}-\mathbf{x}^{o}$ → reference stations

$\bar{\mathbf{x}}-\bar{\mathbf{x}}^{o}$ → other stations

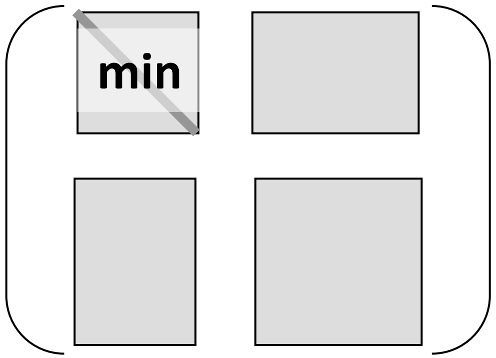
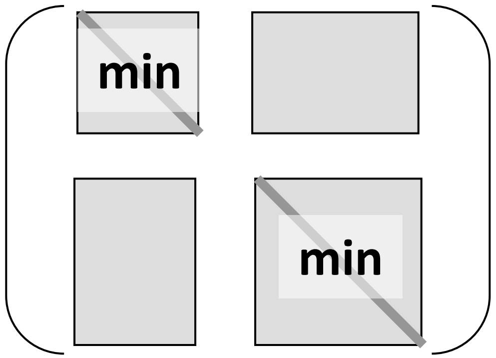
Datum definition

$$\mathbf{E}\mathbf{P}\left(\mathbf{x}-\mathbf{x}^{TRF}\right)=\mathbf{0} \quad\rightarrow\quad \hat{\mathbf{x}},\ \hat{\bar{\mathbf{x}}}$$

$$\mathbf{E}\left(\mathbf{x}-\mathbf{x}^{TRF}\right)=\mathbf{0} \quad\rightarrow\quad \hat{\mathbf{x}},\ \hat{\bar{\mathbf{x}}}$$

Can we get an **improved solution** through a justified choice of the weight matrix **P** ?

ICs and network accuracy in the target frame

	Unweighted ICs	*Weighted ICs*
*Minimize **data noise** effect?*	Yes (at the reference stations)	Yes (at all stations)
*Minimize **datum noise** effect?*	No	
Optimization of network COV matrix $\boldsymbol{\Sigma}_{\hat{X}} = \boldsymbol{\Sigma}_{\hat{X}}^{obs} + \boldsymbol{\Sigma}_{\hat{X}}^{mc}$	$\boldsymbol{\Sigma}_{\hat{X}}^{obs} =$ 	$\boldsymbol{\Sigma}_{\hat{X}} =$ 

Analytical computation of the optimal weight matrix

❶ Unconstrained NEQs: $\mathbf{N}\begin{bmatrix} \mathbf{x}-\mathbf{x}^{o} \\ \cdots\cdots \\ \bar{\mathbf{x}}-\bar{\mathbf{x}}^{o} \end{bmatrix} = \mathbf{u}$

❷ Weighted ICs at the reference stations: $\mathbf{E}\,\mathbf{P}\left(\mathbf{x}-\mathbf{x}^{TRF}\right) = \mathbf{0}$

❸ Optimal weight matrix: $\mathbf{P} = \left(\boldsymbol{\Sigma}+\boldsymbol{\Sigma}_{X}^{TRF}\right)^{-1}$

where: $\left(\mathbf{N}+\mathbf{G}^{T}\mathbf{G}\right)^{-1} = \begin{bmatrix} \boldsymbol{\Sigma} & * \\ * & * \end{bmatrix}$

Note: E and **G** are the usual inner-constraint matrices for the ref stations and for the entire network, respectively

see Kotsakis (2013 JGeod, 2015 IAG Symp)

Numerical tests

- Comparison of **weighted & unweighted IC-based solutions** (i.e. NNT alignment to IGS08) obtained by a multi-year (2007-2014) adjustment of weekly NEQs in a Hellenic GNSS network.

- Particularly, we look into the following aspects:

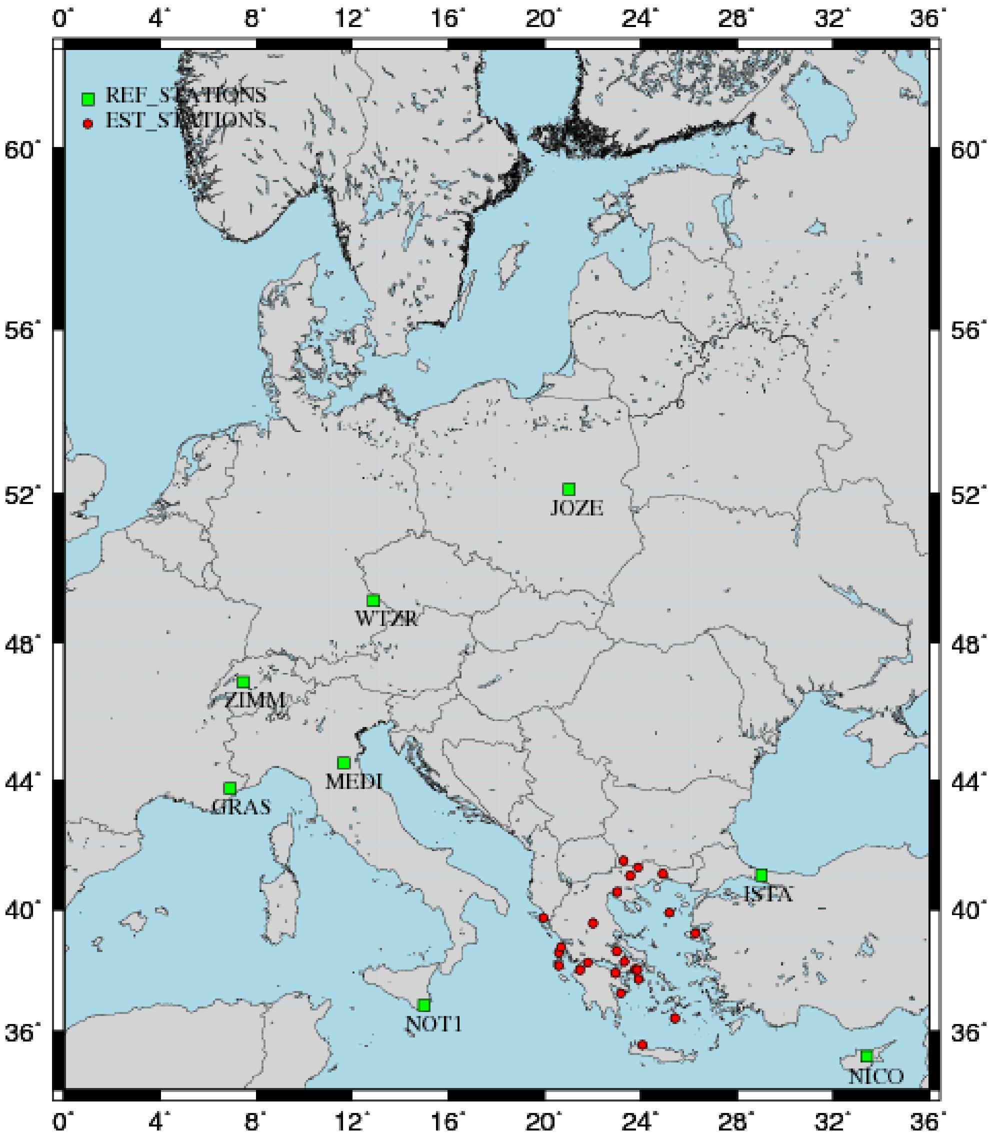
  - accuracy improvement for the estimated positions of each station in IGS08

  - differences of the estimated positions between the two solutions

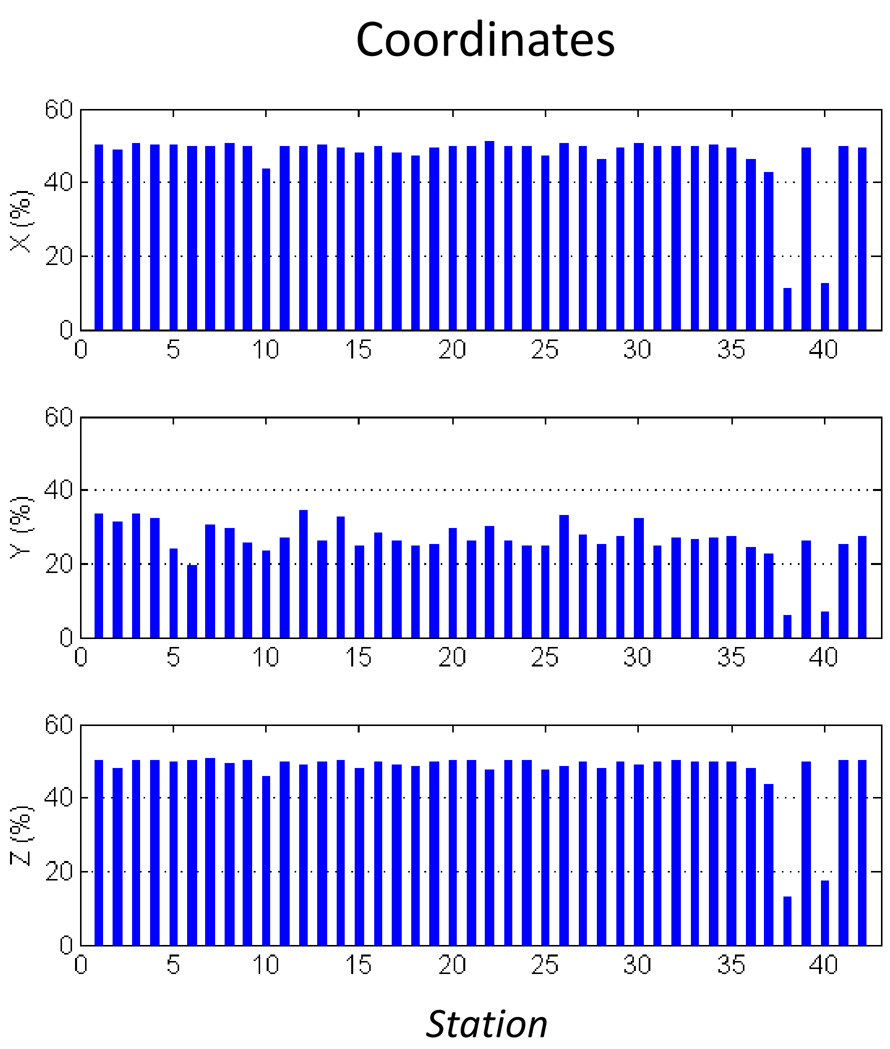
  - behavior of the weight matrix for the reference stations

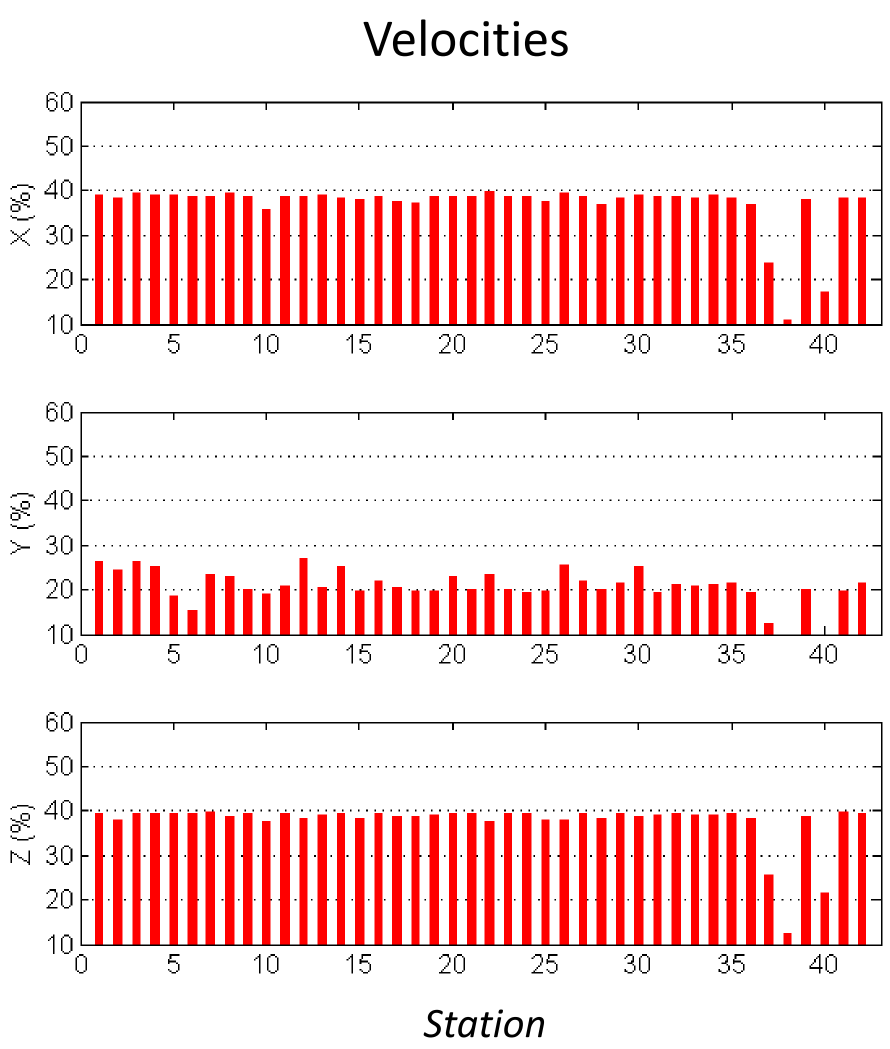
Test GNSS network

8 REF stations, 34 other stations



Per-station **accuracy improvement (%)** of the estimated positions from the weighted IC solution

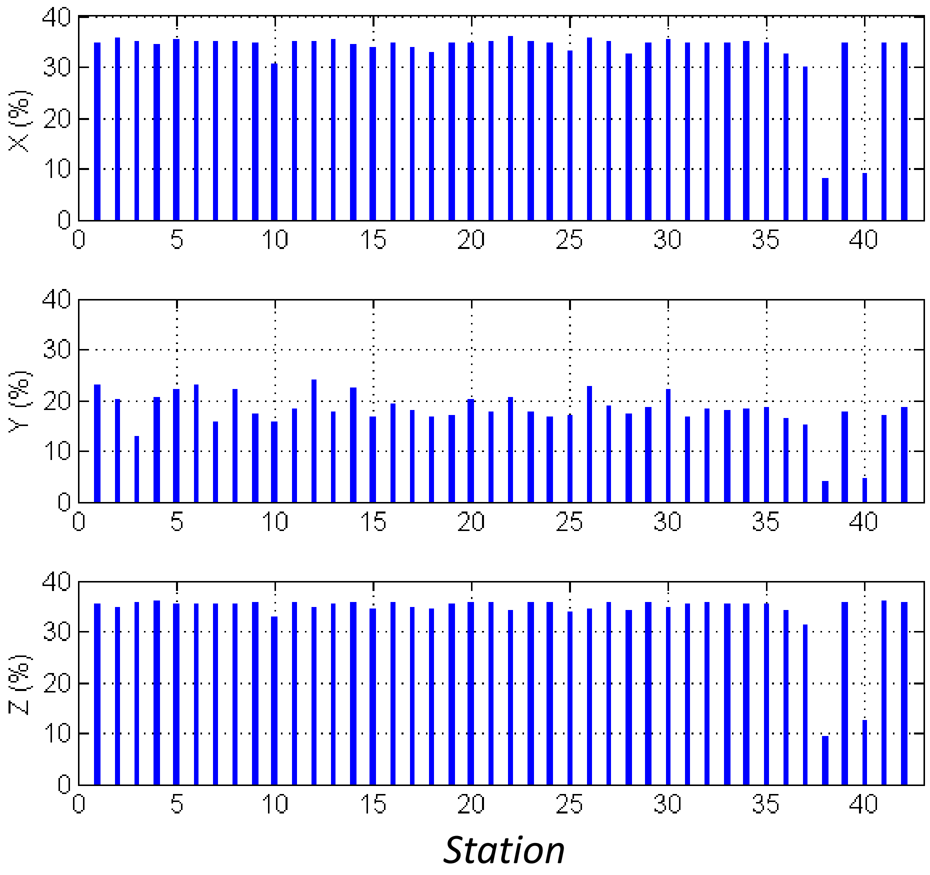




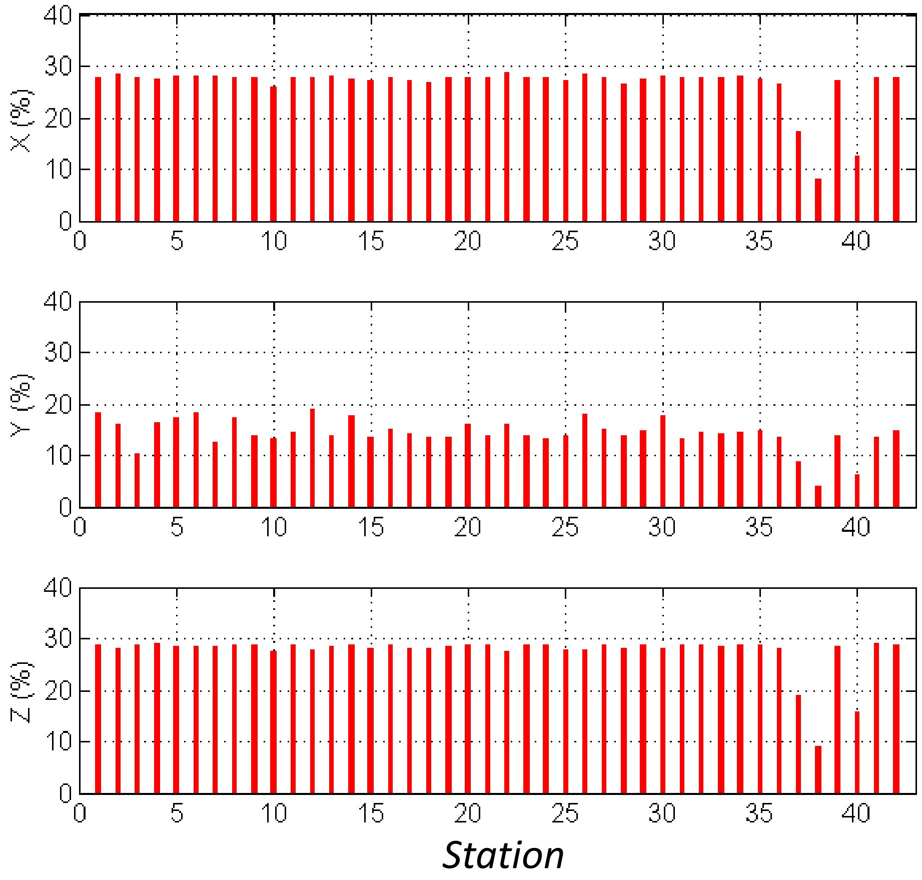
Per-station **accuracy improvement (%)** of the estimated positions from the weighted IC solution

(IGS08 datum noise is omitted in the weighting)

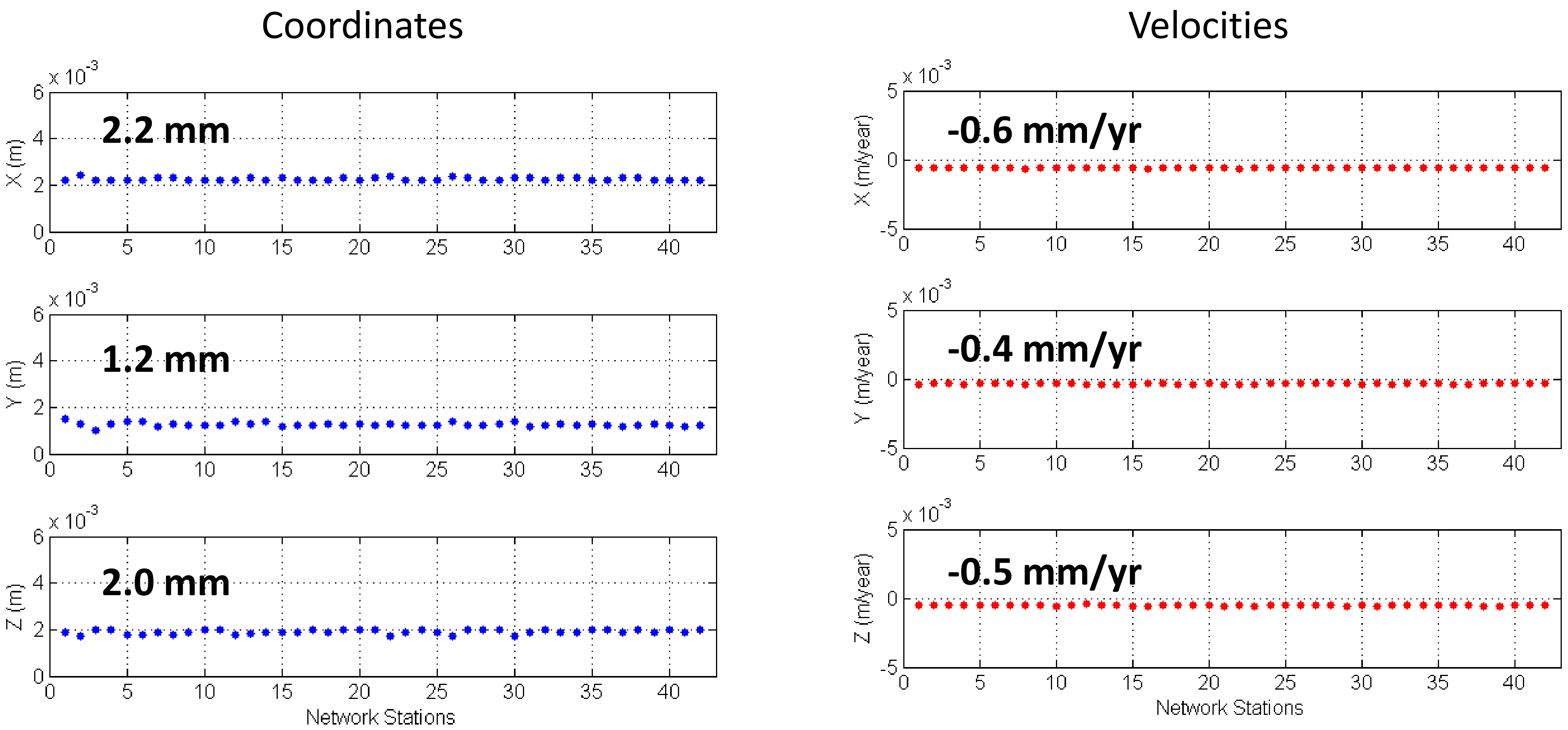
Coordinates



Velocities

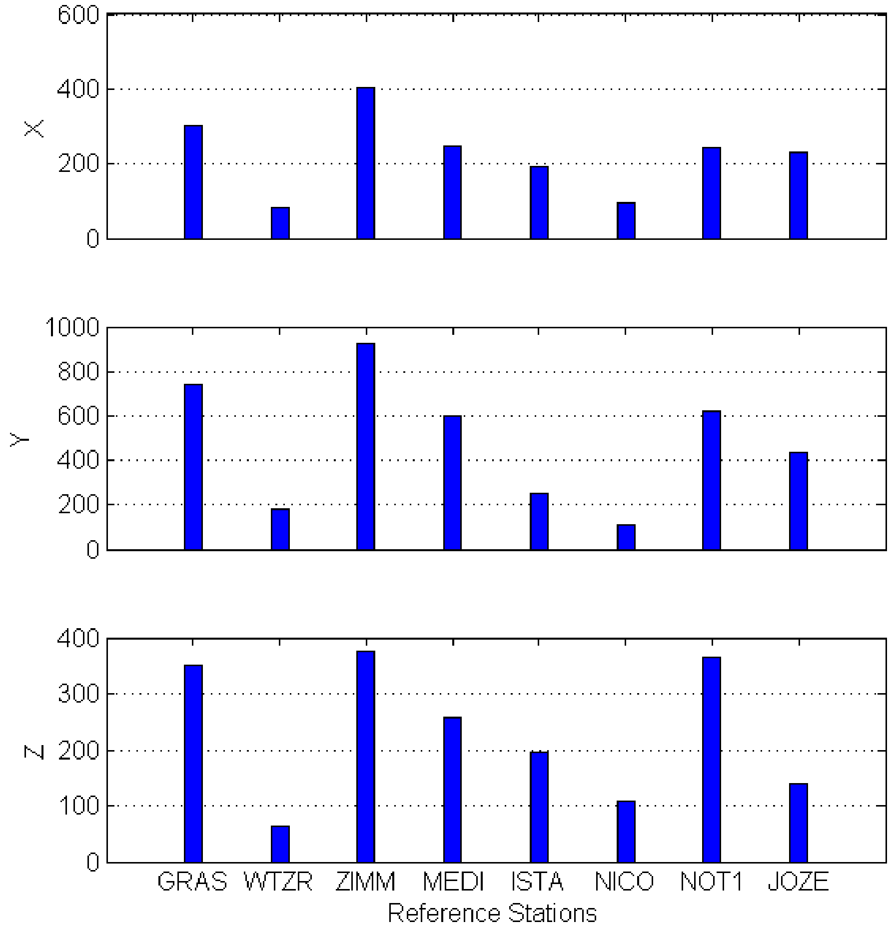


Per-station differences of the estimated positions from the weighted and unweighted IC solutions

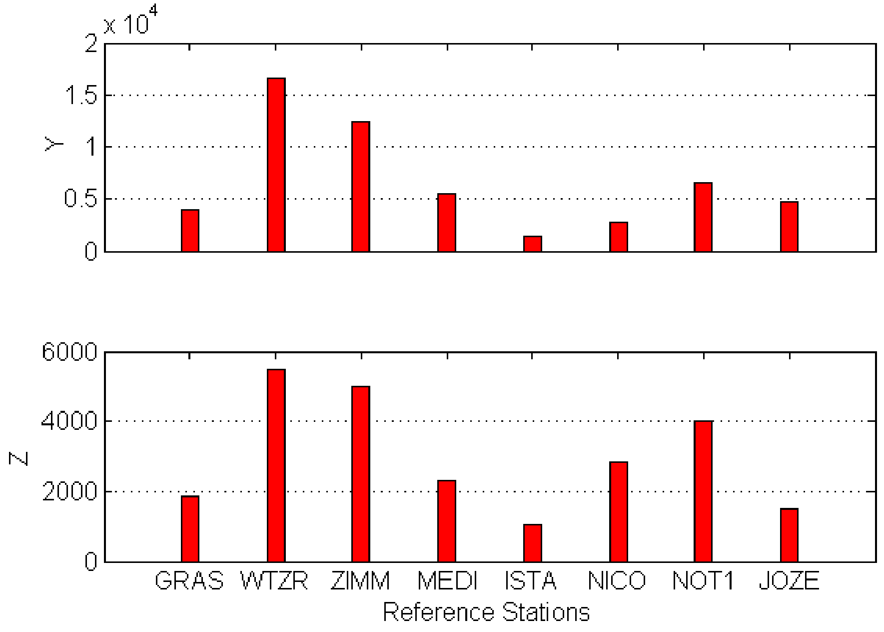


Weights for the 8 reference stations

For their coordinates



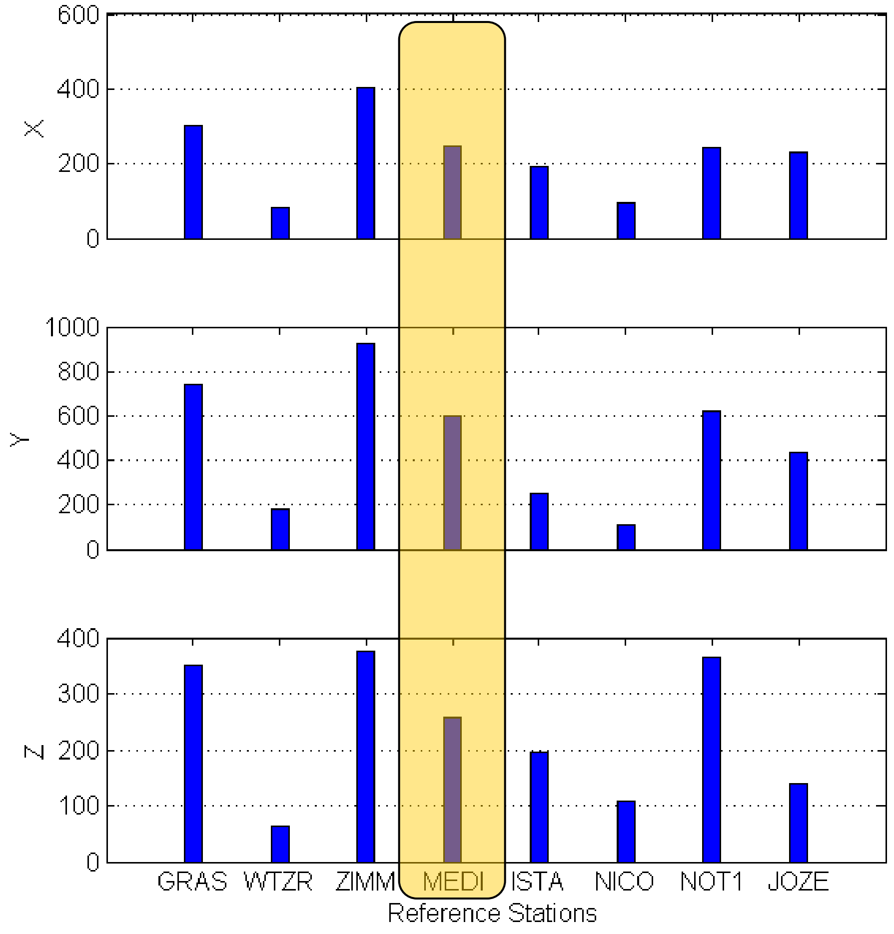
For their velocities



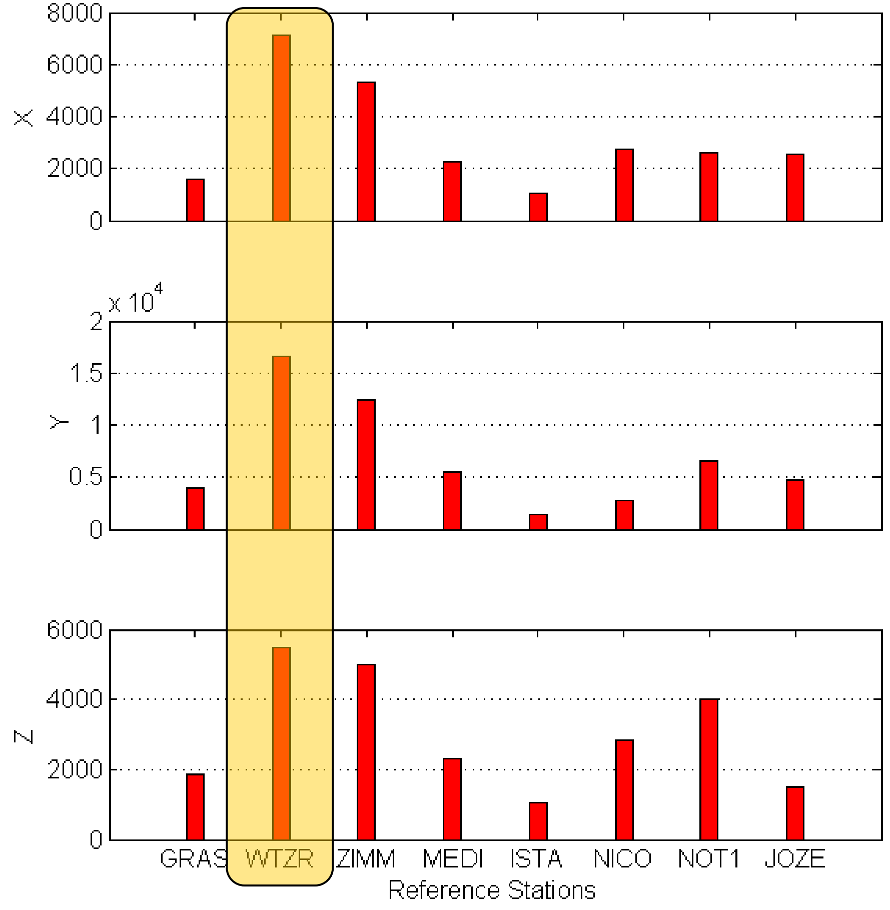
Weights for the 8 reference stations

(note the **intra-station** weight variation)

For their coordinates



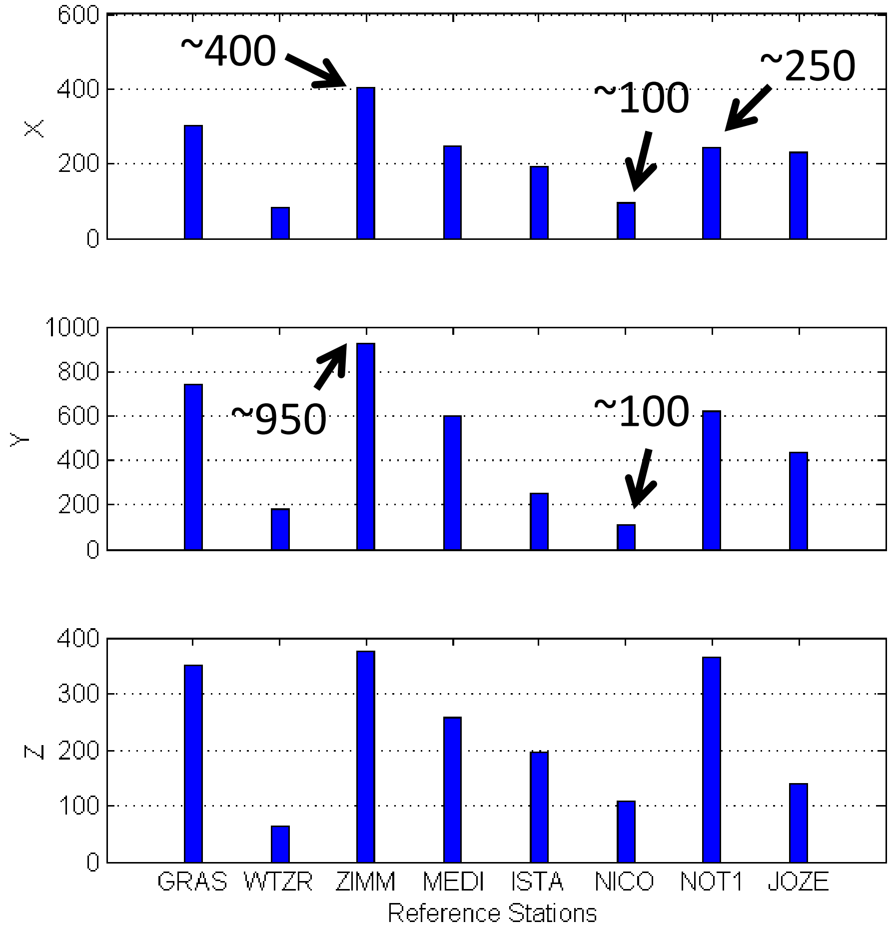
For their velocities



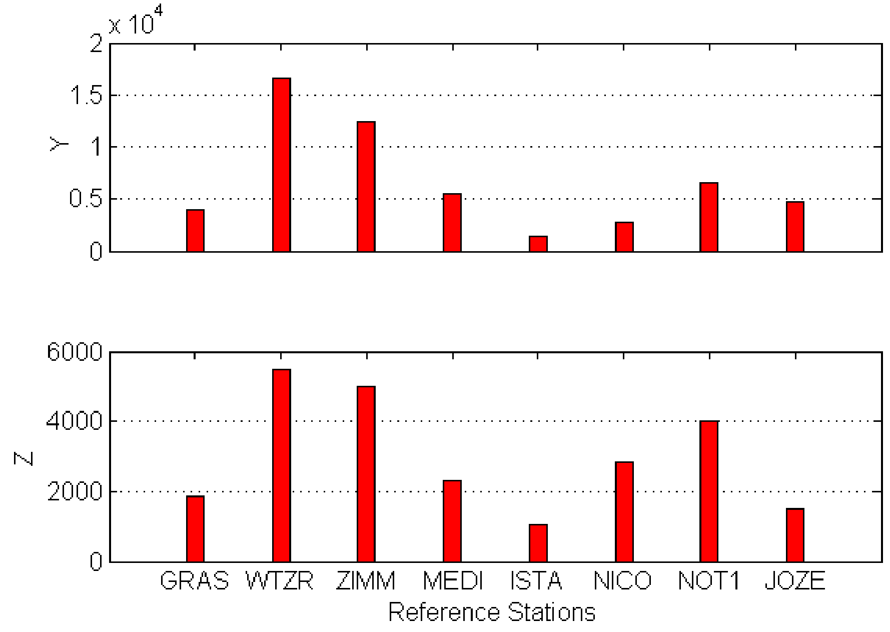
Weights for the 8 reference stations

(note the **inter-station** weight variation)

For their coordinates



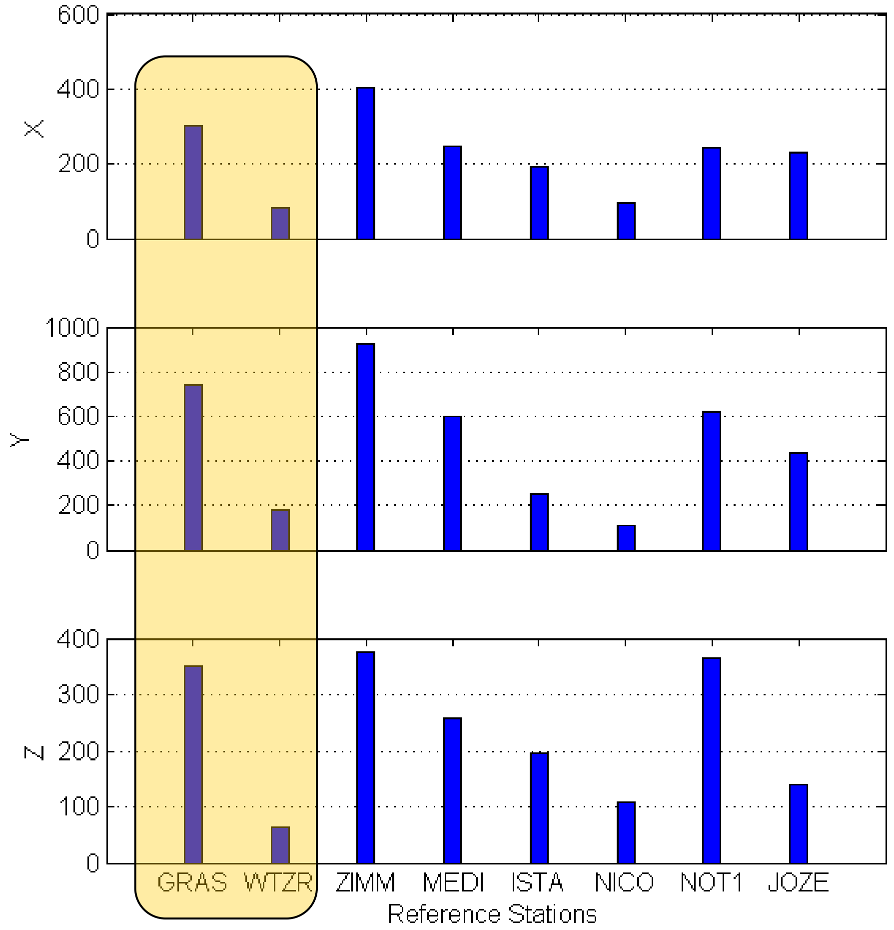
For their velocities



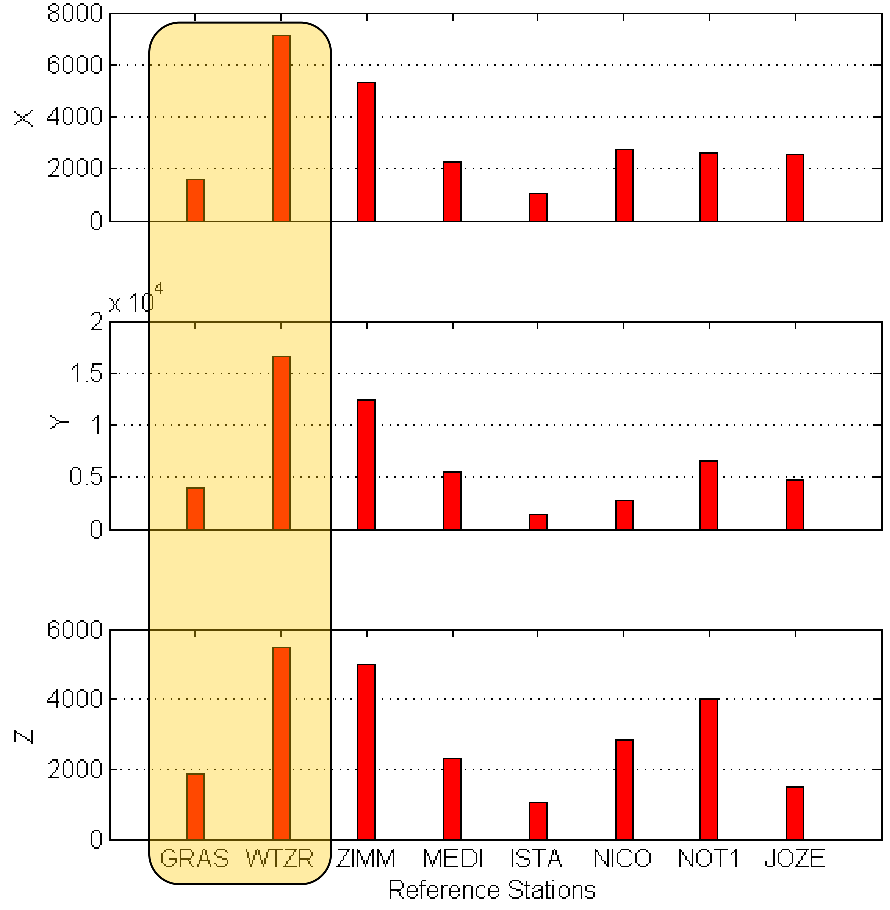
Weights for the 8 reference stations

(note the **inter-station** weight variation)

For their coordinates

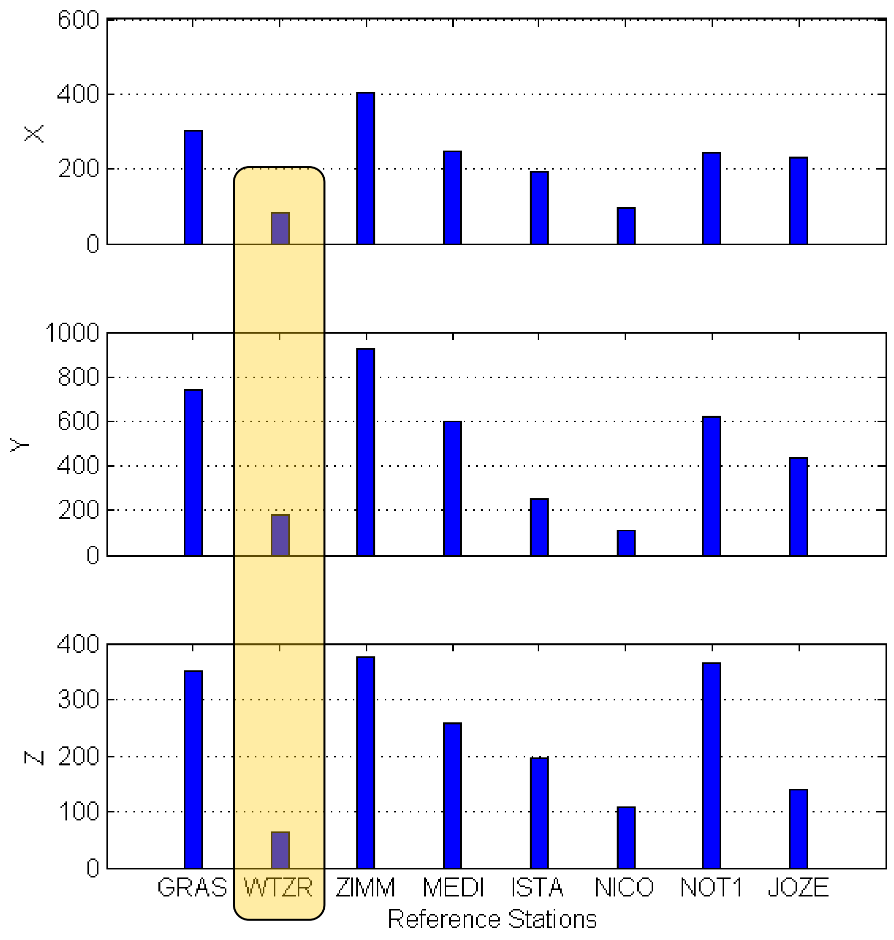


For their velocities

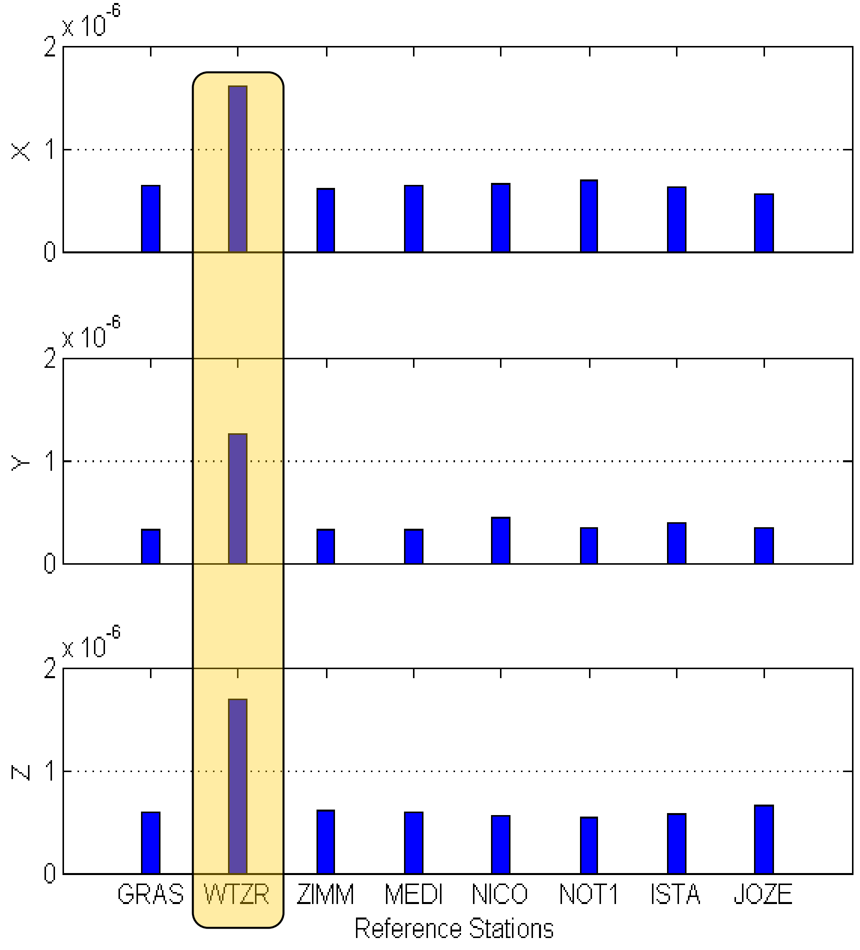


Reference station weighting vs. IGS08 accuracy

Optimal coordinate weights for the ref stations

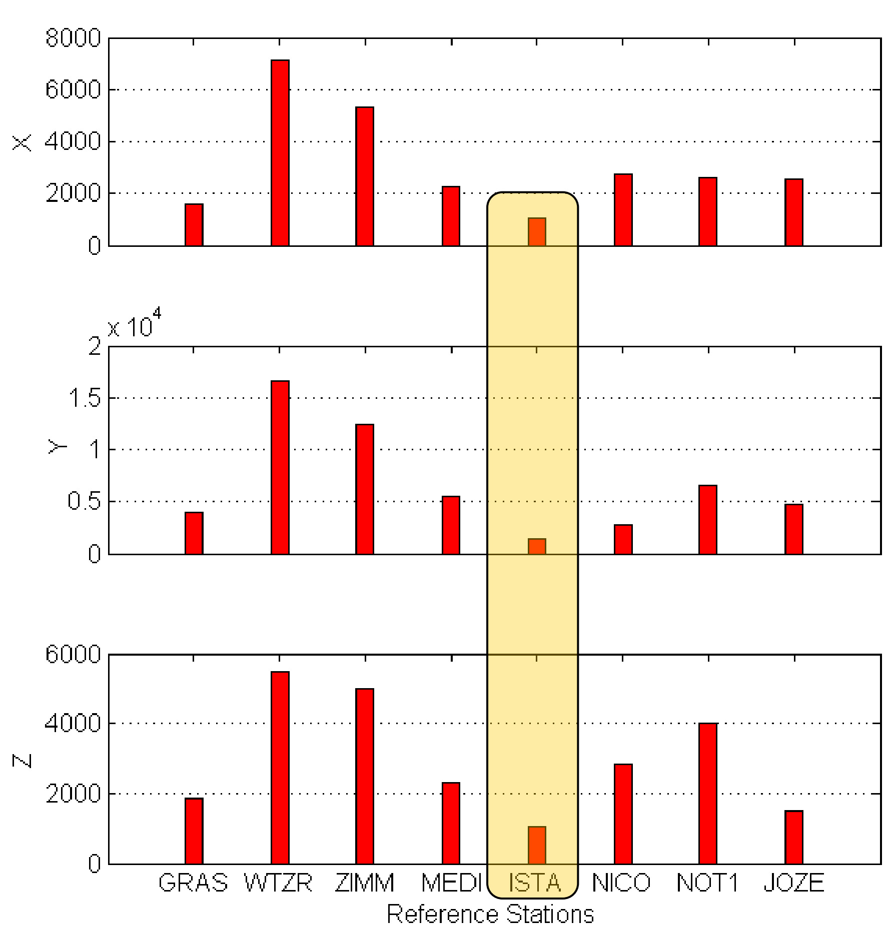


IGS08 coordinate variances

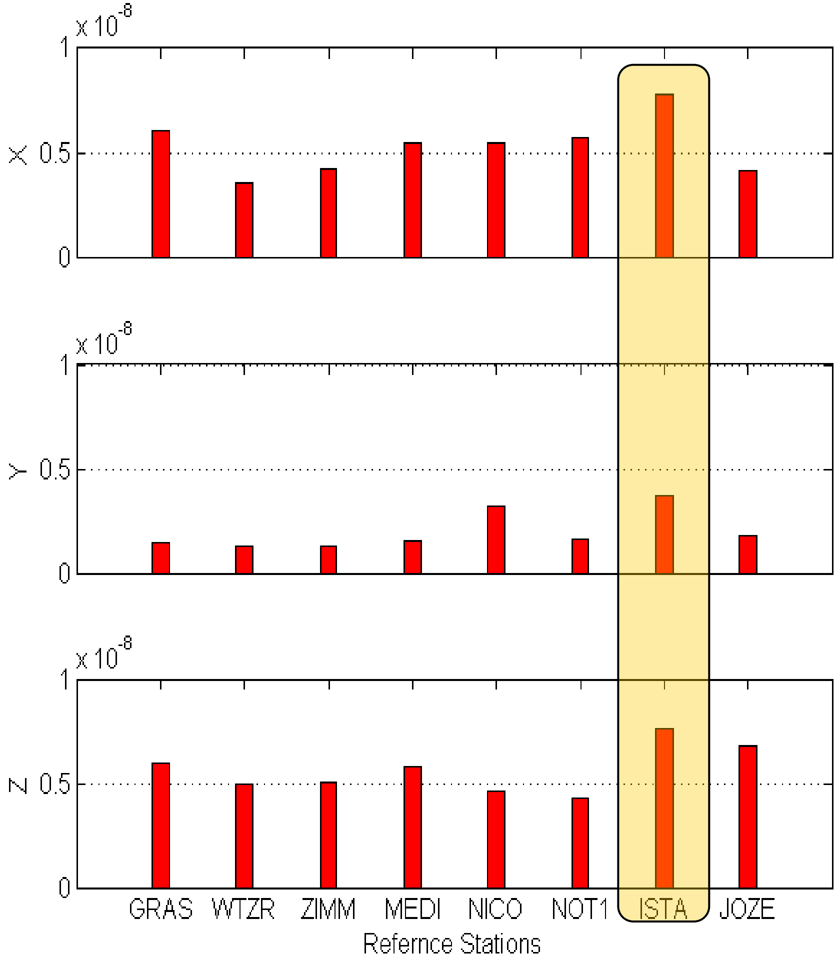


Reference station weighting vs. IGS08 accuracy

Optimal velocity weights for the ref stations



IGS08 velocities variances



Conclusions

- Optimal weighting of reference stations can provide a significant improvement of the estimation accuracy in GNSS network solutions with respect to a desired reference frame.

- The weighting approach presented herein is based on the minimization of the data and datum noise effects over all network stations.

  other optimal weighting schemes are also possible!

- The optimal weight matrix captures the significance of the reference stations for the datum definition in the underlying network
(in terms of geometry, data quality and prior position accuracy)

  it may also be sensitive to hidden outliers within the reference stations
  (see next presentation)