Realization aspects of the International Height Reference System
An exposure of some open problems

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International Height Reference System (IHRS)

**Working definition:**
*(by Ad-hoc group on IHRS, Travaux de l’ IAG, vol. 39)*

The IHRS is a geopotential reference system co-rotating with the Earth in its diurnal motion in space.

The associated coordinates in that system are:

- geopotential values $W(\mathbf{X})$
- geocentric Cartesian coordinates $\mathbf{X}$
  (and their changes in time)
IHRS scientific objectives

- To merge Earth’s geometrical and physical representations in a consistent and useful way.

- To provide an accurate (1 cm or better) and stable physical height frame that is accessible by space geodetic techniques.

- To facilitate the geophysical “predictability” and “interpretability” of:
  - vertical station motions
  - surface gravity variations
  - sea level rise
Heighting in the IHRS context

- The **primary vertical coordinates** are scalar potential differences.
  \[ C(X) = W_o - W(X) \]

- **Physical heights** are derived by suitable metrics.
  \[ H(X) = \frac{W_o - W(X)}{\tilde{g}(X)} \]

The parameter "\( W_o \)" reflects the **vertical datum** of the IHRS and it needs to be clearly specified in its definition.
Conventions for the definition and the realization of IHRS

(IAG Resolution 1, Prague 2015)

1. The **vertical reference level** is an equipotential surface of the Earth’s gravity field with the geopotential value \( W_0 \).
2. Parameters, observations, and data shall be related to the **mean tidal system** and the **mean crust**.
3. Unit of length is the \( m \) and unit of time is the \( sec \) (SI).
4. The **vertical coordinates** are the geopotential numbers with respect to the reference level \( W_0 \).
5. The **spatial reference** of the position \( P \) for the geopotential determination \( W_P = W(X) \) is related to the ITRS.

- \( W_0 = 62\,636\,853.4\, m^2\, s^{-2} \) (datum realization).
Open problems ...

Correlating Earth’s time-variable gravity field and its deforming geometry is a complicated task!
# IHRS in the deforming Earth

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**Choice of geopotential representation**

**Temporal evolution & geoph "predictability"**

**Alignment to ITRS/ITRF**

**Frame densification**
IHRS realization
(semi-dynamic approach)

A mean (static) representation of the gravity field is used.
Physical height changes in IHRS reflect true vertical displacements!
IHRS realization
(semi-dynamic approach)

but temporal variations of observed gravity cannot be fully attributed to the physical height changes in IHRS!
**IHRS’ temporal evolution**  
*(semi-dynamic approach)*

\[
\begin{align*}
\dot{H}_{IHRS} &= \dot{h}_{IHRS} \quad (\dot{N} = 0) \\
\dot{W}_{IHRS} &= \ddot{g}(P) \cdot \dot{X}_{IHRS} \neq \dot{W}_{true}
\end{align*}
\]

\[
X(P') = X(P) + \dot{X}_{IHRS}(t' - t)
\]
Geophysical monitoring
(linearized context)

\[ \dot{g}_{\text{true}} \approx \frac{\partial g}{\partial H} \dot{H}_{\text{IHR}} + \left( \frac{\partial g}{\partial t} \right) \]

\[ \dot{W}_{\text{true}} \approx \vec{g} \cdot \dot{X}_{\text{IHR}} + \left( \frac{\partial W}{\partial t} \right) \]

Inferred from models
Observed (GRACE)
Geophysical monitoring
(time series context)

\[ W_i^{true}(t) = W_i^{IHRS}(t) + \Delta W_i(t) \]

\[ W_i^{true}(t') = W_i^{IHRS}(t') + \Delta W_i(t') \]

e.g. evaluated by GRACE models at current point position

e.g. mass-transport & loading effects on the gravity potential
Two (more practical) questions

1) If IHRS will support the **unification of existing local/regional VDs**, then how are we supposed to deal with the different “potential scales”? e.g. are we allowed to simply merge a leveling-based height frame with IHRS?

2) What will be the value of spirit-leveled data in the realization and temporal evolution of IHRS?
Some comments on $W_o$

- Same parameter – different roles/meanings
  - conventional “zero” vertical level for IHRS
  - best estimate of global MSL from altimetry data
  - $L_G = W_o/c^2$ (IAU 2000 Resolution)
  - Earth reference model (i.e. $W_o \leftrightarrow U_o$)

- Is there any profound reason to update $W_o$ in the context of (future) IHRS realizations?

- Should “$W_o$” be tagged in the IHRS conventions? ($t_o, GM, \omega, \text{other}$)
Conclusions

- IHRS is a much-needed tool to unify the three pillars of geodesy!

- Three crucial items need to be elucidated:
  - choice of geopotential representation
  - its alignment procedure to ITRS/ITRF
  - the time-dependent character of IHRS and its geophysical “predictability”

- and ...
Conclusions

Is the **mean tidal system** the best choice for the definition of the IHRS?