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Unconstrained NEQ with full rank in GNSS networks

Crucial problem or trivial nuisance for TRF applications?

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Introduction

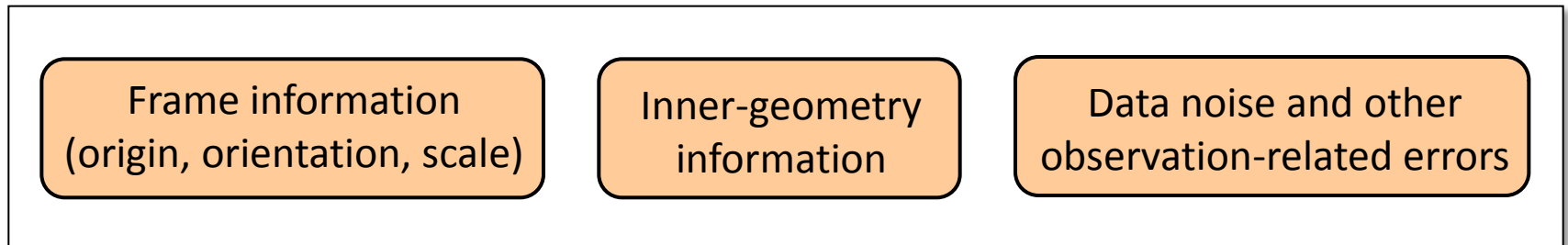
- ❑ The original (unconstrained) NEQ that are formed in daily, weekly or multi-epoch processing of GNSS networks are often invertible; with no datum defect.
- ❑ Applying “minimal constraints” to such full-rank systems creates distortions in the final solution!

Particularly, it affects:

- the estimated network geometry
- well-estimable frame parameters
- other inferences of scientific interest
(assessment of loading effects in GNSS height time-series)

Problem description

Unconstrained NEQ (full-rank)



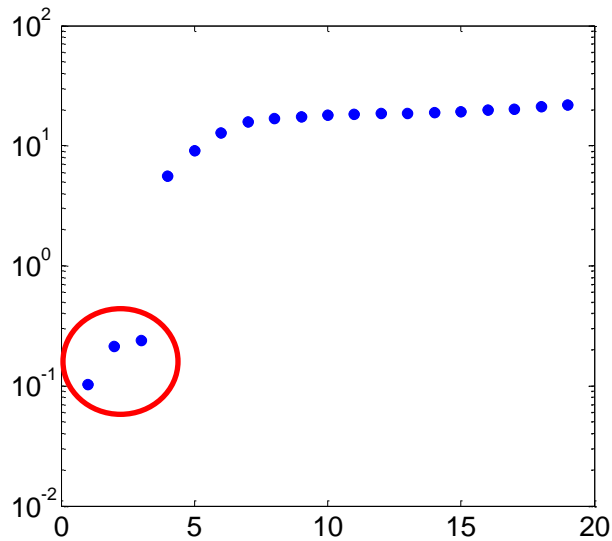
Additive “minimal” constraints (e.g. NNT)



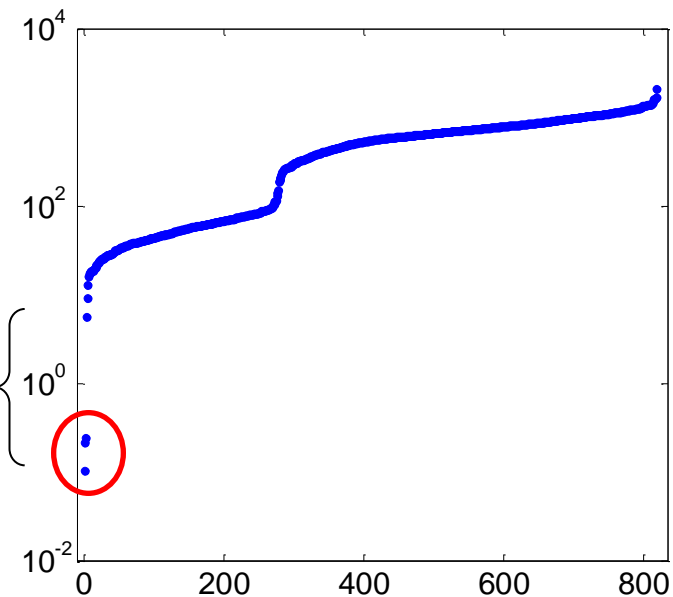
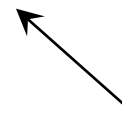
NEQ–MC “interference”

- This is not necessarily bad from an estimation viewpoint!
- Yet, it causes effects that may be unwanted in TRF applications.

Spectrum of weekly unconstrained normal matrix (EPN network)



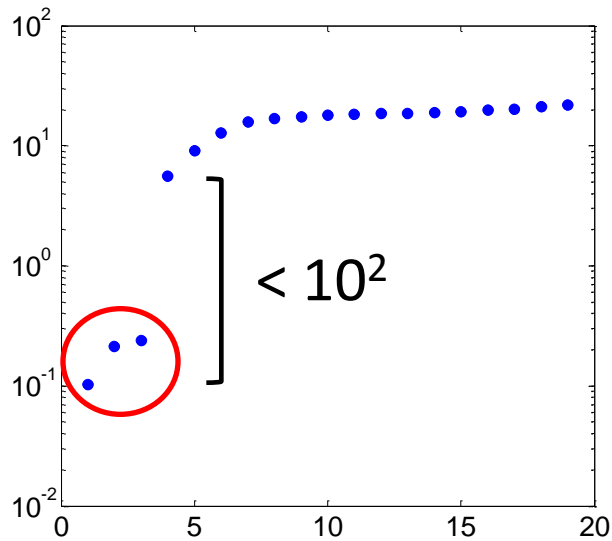
The first 20 eigenvalues



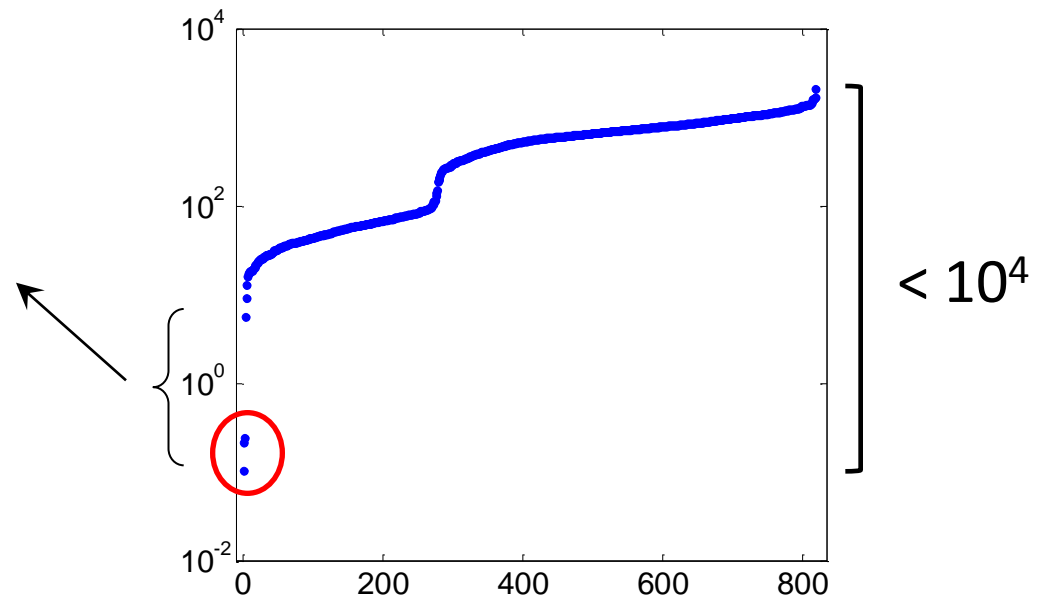
All eigenvalues

EUR18697.snx
(1-7/11/2015)

Spectrum of weekly unconstrained normal matrix (EPN network)



The first 20 eigenvalues



EUR18697.snx
(1-7/11/2015)

Well-conditioned invertible matrix !

Two key questions for the present study

- How significant is the distortion to “minimally constrained” EPN solutions due to absence of true rank defect in the original NEQ ?
- How can we convert unconstrained NEQ to truly singular systems with given rank defect for particular datum parameters, without altering their internal geometrical content ?

Detecting distortion due to absence of NEQ singularity

Compute and compare the following solutions from the original normal system $\mathbf{N}(\mathbf{X}-\mathbf{X}_o)=\mathbf{u}$

Free-net solution

$$\hat{\mathbf{X}} = \mathbf{X}_o + \mathbf{N}^{-1}\mathbf{u}$$

Constrained solution by a set of “minimal constraints” $\mathbf{H}(\mathbf{X}-\mathbf{X}_o)=\mathbf{c}$

$$\hat{\mathbf{X}} = \mathbf{X}_o + \left(\mathbf{N} + \mathbf{H}^T \mathbf{W} \mathbf{H}\right)^{-1} \left(\mathbf{u} + \mathbf{H}^T \mathbf{W} \mathbf{c}\right)$$

- The free-net solution always retains the full data information.
- Comparing the above solutions, after a Helmert transformation is applied between them, can reveal the distortion caused by the MCs.

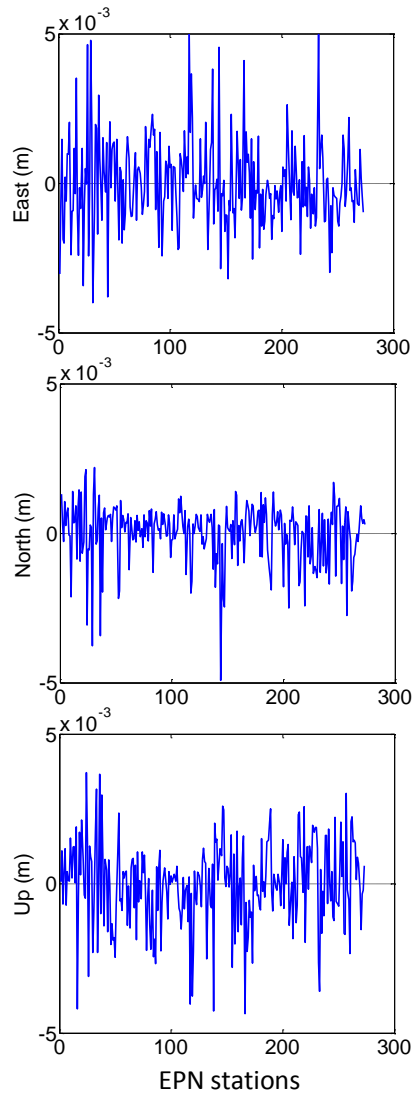
Example

- Experiment with the original (unconstrained) NEQ from several EPN weekly combined solutions.
- For each week, determine the free-net solution and the NNT-constrained solution wrt. IGb08.
 - the weight matrix **W** is set to prior accuracy level of 10^{-5} m for frame origin fixation in each weekly solution
- Fit the above solutions by Helmert transformation, and assess their residuals and their transformation parameters for each week.

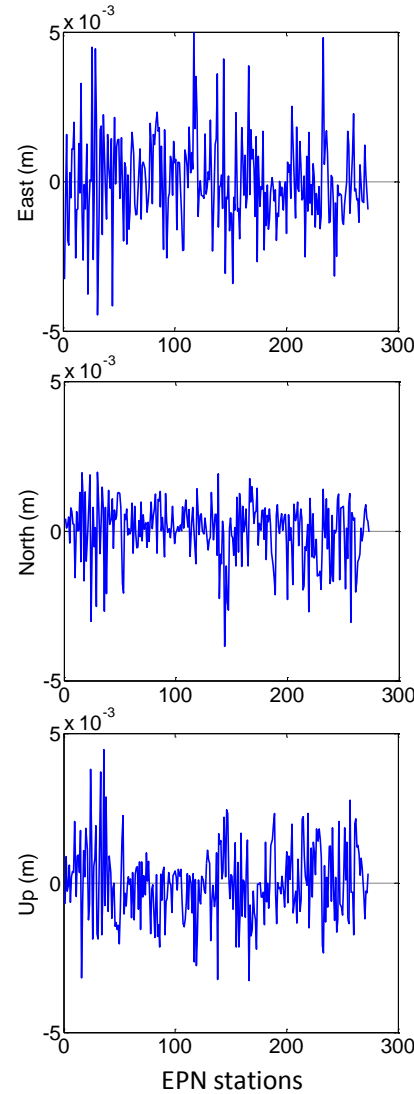
Used reference stations for applying NNT wrt. IGb08

ANKR, BOR1, BRST, BUCU, GLSV, GRAS,
GRAZ, HERS, HERT, HOFN, ISTA, JOZ2, JOZE,
LROC, MAR6, MAS1, MATE, MDVJ, MEDI,
METS, MORP, NICO, NOT1, NYA1, ONSA, PDEL,
POLV, POTS, QAQ1, RABT, RAMO, RIGA, SCOR,
SFER, SOFI, THU3, TLSE, TRO1, UZHL, VILL,
WSRT, WTZR, YEBE, ZECK, ZIMM

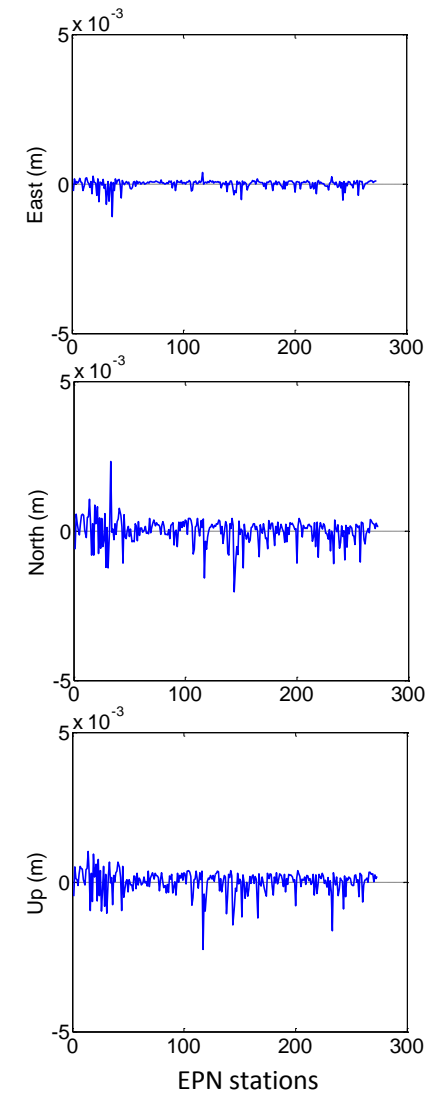
Transformation residuals (week 1869)



Shift-only model



Shift/roto model



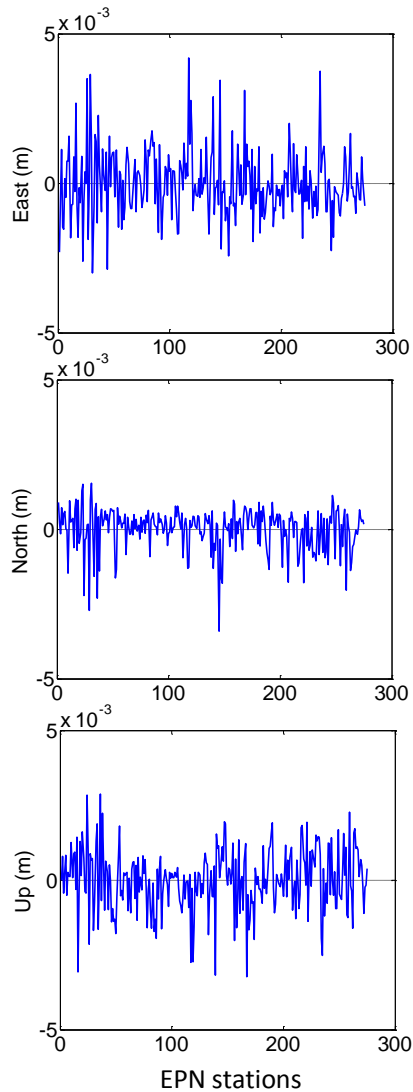
Full-similarity model

Helmert transformation parameters btw free-net and NNT-constrained weekly EPN solutions (week 1869)

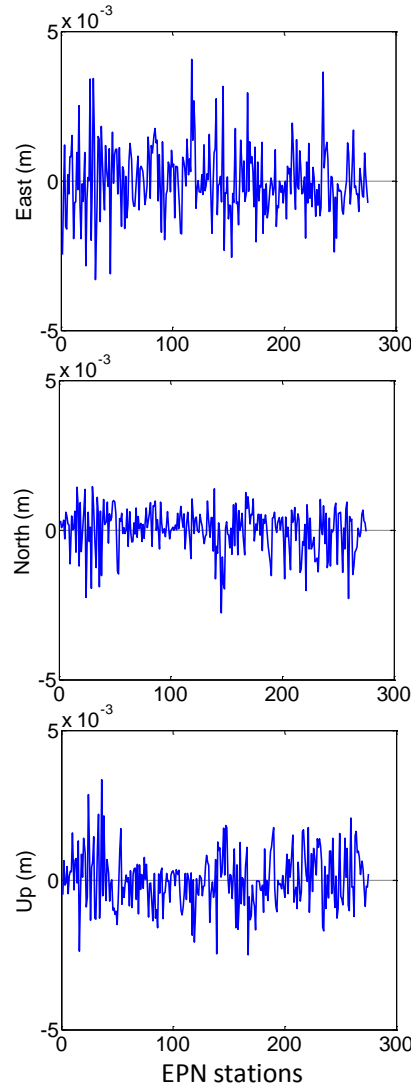
	Tx (cm)	Ty (cm)	Tz (cm)	Rx (mas)	Ry (mas)	Rz (mas)	Scale (ppb)
Shift-only	2.64	1.06	2.94				
Shift/roto	2.59	1.44	2.93	-0.10	-0.01	0.08	
Full-similarity	3.21	1.55	3.66	-0.10	-0.01	0.08	-1.55

(*) It seems that the NNT constraints affect the scale and orientation of the weekly solution in IGb08!

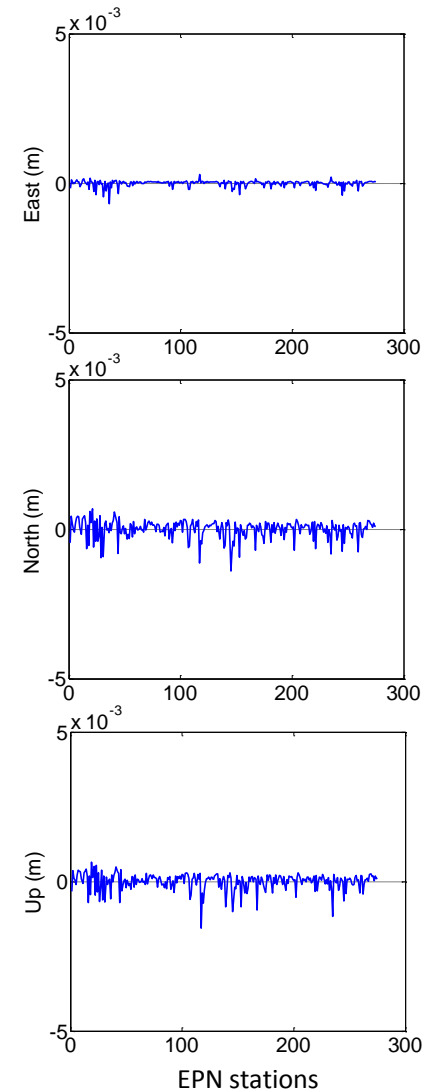
Transformation residuals (week 1870)



Shift-only model

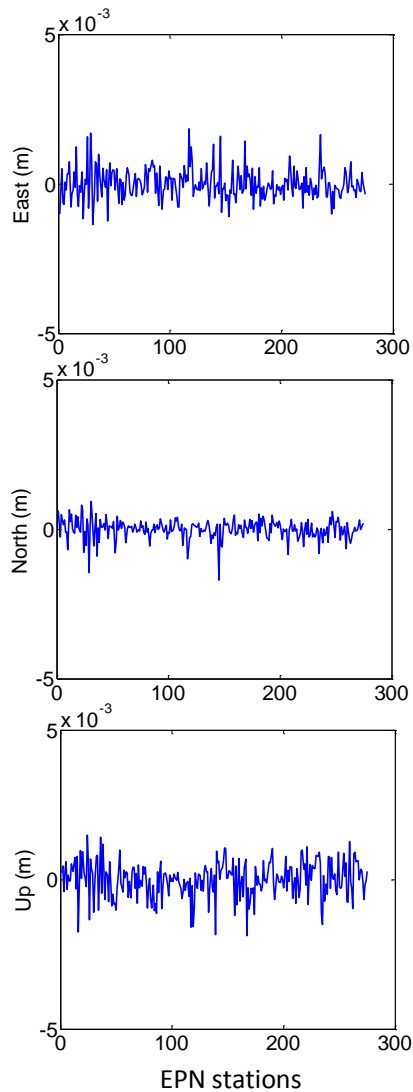


Shift/roto model

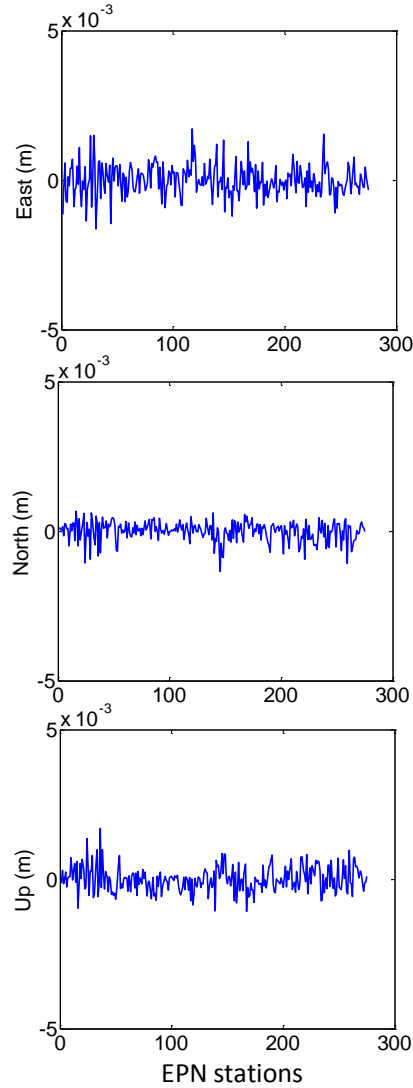


Full-similarity model

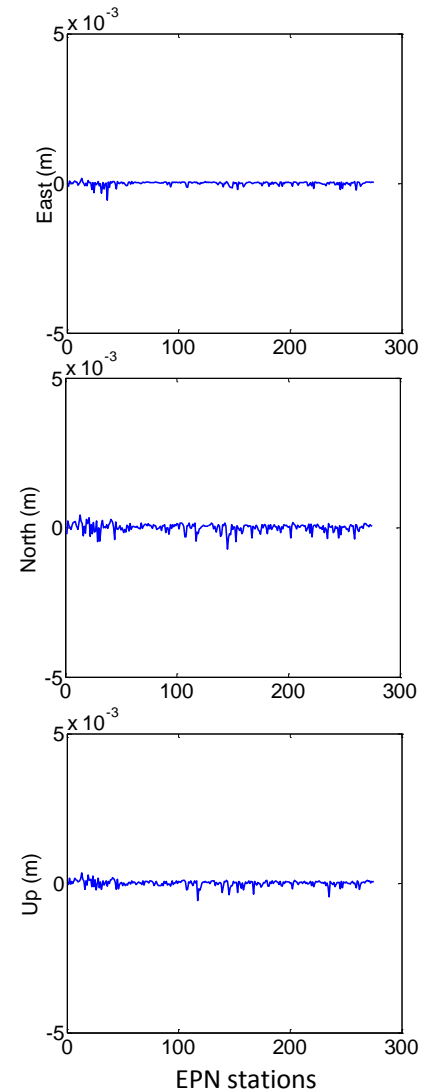
Transformation residuals (week 1871)



Shift-only model

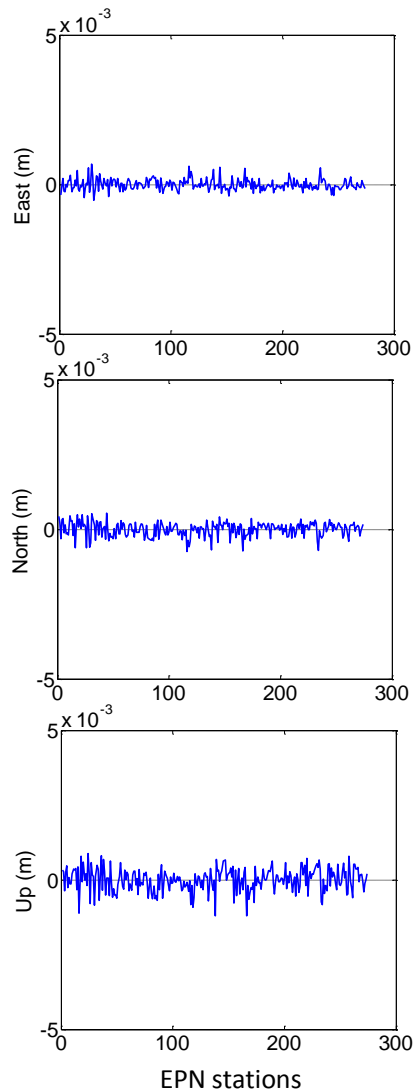


Shift/roto model

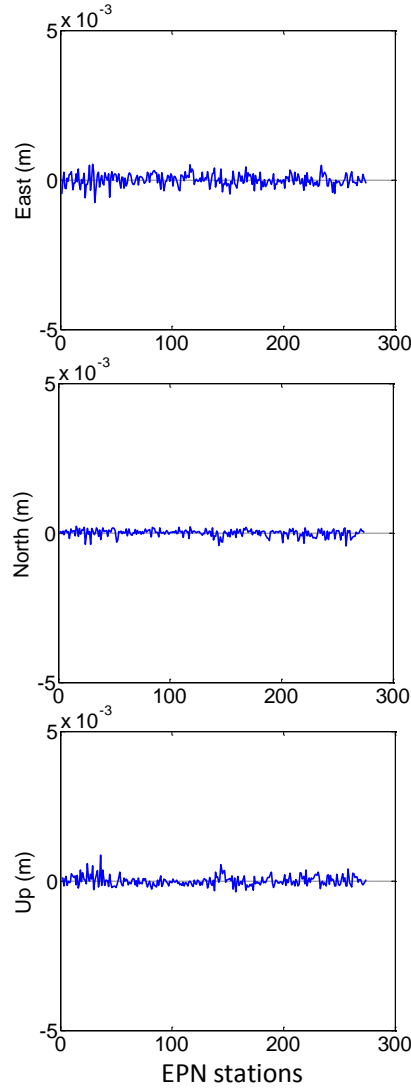


Full-similarity model

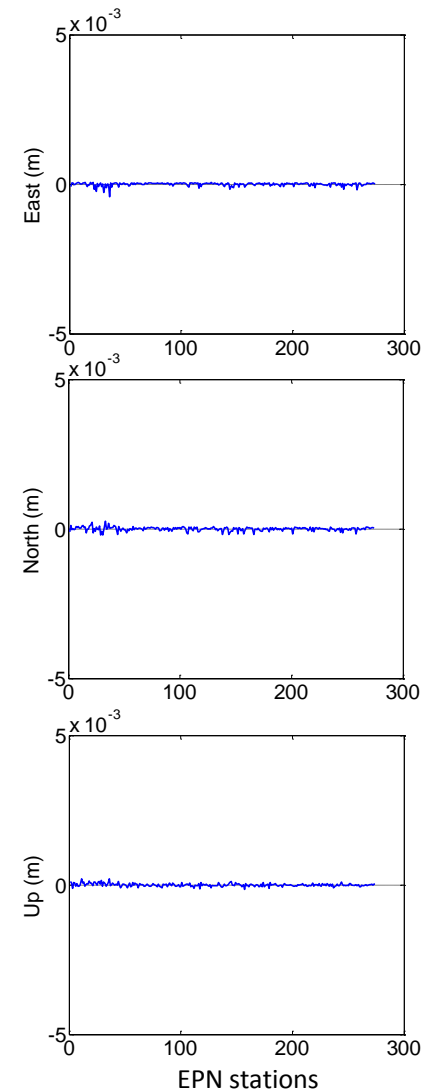
Transformation residuals (week 1872)



Shift-only model

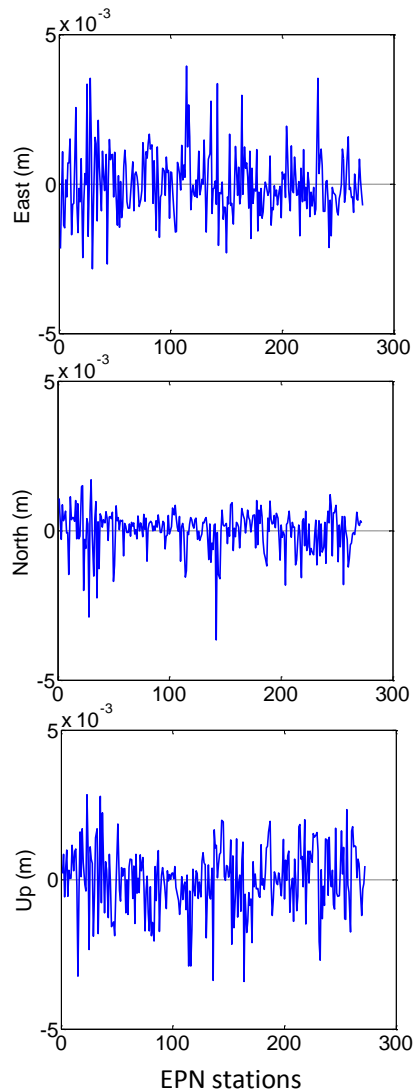


Shift/roto model

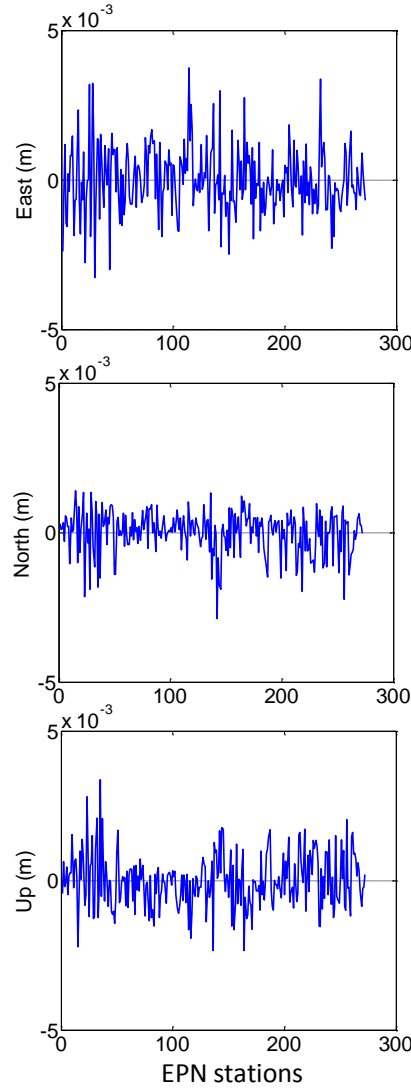


Full-similarity model

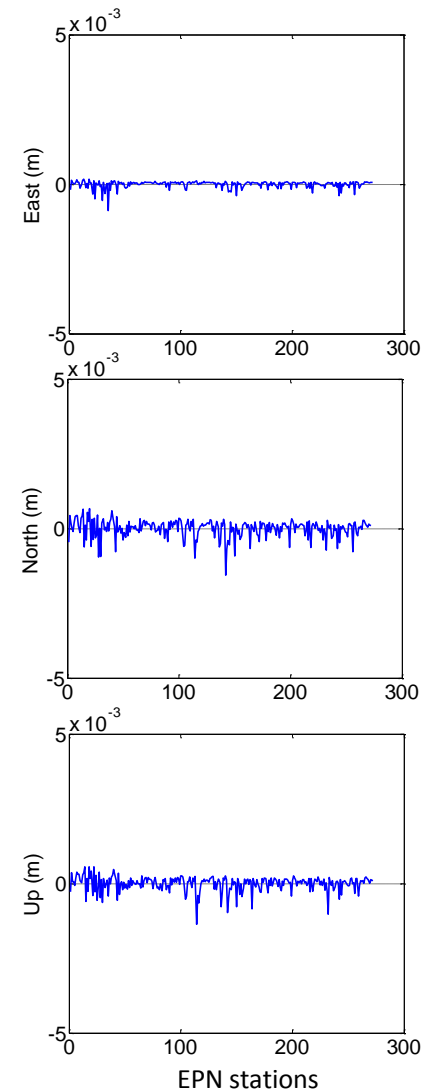
Transformation residuals (week 1873)



Shift-only model



Shift/roto model



Full-similarity model

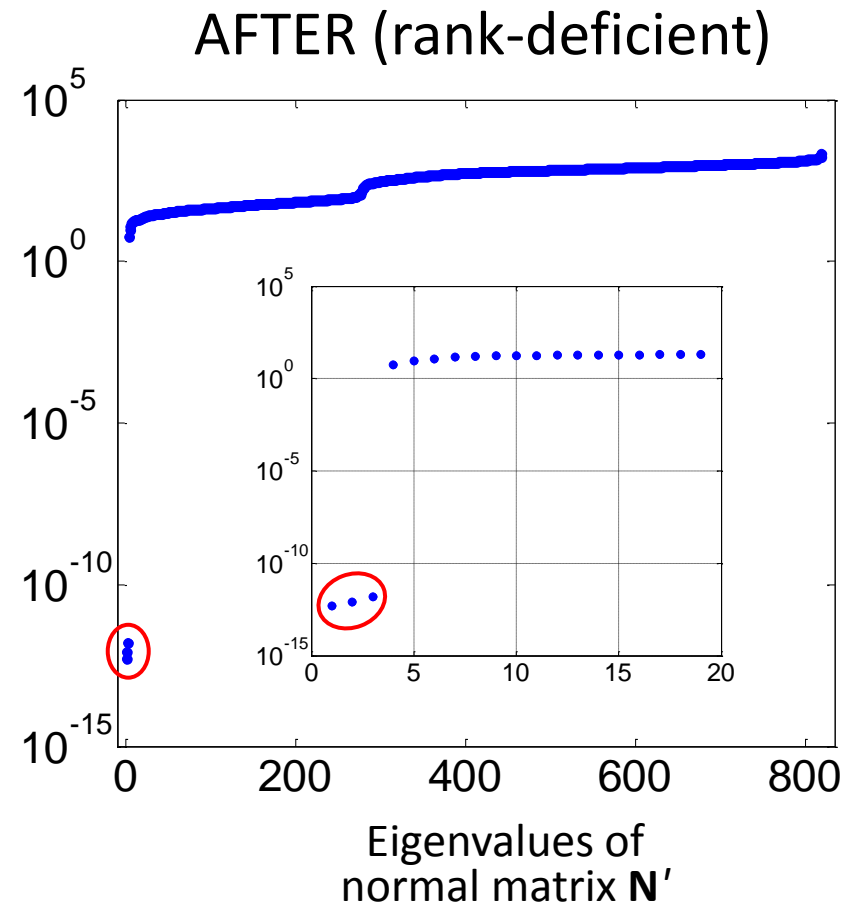
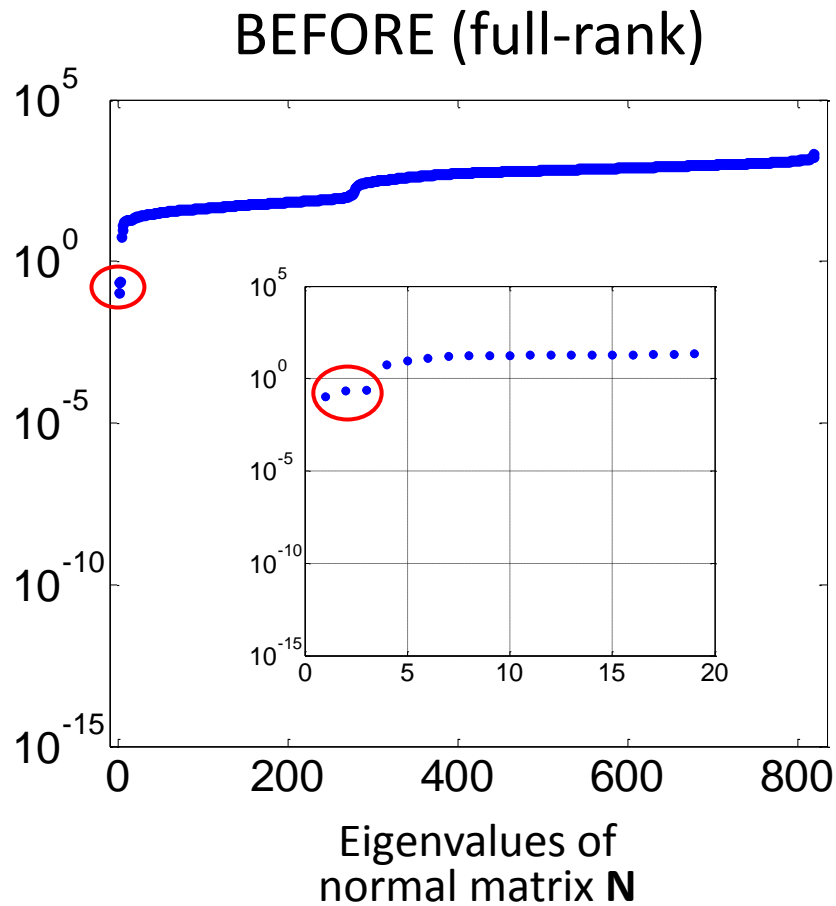
Helmert transformation parameters btw free-net and NNT-constrained weekly EPN solutions

	Tx (cm)	Ty (cm)	Tz (cm)	Rx (mas)	Ry (mas)	Rz (mas)	Scale (ppb)
Week 1869	3.21	1.55	3.66	-0.10	-0.01	0.08	-1.55
Week 1870	2.32	0.98	2.87	-0.06	-0.01	0.05	-1.16
Week 1871	0.79	0.89	1.51	-0.06	-0.03	0.05	-0.53
Week 1872	-0.15	0.77	0.92	-0.05	-0.05	0.04	-0.20
Week 1873	2.07	1.33	2.89	-0.09	-0.03	0.07	-1.12

Removal of frame information from unconstrained NEQ

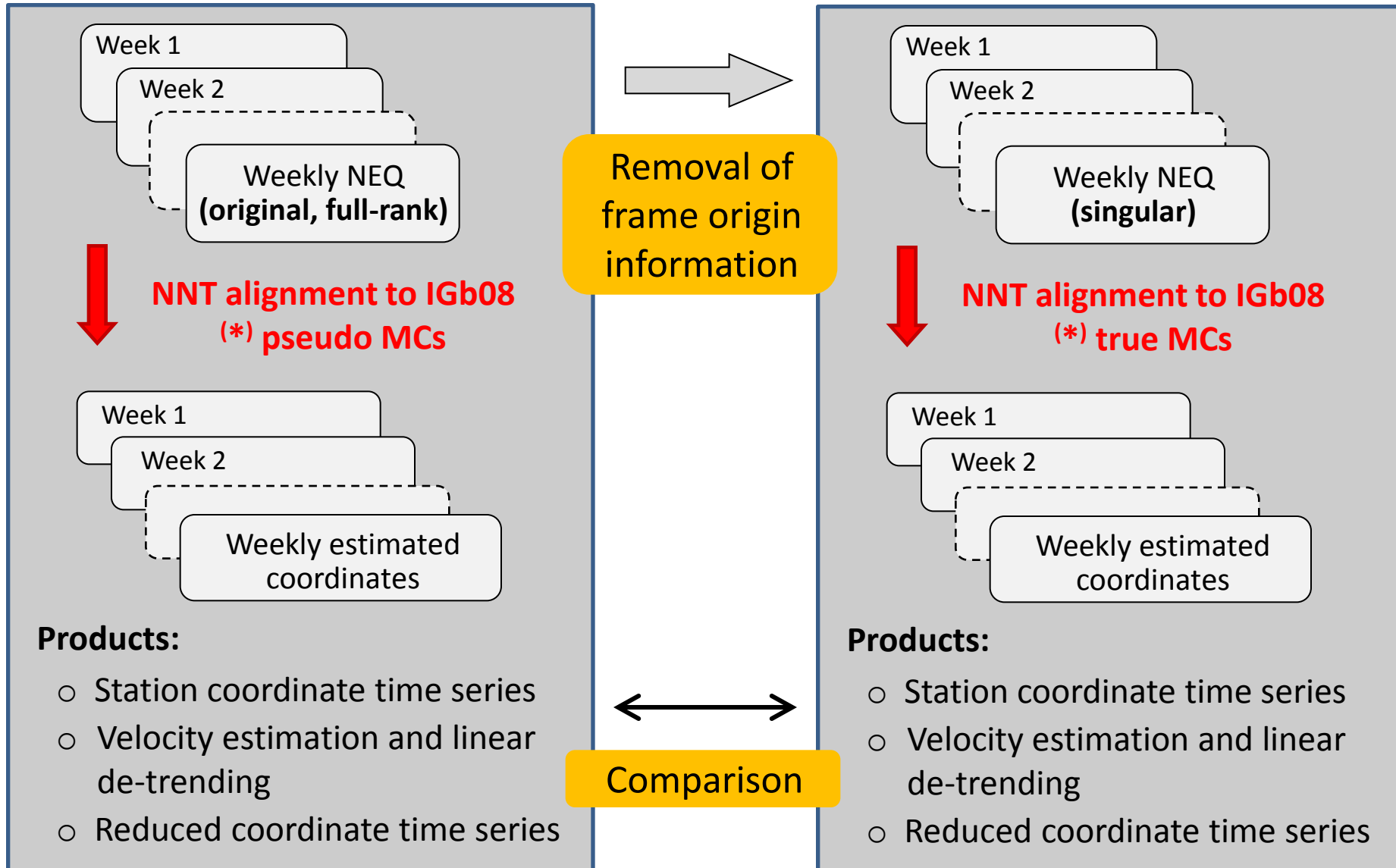
Unconstrained NEQ	$\mathbf{N}(\mathbf{X} - \mathbf{X}_o) = \mathbf{u}$ <p>Original system without rank defect</p>	$\mathbf{N}'(\mathbf{X} - \mathbf{X}_o) = \mathbf{u}'$ <p>Modified system with “proper” rank defect</p>
Helmert transformation matrix & handling of datum parameters	$\mathbf{G} = \begin{bmatrix} \mathbf{E} \\ \tilde{\mathbf{E}} \end{bmatrix} \begin{matrix} \} \boldsymbol{\theta} \\ \} \tilde{\boldsymbol{\theta}} \end{matrix}$ <p>$\boldsymbol{\theta}$: ill-defined datum parameters to be fully removed from the NEQ $\tilde{\boldsymbol{\theta}}$: well-estimable datum parameters from the available data</p>	
Filtering of selected datum parameters (a.k.a. CDR)	$\mathbf{N}' = \left(\mathbf{I} - \mathbf{N}\mathbf{E}^T (\mathbf{E}\mathbf{N}\mathbf{E}^T)^{-1} \mathbf{E} \right) \mathbf{N}$ $\mathbf{u}' = \left(\mathbf{I} - \mathbf{N}\mathbf{E}^T (\mathbf{E}\mathbf{N}\mathbf{E}^T)^{-1} \mathbf{E} \right) \mathbf{u}$	
Preservation of original NEQ information	$\mathbf{N}'(\mathbf{N}^{-1}\mathbf{u}) = \mathbf{u}'$ <p>(*) the free-net solution of the original NEQ satisfies also the modified singular NEQ</p>	

EPN weekly combined NEQ

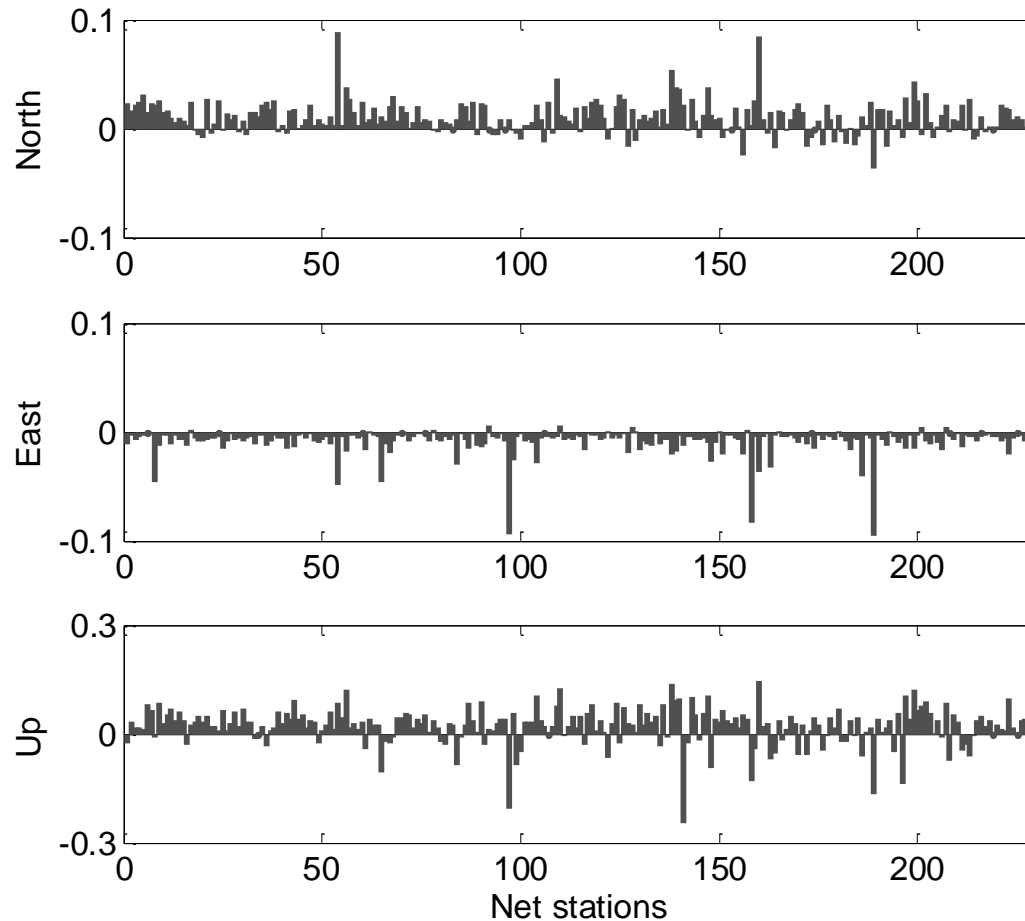


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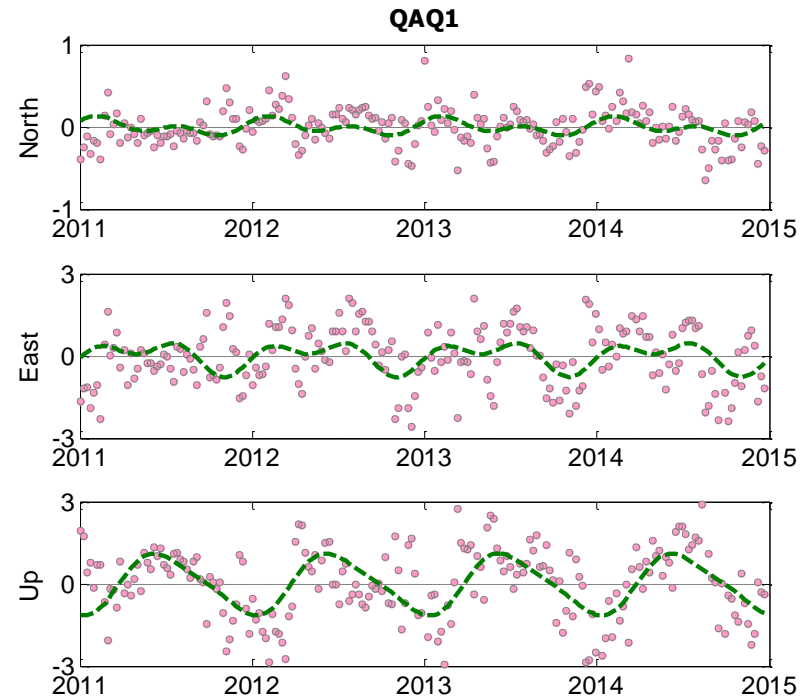
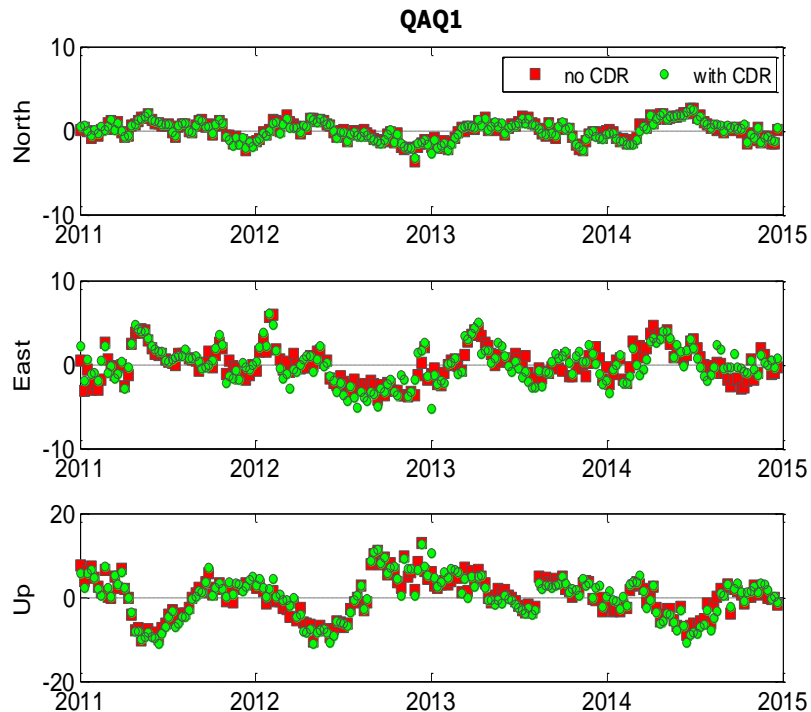
Testing on EPN weekly NEQ (2011-2015)



Differences (in mm/year) of the estimated EPN station velocities

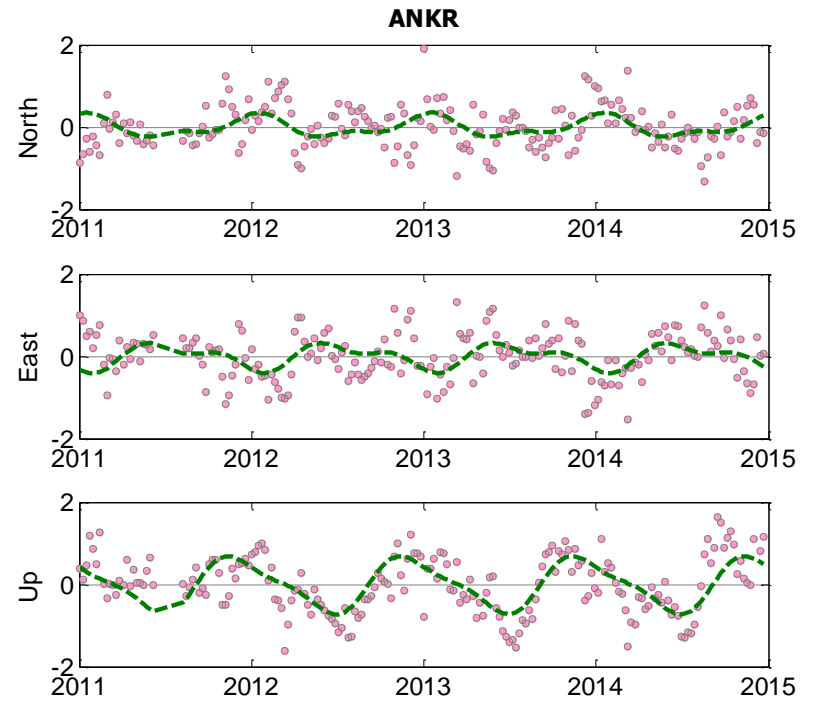
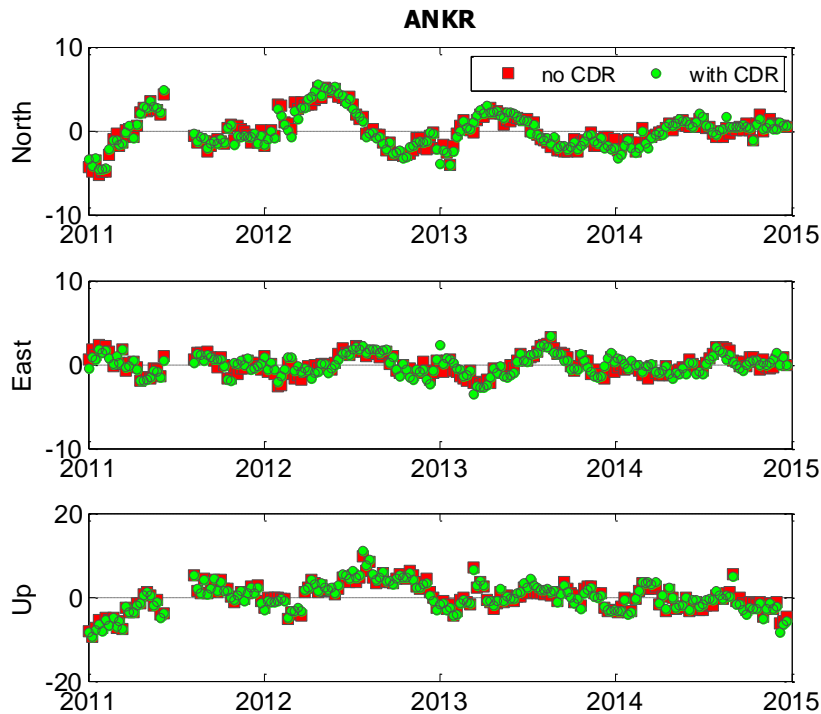


Reduced coordinate time series and their differences (in mm)



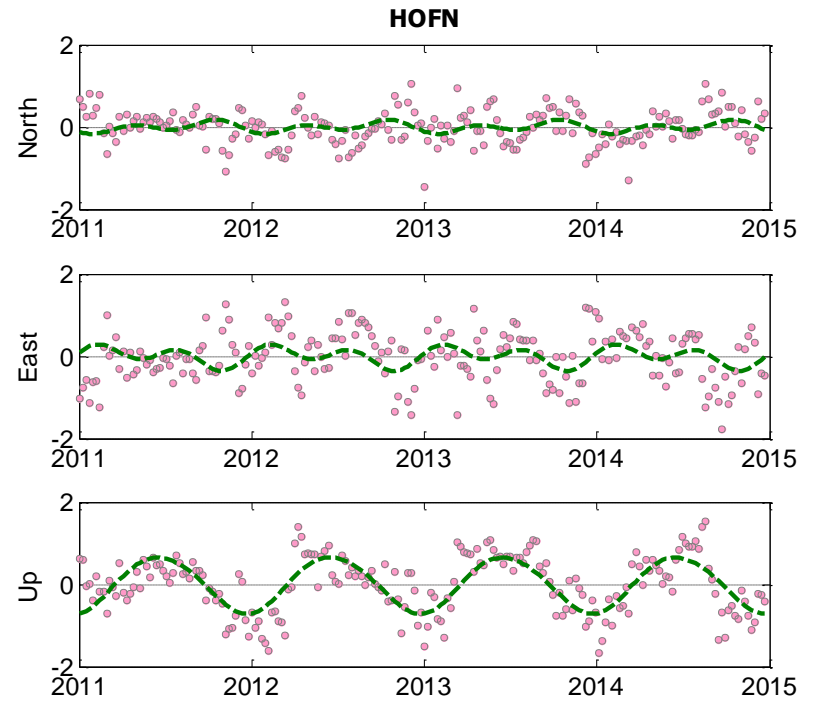
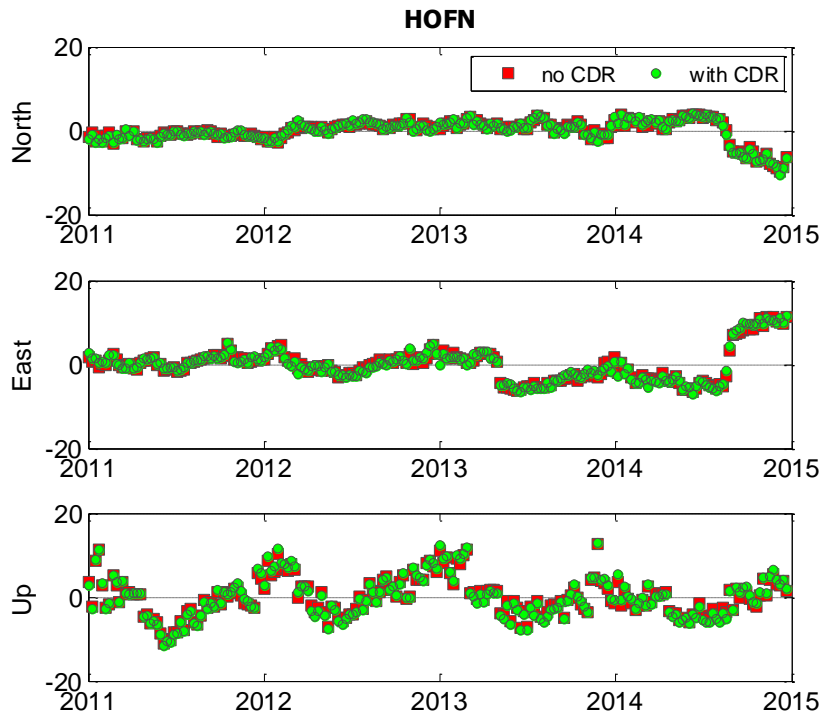
systematic effect on the annual and semi-annual variations
(especially in the height time-series!)

Reduced coordinate time series and their differences (in mm)



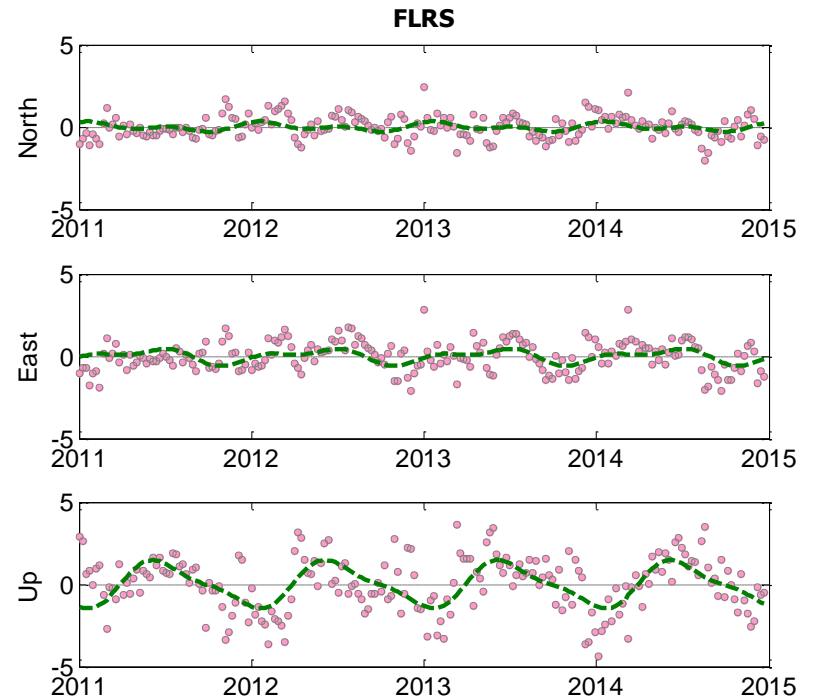
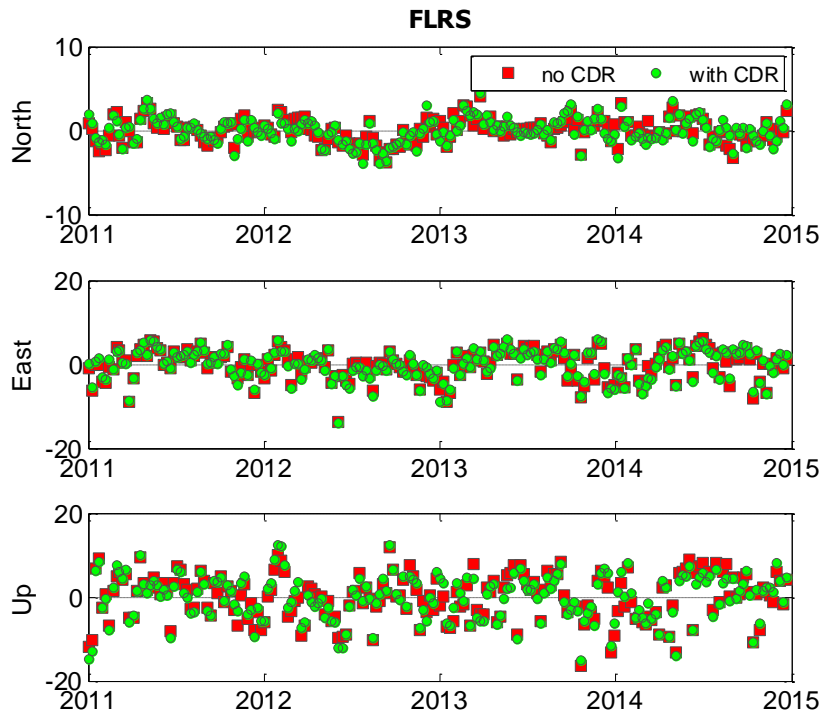
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Reduced coordinate time series and their differences (in mm)



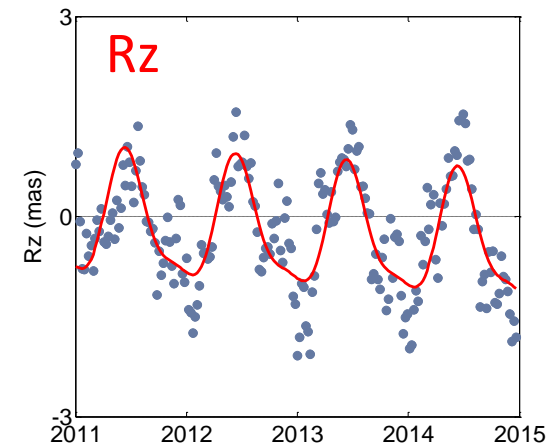
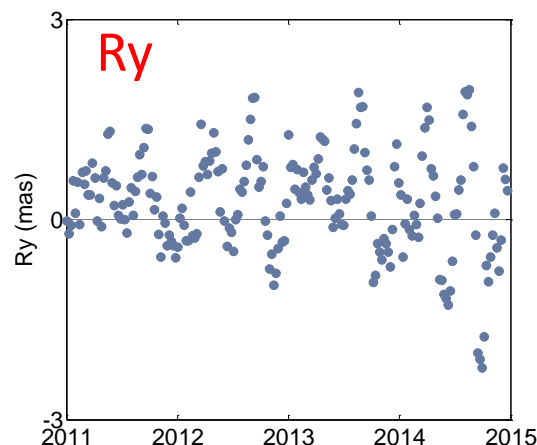
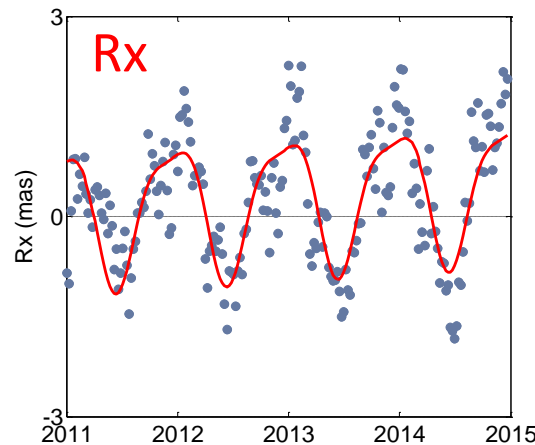
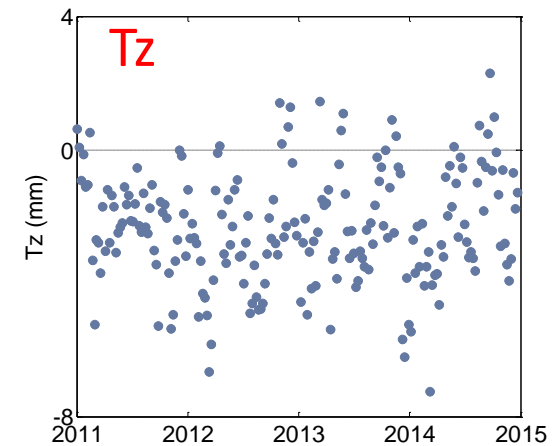
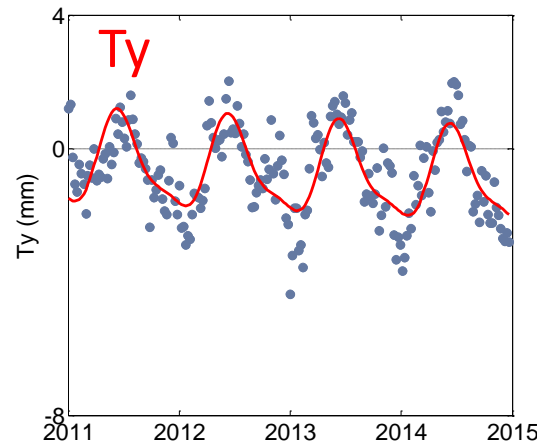
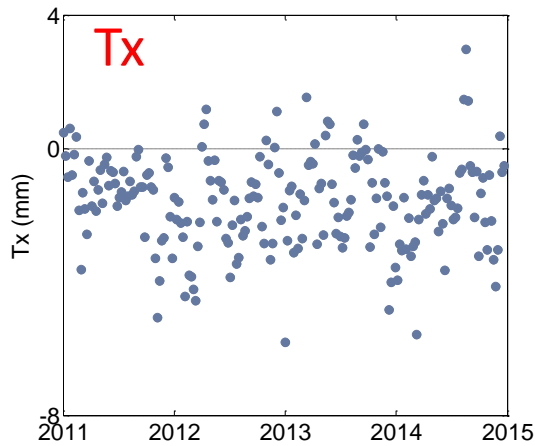
systematic effect on the annual and semi-annual variations
(especially in the height time-series!)

Reduced coordinate time series and their differences (in mm)

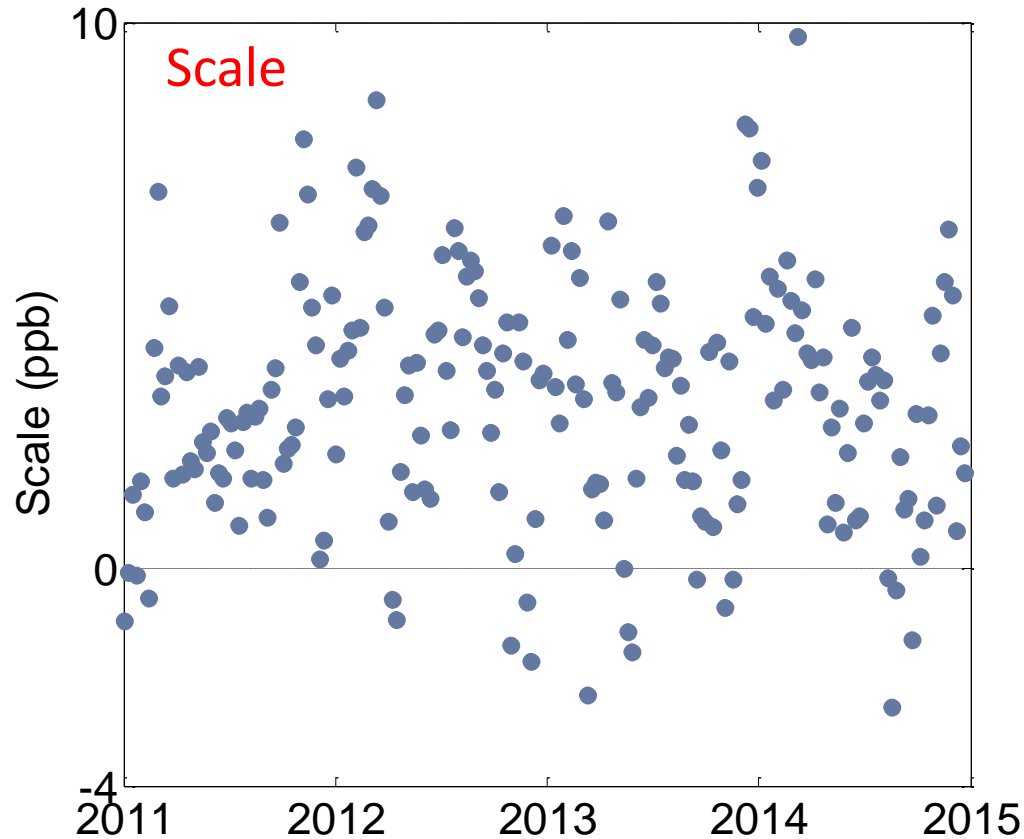


systematic effect on the annual and semi-annual variations
(especially in the height time-series!)

Helmert transformation parameters (withCDR vs. noCDR weekly solutions)



Helmert transformation parameters (with CDR vs. noCDR weekly solutions)



Conclusions

- The implementation of “minimal constraints” to unconstrained full-rank NEQ can create various types of distortion in the computed solutions.
- In the case of EPN’s weekly combined NEQ:
 - geometrical distortions of **several mm** were identified in the NNT-constrained weekly solutions
 - a significant part of these distortions seems to be equivalent to a scale-like bias of several ppb
 - vertical velocities inferred from the corresponding time series showed a systematic effect of more than **0.1-0.2 mm/yr**
 - additional systematic effects occur in the annual/semi-annual periodic variations in the reduced time series

Conclusions

- The aforementioned distortions can be prevented by a simple “pre-filtering” of selected frame parameters from the original normal system.
 - the removed information may refer only to ill-defined frame parameters (as in the present study) or it could be related to a complete de-stripping of the original NEQ’s datum content!
 - this is useful not only in GNSS network analysis, but also in multi-technique combination strategies for TRF estimation at the NEQ level