Solving the reference station weighting problem in minimally constrained networks

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EUREF Annual Symposium
Vilnius, Lithuania, June 3-7, 2014

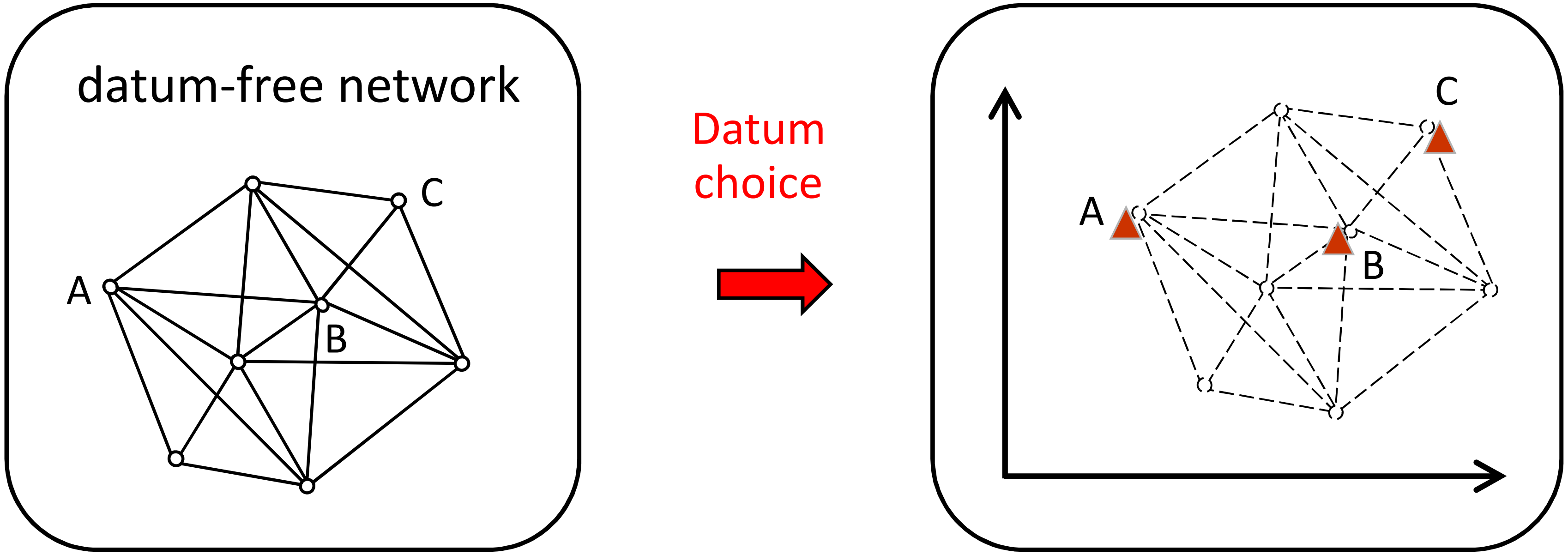
Introduction

- Minimally constrained (MC) network adjustment is a standard tool for geodetic frame realizations.

- Optimal weighting for the reference stations (within the MCs) has not been dealt with.

- The aim of this paper is to resolve the reference station weighting problem in the MC framework based on an optimal statistical setting.

Rationale



Minimal constraints on reference stations

$\mathbf{E}(\mathbf{x}-\mathbf{x}^{\mathrm{ref}}) = \mathbf{0}$ or, more generally $\mathbf{E}\mathbf{P}(\mathbf{x}-\mathbf{x}^{\mathrm{ref}}) = \mathbf{0}$

Un-resolved issue: choice of the <u>weight matrix</u> P

The matrix E

$$\mathbf{E} = \left[\begin{array}{ccccccc} 1 & 0 & 0 & \cdots & 1 & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 & 1 \\ \hdashline 0 & z_1 & -y_1 & \cdots & 0 & z_m & -y_m \\ -z_1 & 0 & x_1 & \cdots & -z_m & 0 & x_m \\ y_1 & -x_1 & 0 & \cdots & y_m & -x_m & 0 \\ \hdashline x_1 & y_1 & z_1 & \cdots & x_m & y_m & z_m \end{array}\right]$$

Example

Classic form of NNT/NNR conditions	$$\sum_i (\mathbf{x}_i - \mathbf{x}_i^{\mathrm{ref}}) = \mathbf{0}$$ $$\sum_i \mathbf{x}_i^{\mathrm{o}} \times (\mathbf{x}_i - \mathbf{x}_i^{\mathrm{ref}}) = \mathbf{0}$$
Weighted form of NNT/NNR conditions	$$\sum_i p_i (\mathbf{x}_i - \mathbf{x}_i^{\mathrm{ref}}) = \mathbf{0}$$ $$\sum_i \mathbf{x}_i^{\mathrm{o}} \times p_i (\mathbf{x}_i - \mathbf{x}_i^{\mathrm{ref}}) = \mathbf{0}$$

Simplified scheme: diagonal weight matrix with a single scalar weight for each reference station

Example

Classic form of NNT/NNR conditions

$$\sum_i (\mathbf{x}_i - \mathbf{x}_i^{\mathrm{ref}}) = \mathbf{0}$$

$$\sum_i \mathbf{x}_i^{\mathrm{o}} \times (\mathbf{x}_i - \mathbf{x}_i^{\mathrm{ref}}) = \mathbf{0}$$

Weighted form of NNT/NNR conditions

$$\sum_i \mathbf{P}_i \, (\mathbf{x}_i - \mathbf{x}_i^{\mathrm{ref}}) = \mathbf{0}$$

$$\sum_i \mathbf{x}_i^{\mathrm{o}} \times \left(\mathbf{P}_i \, (\mathbf{x}_i - \mathbf{x}_i^{\mathrm{ref}})\right) = \mathbf{0}$$

Simplified scheme: block-diagonal weight matrix with a single weight matrix for each reference station

Frame optimality in classic (un-weighted) MC adjustment

- The realized frame of the adjusted network is optimized at the stations participating in the MCs

  (what about the other network stations?)

- The optimality of the realized frame considers only the data noise effect in the estimated coordinates

  (what about the “datum noise” effect?)

- Optimization of derived frame-dependent quantities (e.g. horizontal coordinates) is not guaranteed !

What do "classic" MCs optimize?

Rank-deficient NEQs: $\mathbf{N}\begin{bmatrix}\delta\mathbf{x} \\ \delta\mathbf{x}'\end{bmatrix} = \mathbf{u}$ reference stations

MCs applied to reference stations: $\mathbf{E}\,(\mathbf{x} - \mathbf{x}^{\mathrm{ref}}) = \mathbf{0}$

Covariance matrix of MC solution:

$$\boldsymbol{\Sigma} = \mathbf{N}^{-} = \left[\begin{array}{c|c} \boldsymbol{\Sigma}_{\hat{\mathbf{x}}} & \boldsymbol{\Sigma}_{\hat{\mathbf{x}}\hat{\mathbf{x}}'} \\ \hline \boldsymbol{\Sigma}_{\hat{\mathbf{x}}'\hat{\mathbf{x}}} & \boldsymbol{\Sigma}_{\hat{\mathbf{x}}'} \end{array}\right]$$

Minimum trace

Data noise effect

Minimization of data noise effect only at the reference stations!

What can "weighted" MCs optimize?

$$\mathbf{N}\begin{bmatrix}\delta\mathbf{x}\\ \delta\mathbf{x}'\end{bmatrix} = \mathbf{u}$$

reference stations

$$\mathbf{E}\,\mathbf{P}(\mathbf{x}-\mathbf{x}^{\mathrm{ref}}) = \mathbf{0}$$

Minimization of data noise over *any station group*

$$\boldsymbol{\Sigma} = \mathbf{N}^{-} = \left[\begin{array}{c:c}\boldsymbol{\Sigma}_{\hat{\mathbf{x}}} & \boldsymbol{\Sigma}_{\hat{\mathbf{x}}\hat{\mathbf{x}}'}\\ \hdashline \boldsymbol{\Sigma}_{\hat{\mathbf{x}}'\hat{\mathbf{x}}} & \boldsymbol{\Sigma}_{\hat{\mathbf{x}}'}\end{array}\right]$$

minimum trace

Minimization of data/datum noise over *any station group*

$$\boldsymbol{\Sigma} = \mathbf{N}^{-} + \boldsymbol{\Sigma}^{\mathrm{ref}} = \left[\begin{array}{c:c}\boldsymbol{\Sigma}_{\hat{\mathbf{x}}} & \boldsymbol{\Sigma}_{\hat{\mathbf{x}}\hat{\mathbf{x}}'}\\ \hdashline \boldsymbol{\Sigma}_{\hat{\mathbf{x}}'\hat{\mathbf{x}}} & \boldsymbol{\Sigma}_{\hat{\mathbf{x}}'}\end{array}\right]$$

minimum trace

Data noise effect

Datum noise effect

What can "weighted" MCs optimize?

$$\mathbf{N}\begin{bmatrix}\delta\mathbf{x} \\ \delta\mathbf{x}'\end{bmatrix} = \mathbf{u}$$

reference stations

$$\mathbf{E}\,\mathbf{P}(\mathbf{x}-\mathbf{x}^{\mathrm{ref}}) = \mathbf{0}$$

Minimization of data/datum noise on *other derived frame-dependent quantities*

$$\hat{\mathbf{q}} = \mathbf{f}(\hat{\mathbf{x}}, \hat{\mathbf{x}}')$$

e.g. horizontal coordinates, geometric heights

$$\boldsymbol{\Sigma}_{\hat{\mathbf{q}}} = \mathbf{Q}\left[\begin{array}{c|c}\boldsymbol{\Sigma}_{\hat{\mathbf{x}}} & \boldsymbol{\Sigma}_{\hat{\mathbf{x}}\hat{\mathbf{x}}'} \\ \hline \boldsymbol{\Sigma}_{\hat{\mathbf{x}}'\hat{\mathbf{x}}} & \boldsymbol{\Sigma}_{\hat{\mathbf{x}}'}\end{array}\right]\mathbf{Q}^T$$

minimum trace

Datum choice problem

Rank-deficient NEQs

$$\mathbf{N}\begin{bmatrix}\delta\mathbf{x}\\ \delta\mathbf{x}'\end{bmatrix} = \mathbf{u}$$

reference stations

Arbitrary MCs

$$\mathbf{H}\left(\mathbf{x}-\mathbf{x}^{\mathrm{ref}}\right)=\mathbf{0}$$

Optimization problem to be solved

$$\min_{\mathbf{H}} \; trace \; \mathbf{S}\left[\begin{array}{c:c}\boldsymbol{\Sigma}_{\hat{\mathbf{x}}} & \boldsymbol{\Sigma}_{\hat{\mathbf{x}}\hat{\mathbf{x}}'}\\ \hdashline \boldsymbol{\Sigma}_{\hat{\mathbf{x}}'\hat{\mathbf{x}}} & \boldsymbol{\Sigma}_{\hat{\mathbf{x}}'}\end{array}\right]\mathbf{S}^T$$

Total CV matrix
of MC solution

where **S** is a “station selection” matrix, a Jacobian matrix, or a combination of such matrices

Problem solution

Frame/network optimality principle

$$\min_{\mathbf{H}} \; trace \; \mathbf{S} \left[\begin{array}{c|c} \boldsymbol{\Sigma}_{\hat{\mathbf{x}}} & \boldsymbol{\Sigma}_{\hat{\mathbf{x}}\hat{\mathbf{x}}'} \\ \hline \boldsymbol{\Sigma}_{\hat{\mathbf{x}}'\hat{\mathbf{x}}} & \boldsymbol{\Sigma}_{\hat{\mathbf{x}}'} \end{array} \right] \mathbf{S}^T$$

Optimal MC matrix (applied to reference stations)

$$\mathbf{H} = \mathbf{E}\Big(\underbrace{\mathbf{Q}_{\mathbf{x}} + \boldsymbol{\Sigma}_{\mathbf{x}^{\mathrm{ref}}}}_{\text{optimal weight matrix}}\Big)^{-1}$$

where:

$$(\mathbf{N} + \mathbf{S}^T\mathbf{S}\tilde{\mathbf{E}}^T\tilde{\mathbf{E}}\mathbf{S}^T\mathbf{S})^{-1} = \left[\begin{array}{c|c} \mathbf{Q}_{\mathbf{x}} & \# \\ \hline \# & \# \end{array} \right]$$

inner-constraint matrix
for the entire network (**N**$\tilde{\mathbf{E}}^T$=**0**)

(*) *see Kotsakis (2013, JGeod)*

Numerical tests

- EPN network – EUR**1780**7.SNX

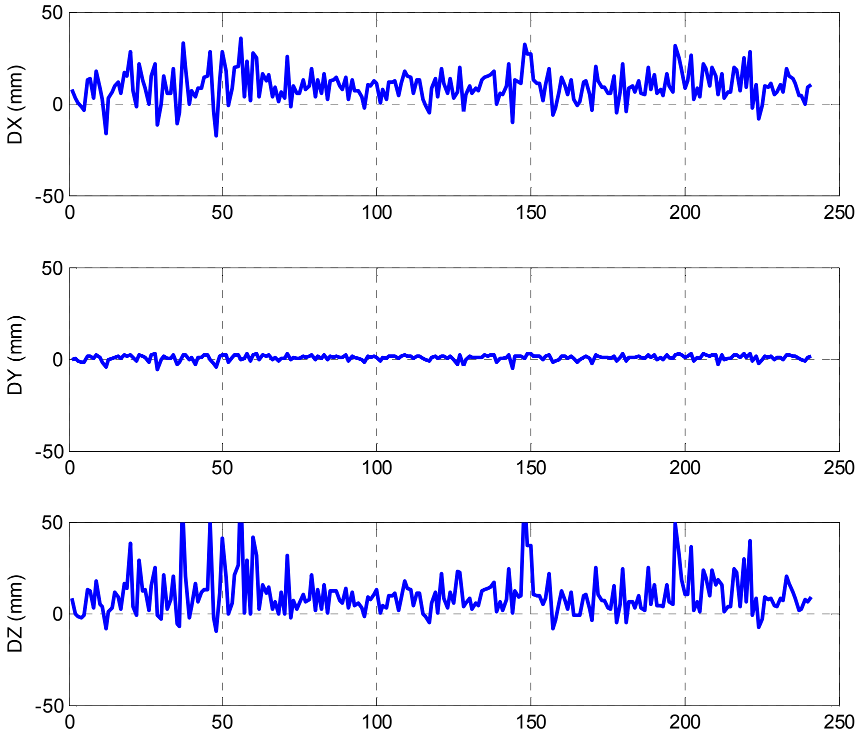
- Obtain weekly NEQs + remove inherent datum info

$$\mathbf{N}\begin{bmatrix}\boldsymbol{\delta}\mathbf{x} \\ \boldsymbol{\delta}\mathbf{x}'\end{bmatrix} = \mathbf{u}\ , \qquad \mathbf{N}\,\tilde{\mathbf{E}}^{T} = \mathbf{0}$$

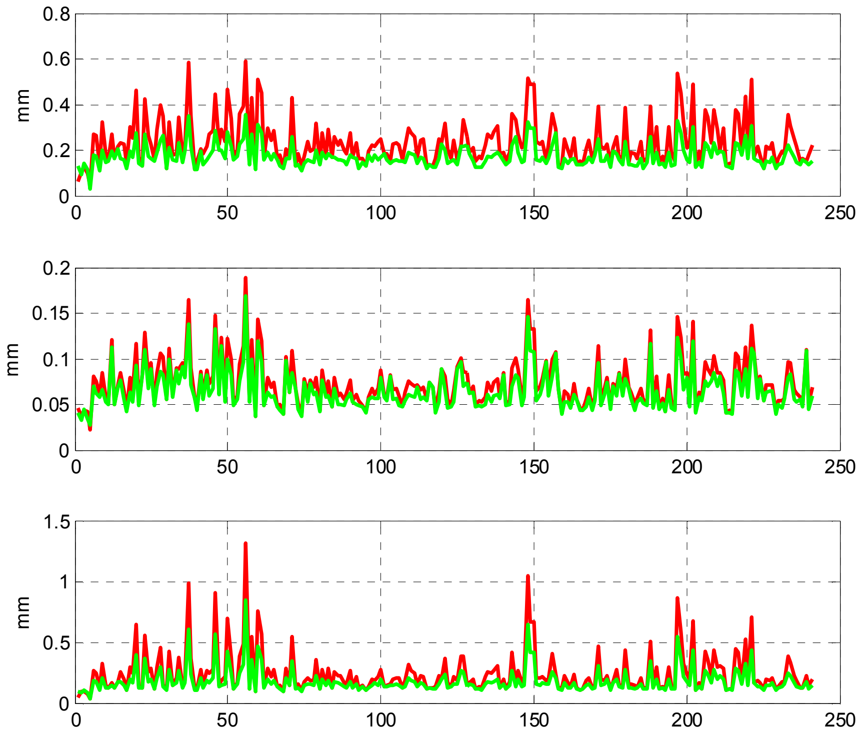
- Compare the weighted and un-weighted MC solutions (IGb08 frame)

Comparison between weighted & un-weighted MC solutions

CRD differences



STDs of estimated CRDs



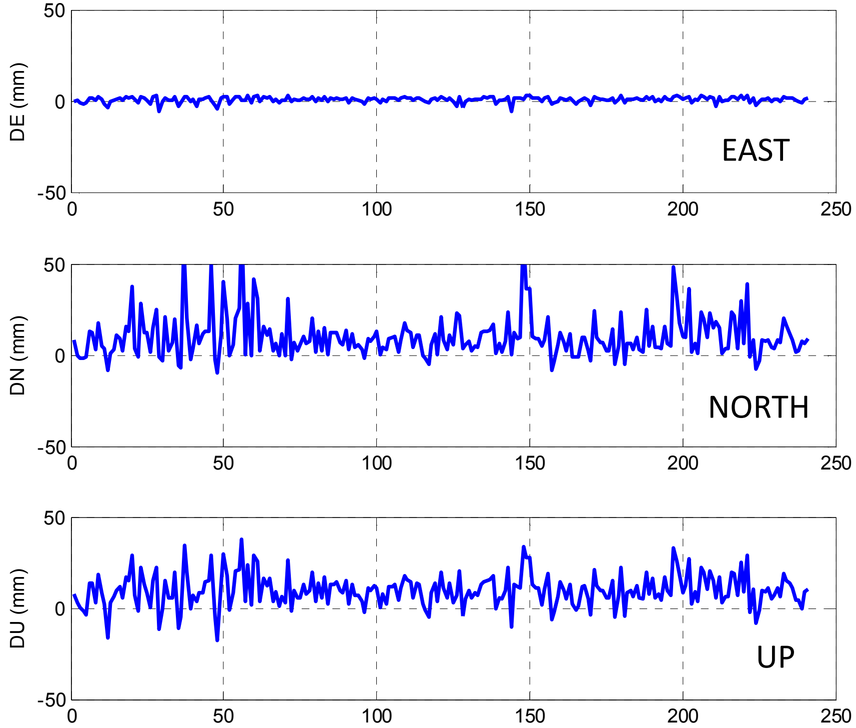
5 reference stations, **S** = **I**

Un-weighted MCs

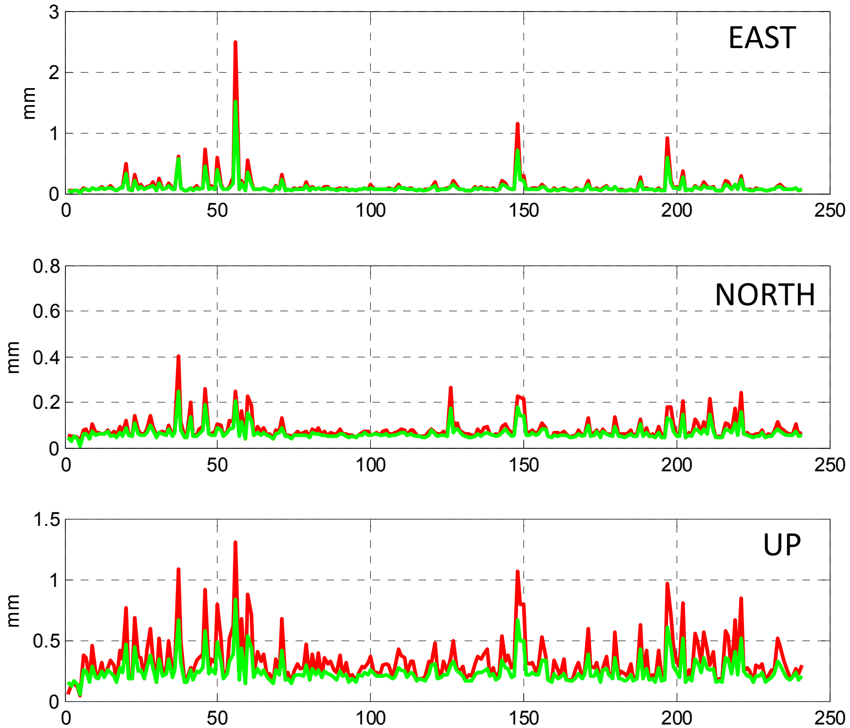
Weighted MCs

Comparison between weighted & un-weighted MC solutions

CRD differences



STDs of estimated CRDs



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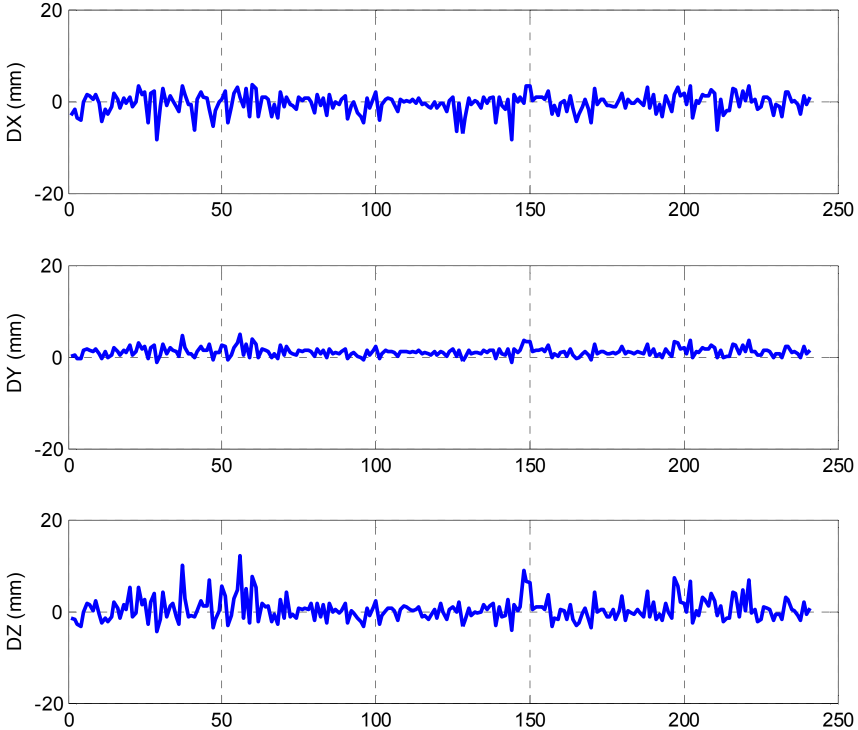
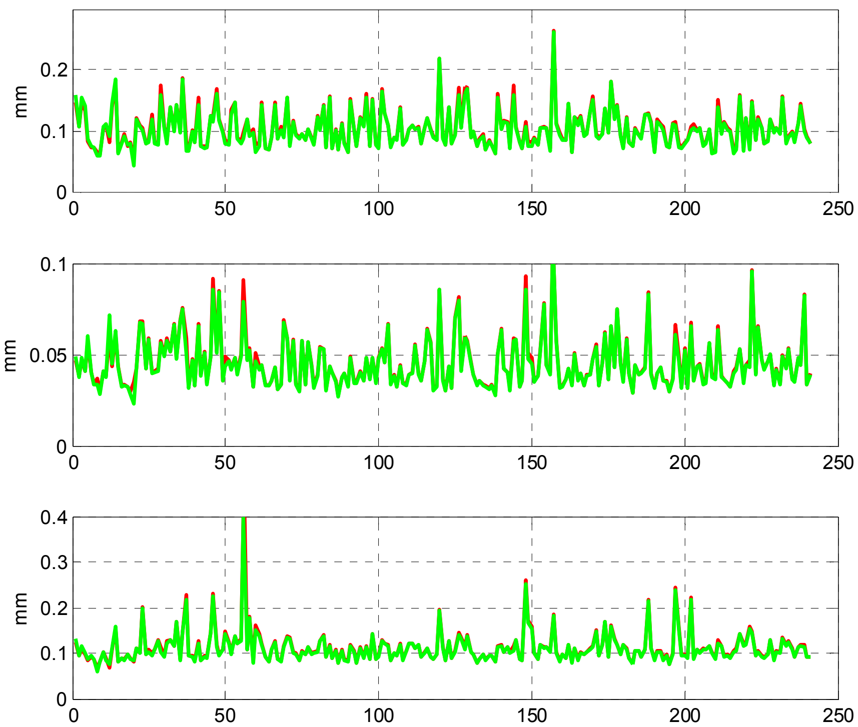


STDs of estimated CRDs



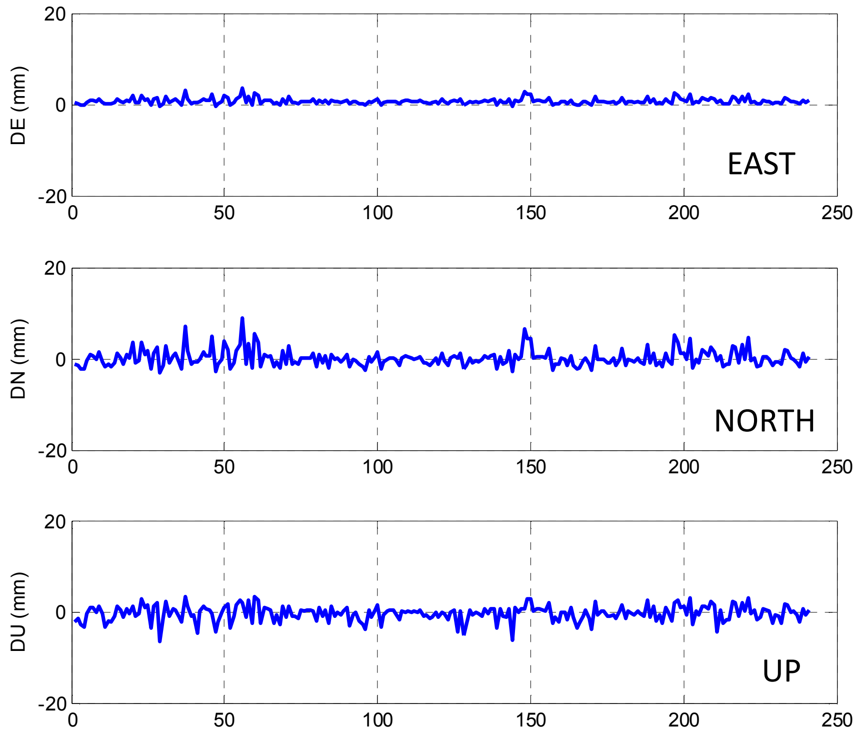
20 reference stations, **S** = **I**

Un-weighted MCs

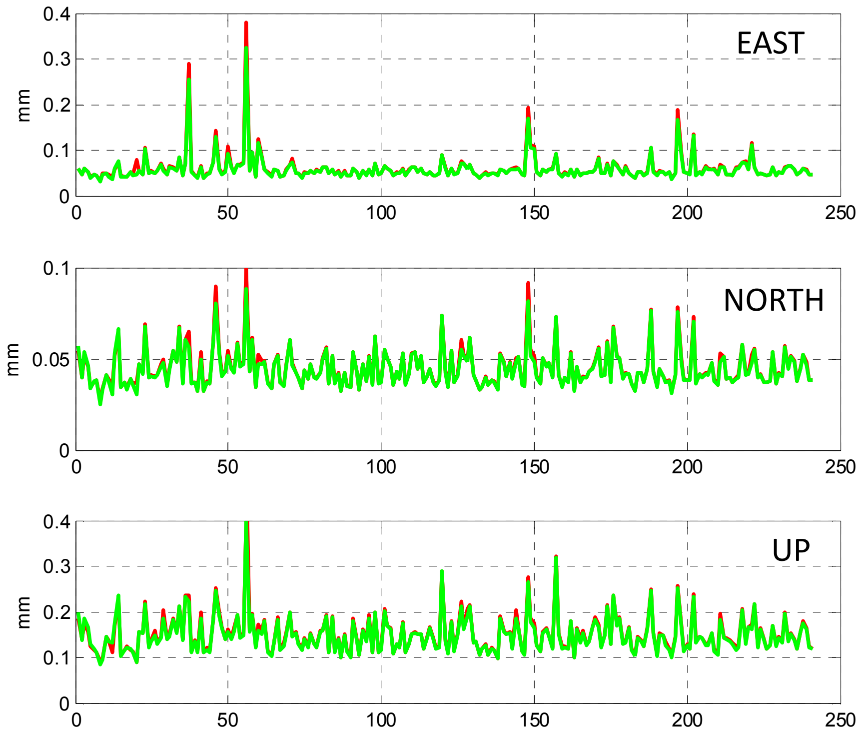
Weighted MCs

Comparison between weighted & un-weighted MC solutions

CRD differences



STDs of estimated CRDs



Un-weighted MCs

Weighted MCs

20 reference stations, **S** = **I**

Conclusions

- Reference station weighting (within the MCs) can lead to different types of frame optimality

- Reference station weighting can be used to optimize the accuracy of a MC solution in terms of

  - the data and datum noise effects
  - the network stations over which these effects are considered

- Detailed numerical testing will be presented in a forthcoming paper

Thanks for your attention !