On the (unexpected) usefulness of theory in geodetic problems

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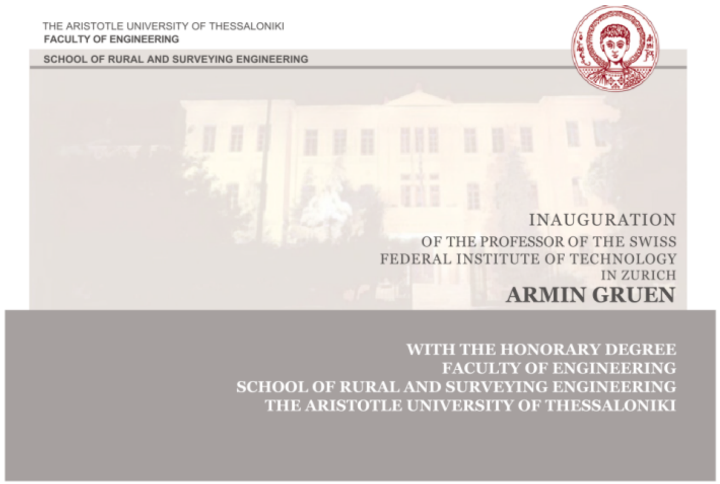

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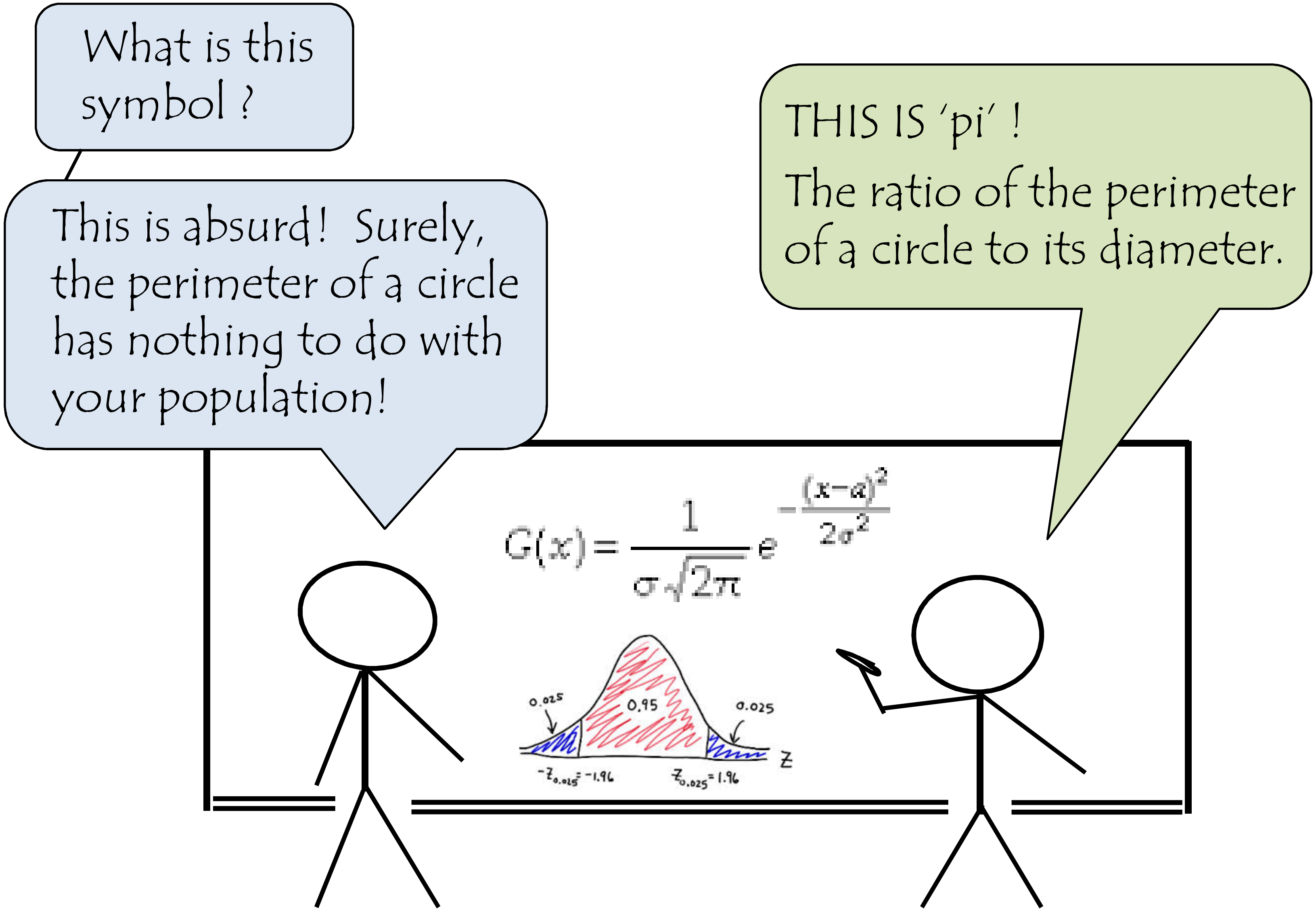
Introduction

There is a story about two friends, who were classmates in high school, talking about their jobs.

One of them became a statistician and is now working on population trends.

The statistician tries to explain the role of the Gaussian distribution to his friend, who seems to be puzzled by many queries…

Introduction (cont'd)



The moral of the story…

Theoretical concepts defined and developed in one context, often turn out to have a highly effective application in another context !

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...and the focus of our talk

Theoretical concepts defined and developed in one context, often turn out to have a highly effective application in another context!

Such (unexpected) theoretical windfalls:

- led to key developments and novel solutions for practical problems
- stimulate challenging directions of future scientific work in geodesy and geospatial science

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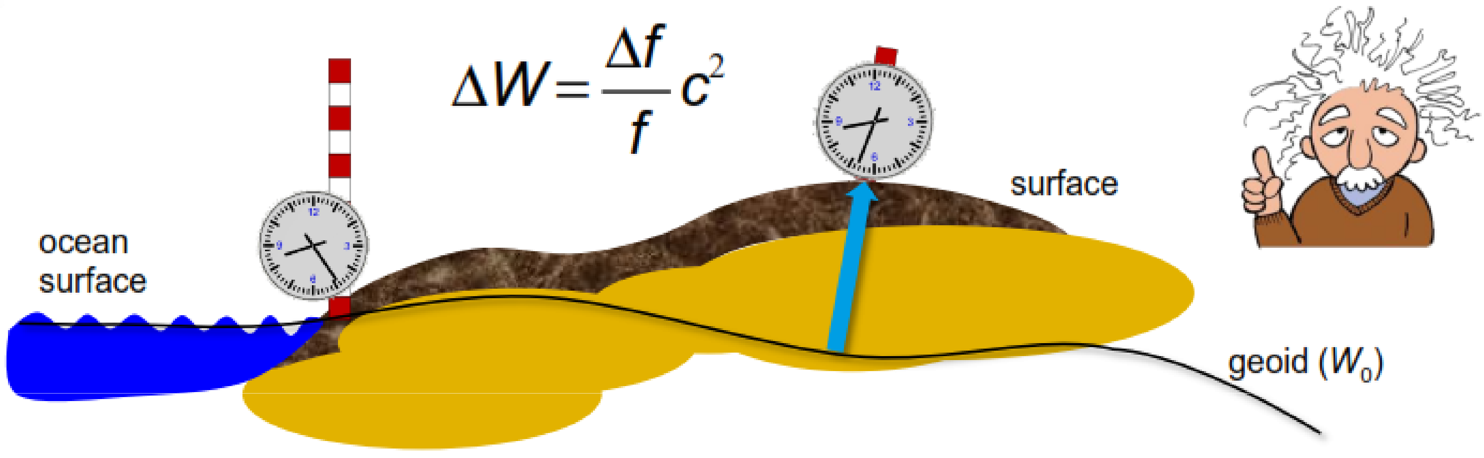
Example: chronometric leveling



after Flury (2015)

- Optimal atomic clocks approaching **10^{-18}** (~ 1 cm).
- Potential/height determination over very large distances.
- A. Bjerhammar (1975), M. Vermeer (1983).

Motivated by a classic paper

"*The Unreasonable Effectiveness of Mathematics in the Natural Sciences*", Comm Pur Appl Math, vol. XIII, pp. 1-14, 1960.

Eugene Wigner (1902-1995)
Princeton University
Physics Nobel Prize Winner (1963)

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Why bother with theory ?

- In a meeting of geodesists and surveyors, such question is easy to dismiss.
- Yet, is theoretical work really useful?

  practitioners: any time spent on theory is wasted

  theoreticians: it’s the only thing worth spending time on
- Researchers often rely on the assertion that something “is so” or “it just works”.
- Traditional “application-of-theory” model prevails in most academic programs.

A rightful tricky argument

"**In theory** this is useful but **in practice** I would not expect more than marginal changes."

So, why bother with theory ?

For the **“why”** question – understanding matters!

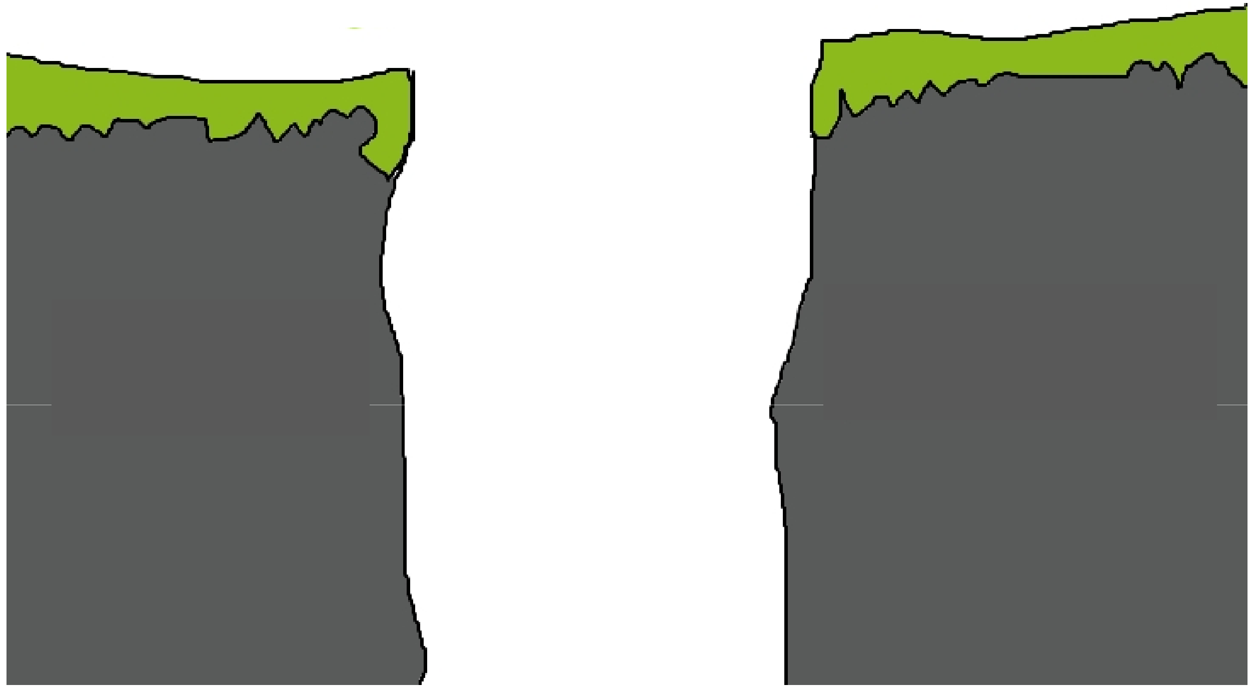
- In mid-sixties, at the OSU, R. Rapp argued that H. Moritz’s least-squares prediction theory is wrong.
- The reason was that he was getting imaginary standard errors in his computations.
- It turned out that he used a polynomial covariance function which is of course not positive-definite !

So, why bother with theory ?

For the **“how”** question – improvement matters!

- New problems often require the revision of old theories and of their related tools.
- Also, new theoretical insight to old problems may lead to more satisfactory or powerful solutions.
- Proper theory/practice interaction is required.

The theory-practice gap



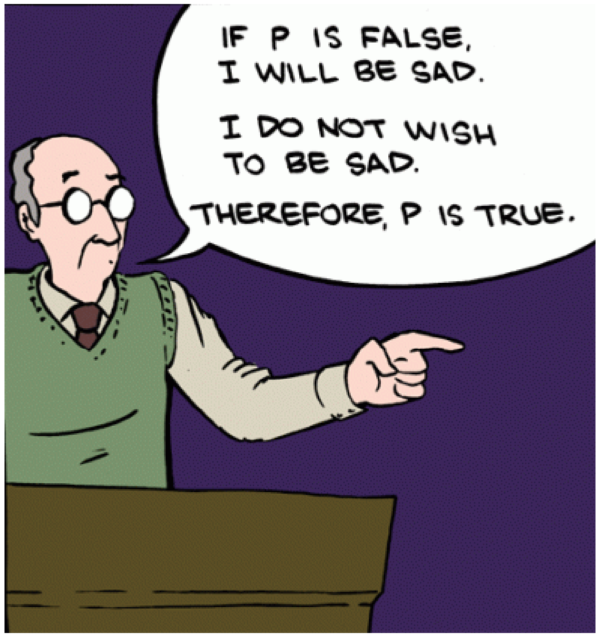
THEORY is when you know everything and nothing works.

PRACTICE is when everything works and no one knows why.

Merging theory & practice

The theoretician's way

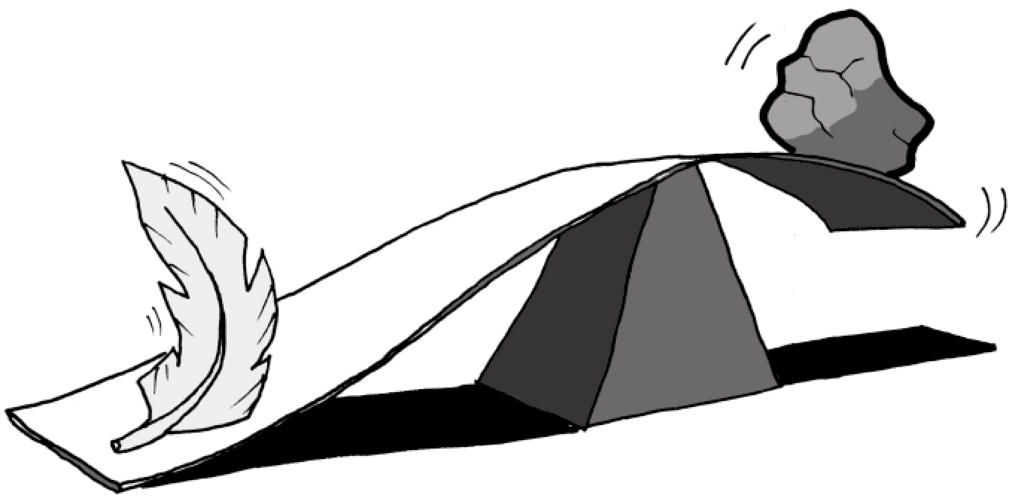
Reformulate the problem to our liking – we may end up with useless theories.



Merging theory & practice

The practitioner's way

Overuse the theoretical tools and software in our disposal – we may end up with erroneous results.



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Merging theory & practice

The "useful" way

The theory should be selected, or appropriately modified, to fit the problem we need to solve.

Not the other way around!

Classic geodetic examples:

Least-squares collocation

Bruns' 3D concept of geodesy

Some theory/practice “thrusters”

- Technological advancements
- New data types

Also,

- Solving an easy problem in a hard way
  (see, e.g., H. Moritz’s PhD thesis)
- Solving an old problem in an alternate way
  (see, e.g., next example)

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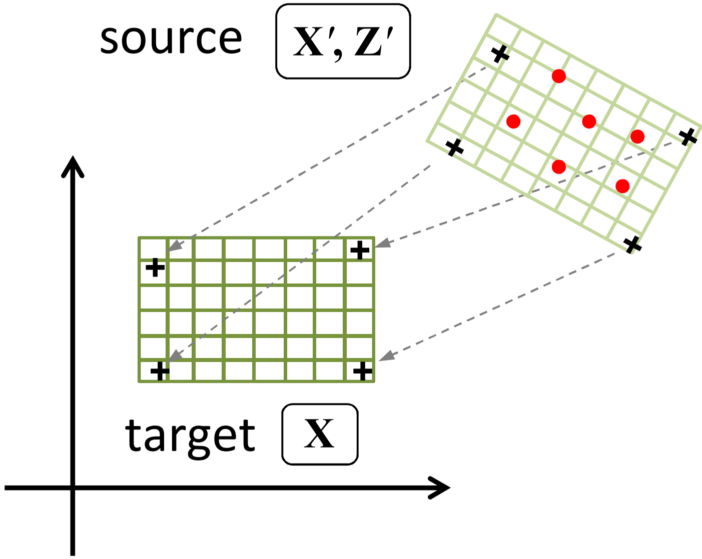
Example of an "old" problem



Datum transformation (geodesy)

Image registration (photogram.)

Map conflation (cartography)

Georeferencing

Procrustes analysis

Standard solution:

$$\hat{\boldsymbol{\theta}} = \left(\mathbf{G}^T \mathbf{P} \mathbf{G}\right)^{-1} \mathbf{G}^T \mathbf{P} (\mathbf{X} - \mathbf{X}')$$ estimated parameters

$$\begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{z}} \end{bmatrix} = \begin{bmatrix} \mathbf{X}' \\ \mathbf{Z}' \end{bmatrix} + \begin{bmatrix} \mathbf{G} \\ \tilde{\mathbf{G}} \end{bmatrix} \hat{\boldsymbol{\theta}}$$ transformed coordinates

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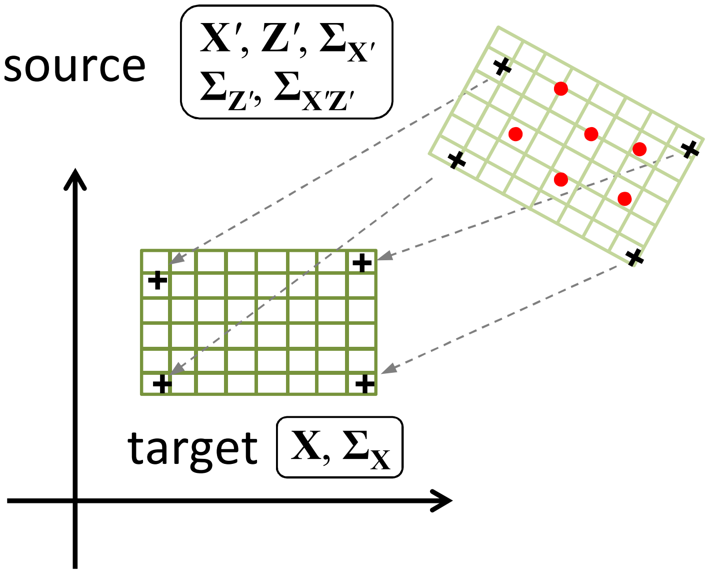
Example of an "old" problem



Data stacking

$$\begin{aligned} \mathbf{X}' &= \mathbf{x} - \mathbf{G}\boldsymbol{\theta} + \mathbf{v}_{\mathbf{X}'} \\ \mathbf{Z}' &= \mathbf{z} - \tilde{\mathbf{G}}\boldsymbol{\theta} + \mathbf{v}_{\mathbf{Z}'} \\ \mathbf{X} &= \mathbf{x} + \mathbf{v}_{\mathbf{X}} \end{aligned}$$

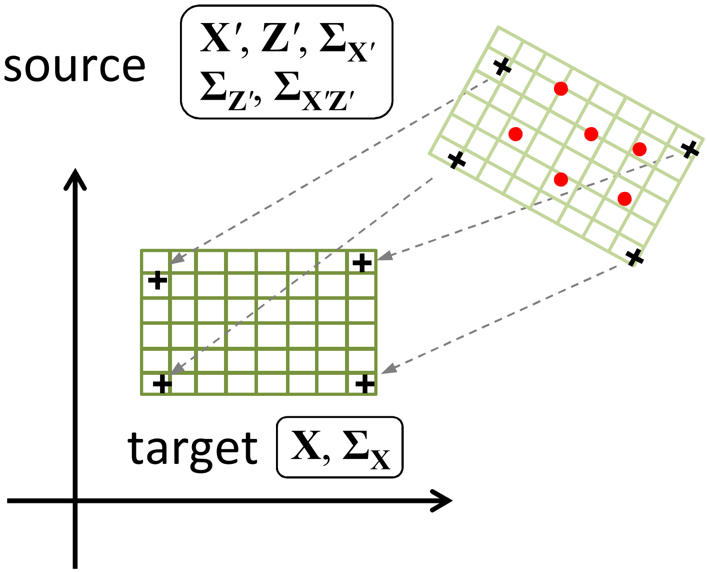
Least-squares solution:

$$\begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{z}} \end{bmatrix} = \begin{bmatrix} \mathbf{X}' \\ \mathbf{Z}' \end{bmatrix} + \begin{bmatrix} \mathbf{G} \\ \tilde{\mathbf{G}} \end{bmatrix} \hat{\boldsymbol{\theta}} + \underbrace{\begin{bmatrix} \boldsymbol{\Sigma}_{\mathbf{X}'} \\ \boldsymbol{\Sigma}_{\mathbf{Z}'\mathbf{X}'} \end{bmatrix} (\boldsymbol{\Sigma}_{\mathbf{X}} + \boldsymbol{\Sigma}_{\mathbf{X}'})^{-1} \left(\mathbf{X} - \mathbf{X}' - \mathbf{G}\hat{\boldsymbol{\theta}}\right)}_{\text{Correction term}}$$

Correction term

Optimal rubber sheeting !

Example of an “old” problem



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Relevant for PHOTO/GIS applications: the incorporation of a prior statistical shape model ($\Sigma_{\mathbf{x}}$, $\Sigma_{\mathbf{z}}$, $\Sigma_{\mathbf{zx}}$) in the adjustment procedure!

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Epilogue

He who loves practice without theory is like the sailor who boards ship without a rudder and compass and never knows where he may cast.

Leonardo da Vinci
Italian Painter

Thank you for your attention !

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