Survey Review, **42**, 318 pp. 327-344 (October 2010)

QUALITY ANALYSIS OF GLOBAL GEOPOTENTIAL MODELS AT 1542 GPS/LEVELLING BENCHMARKS OVER THE HELLENIC MAINLAND

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ABSTRACT

The aim of this study is to present the results of several 'external' quality tests for the most recent (at the time of writing this paper) global geopotential models (GGMs) using precise GPS and leveled orthometric heights over the area of Greece. The tested GGMs include the GRACE-based combined model GGM03C, the latest EIGEN-type combined models EIGEN-GL04C and EIGEN-GL05C, the ultra-high resolution model EGM08 that was released last year by the US National Geospatial-Intelligence Agency, and also the older NASA/NIMA/OSU's EGM96 model. The evaluation tests are based on comparisons of absolute and relative geoid undulations that are computed from the selected GGMs and the external GPS/levelling data. The test network covers the entire part of the Hellenic mainland and it consists of more than 1500 benchmarks which belong to the Hellenic national triangulation network, with direct levelling ties to the Hellenic vertical reference frame. The spatial positions of these benchmarks have been recently re-determined at cm-level accuracy (with respect to ITRF00) through a nation-wide GPS campaign that was organized in the frame of the HEPOS project. Our results show that the EGM08 model offers a remarkable improvement for the agreement among geoidal, ellipsoidal and orthometric heights in the mainland part of Greece, compared to the performance of other combined GGMs over the same area. Finally, our study gives a preliminary (yet realistic) accuracy assessment for GGM/GPS-aided orthometric height determination, over different baseline lengths, throughout the Hellenic mainland.

KEYWORDS: Geopotential models. Geoid undulation. Orthometric and ellipsoidal heights. GPS levelling

INTRODUCTION

Following the launch of the CHAMP and GRACE satellite-gravity space missions in 2001-2002, there have been developed and publicly released more than 25 global geopotential models (GGMs) for Earth's mean gravity field [**31**]. Typically given in the form of a spherical harmonic series (SHS) expansion for the external gravitational potential [**10**], [**25**], these models provide valuable tools for several geodetic and surveying applications. From a geodetic positioning perspective, for example, modern geopotential models play a key role for the unification of national height systems and the support of vertical datum modernization efforts based on precise GPS positioning [**1**], [**4**], [**6**], [**21**], [**22**], [**30**]. In view of the recent progress and upcoming improvements in gravity field mapping (e.g. GOCE mission), it is not actually unreasonable to claim that a cm-level world vertical datum may be eventually realizable through a global geoid model obtained from a high-resolution and high-accuracy GGM [**12**].

Among the up-to-date developments in geopotential modeling, the most prominent achievement is certainly the release of the Earth Gravitational Model EGM08 by the US National Geospatial-Intelligence Agency [**24**]. Complete to degree and order 2159, with additional spherical harmonic coefficients (SHCs) extending up to degree 2190

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DOI 10.1179/003962610X12747001420500

and order 2159, EGM08 offers an unprecedented level of spatial resolution (~ 9 km) for the recovery of any gravity field functional over the entire globe. Together with other satellite-only and combined GGMs that have been available after the GRACE and CHAMP missions, the EGM08 model represents the state-of-art in global gravity field mapping and it contributes significantly to the continuing efforts of the geodetic community for a highly accurate reference model of Earth's gravity field.

Besides an internal quality analysis that is usually performed through the formally propagated statistical errors of their SHCs into different gravity field signals (see e.g. [**18**], [**23**]), GGMs need also to be tested with 'external' geodetic data to obtain an independent (and, hopefully, more realistic) estimate of their actual accuracy at different spatial scales. Specifically, external comparisons with ellipsoidal and orthometric heights over regional networks of GPS/levelling benchmarks is a standard evaluation technique for GGM-based geoids, through which certain deficiencies and other regional systematic problems can be identified in current geopotential models [**5**], [**14**], [**26**], [**27**]. Although the results from such evaluation studies depend on several factors (e.g. quality of the external height data, consistency of their inherent reference frames and other modeling assumptions, underlying testing methodology, etc.), they often can lead to a reliable assessment of the GGM accuracy level over different areas. Furthermore, the statistical agreement between absolute and/or relative GPS-based and GGM-based geoid undulations, as well as the spatial pattern of their differences, provide useful information for the performance of GGM/GPS-aided levelling and the feasibility of fitting auxiliary parametric models to improve the consistency among ellipsoidal heights, local orthometric heights and GGM geoid undulations within specific geographical regions [**3**], [**4**],[**6**], [**13**], [**21**].

The objective of this paper is to present a quality analysis of the most recent combined GGMs (including the older EGM96 model for chronological comparison purposes) over the entire Hellenic mainland using precise GPS and leveled orthometric heights. All evaluation tests refer to a network of 1542 GPS/levelling benchmarks which covers the mainland part of Greece with a relatively uniform spatial distribution (see Figure 1). This is actually the first time that such an extensive GGM-evaluation task is carried out in Greece with the aid of precise GPS positioning. The test network consists of control points that belong to the Hellenic national triangulation frame and they are directly tied to the Hellenic vertical reference frame through spirit or precise trigonometric levelling. These control points were recently re-surveyed through a nation-wide GPS campaign in the frame of the HEPOS project [**9**] and their spatial positions have been estimated anew at cm-level accuracy with respect to ITRF00. Although a large number of additional GPS/levelling benchmarks is currently available over the Greek islands (which were also measured in the frame of the HEPOS project), they have been deliberately excluded from our current analysis to avoid misleading systematic effects in the GGM evaluation results due to the unknown vertical datum differences that exist between the various islands and the mainland region.

An important aspect of our study is the extensive national coverage and high spatial density of the test network, with an average distance of about 7 km between adjacent points throughout Greece. These characteristics have been most helpful in identifying the major improvement that the EGM08 model yields, over other existing GGMs, for the representation of gravity field features in the Hellenic mainland (particularly for the mountainous areas). In addition, the present study provides a preliminary accuracy assessment of EGM08 and other geopotential models for the forward determination of orthometric height differences (over different baseline lengths) using GGM/GPS-aided levelling in Greece.

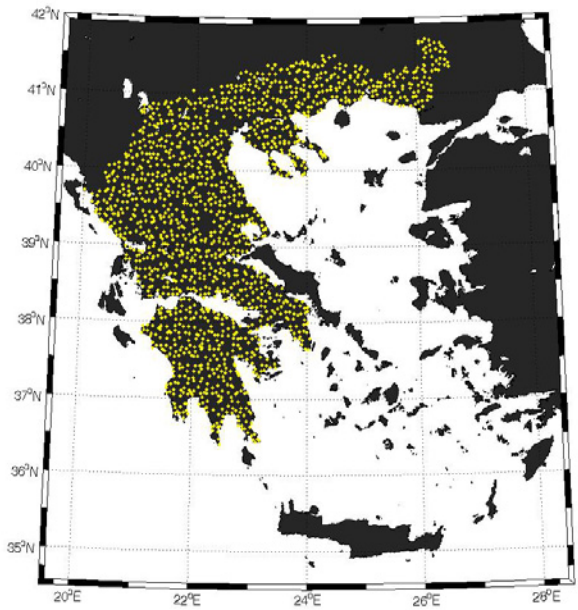

Fig. 1. Geographical distribution of the 1542 GPS/levelling benchmarks over the Hellenic mainland

ELLIPSOIDAL AND ORTHOMETRIC HEIGHTS

After several years of undergoing efforts for the enhancement of the spatial data infrastructure in Greece, a national GPS campaign took place in 2007 to acquire a sufficient number of control points with accurately known 3D spatial positions in an ITRF-type coordinate system. These activities have been initiated by the Ministry for the Environment, Planning and Public Works and the financial support of the EU and the Hellenic State, and they are part of the HEPOS (Hellenic Positioning System) project that will lead to the launch of a modern satellite-based positioning service for cadastral, mapping, surveying and other geomatics applications in Greece [**9**]. The entire project is coordinated by Ktimatologio S.A, a state-owned private sector firm that is responsible for the operation of the Hellenic Cadastre.

The aforementioned GPS campaign involved more than 2450 geodetic benchmarks within the existing national triangulation network, part of which are the 1542 points shown in Figure 1. The scope of the campaign was to provide a sufficient number of well-distributed control stations for the determination of a precise datum transformation model between the official Hellenic Geodetic Reference Frame of 1987 and other ITRF/ETRF-type frames. The fieldwork was performed within a 6-month period (March to September 2007) using twelve dual-frequency Trimble 5700/5800 GPS receivers with Zephyr or R8 internal antennas. Thirty three points were used as 'base' reference stations with 24-hour continuous GPS observations, while the rest of the points were treated as 'rover' stations with observation periods ranging between 1-3 hours. In all cases, a 15-sec sampling rate and an 15° elevation cut-off angle were used for the data collection.

After the processing of the GPS carrier phase observations using EUREF/EPN ties and IGS precise orbits, the geocentric Cartesian coordinates of all stations (including the 1542 points shown in Figure 1) were determined in ITRF00 (epoch: 2007.236) and their geometric heights were then derived with respect to the GRS80 ellipsoid. The formal accuracy of the ellipsoidal heights ranged between 2-5 cm, while the horizontal positioning accuracy with respect to ITRF00 was marginally better by 1-2 cm (1σ level). For more details, see [**32**] and the references provided therein.

Helmert-type orthometric heights at the 1542 test points have been also known through levelling survey ties to surrounding benchmarks of the national vertical reference frame. These local ties were performed in previous years by the Hellenic Military Geographic Service (HMGS) using spirit and/or precise trigonometric levelling techniques. It should be mentioned that a large number of these test points is located in mountainous areas (24% of them have orthometric heights $H > 800$ m).

The quality of the available orthometric heights in our test network is affected by two main factors: the internal accuracy and consistency of the Hellenic vertical reference frame (HVRF), and the precision of the local levelling ties to the surrounding HVRF benchmarks. Due to the absence of sufficient public documentation on behalf of HMGS, the actual absolute accuracy of these orthometric heights is largely unknown. Their values refer, in principle, to the equipotential surface of Earth's gravity field that coincides with the mean sea level at the HVRF's fundamental tide-gauge reference station located in Piraeus port (unknown W_o value, period of tide gauge measurements: 1933-1978); for more details, see [**2**], [**28**].

Based on the known ellipsoidal and orthometric heights, GPS-based geoid undulations have been computed at the 1542 test points from the equation [**10**]

$$N^{GPS} = h - H \tag{1}$$

The above values provide the 'external' dataset for the GGM evaluation tests that will be performed in the following sections. Note that low-pass filtering or other smoothing techniques have not been applied to the GPS/H geoid heights, and thus the effect of the GGMs' *omission error* should be directly reflected in our evaluation results.

Table 1. *GGMs used for the evaluation tests at the 1542 GPS/levelling benchmarks*

Models	n_{max}	*Data*	*Reference*
EGM08	2190	S (GRACE), G, A	[**24**]
EIGEN-GL05C	360	S (GRACE, LAGEOS), G, A	[**7**]
EIGEN-GL04C	360	S (GRACE, LAGEOS), G, A	[**8**]
GGM03C	360	S (GRACE), G, A	[**29**]
EGM96	360	S, G, A	[**17**]

S: satellite tracking data, G: gravity data, A: satellite altimetry data

GLOBAL GEOPOTENTIAL MODELS (GGMS) USED IN THIS STUDY

Geoid undulations were computed at the 1542 GPS/levelling benchmarks using several different GGMs. For the evaluation tests presented in this paper, we consider the most recent (at the time of writing this paper) high-resolution GGMs that have been developed from the combined contribution of various types of satellite tracking data, gravity data and satellite altimetry data, see Table 1. The older EGM96 model is also included in our comparisons to demonstrate its advantageous performance over the latest EIGEN-type combined models for the area of Greece (see detailed results in the following sections).

Computation of GGM geoid undulations

The GGM-based geoid undulations have been determined through the formula [**25**]

$$N = \zeta + \frac{\Delta g^{FA} - 0.1119H}{\bar{\gamma}} H + N_o \tag{2}$$

where ζ and Δg^{FA} correspond to the height anomaly and free-air gravity anomaly signals that are computed from corresponding SHS expansions (from n = 2 up to n_{max}) based on the coefficients of each model and the GRS80 normal gravity field parameters. The additive term N_o denotes the contribution of the zero-degree harmonic to the GGM geoid undulations with respect to the GRS80 reference ellipsoid. It has been computed from the following equation [**10**], [**25**]

$$N_o = \frac{GM - GM_o}{R\gamma} - \frac{W_o - U_o}{\gamma} \tag{3}$$

where the parameters GM_o and U_o correspond to the Somigliana-Pizzeti normal gravity field generated by the GRS80 ellipsoid, i.e. GM_o = 398600.5000×10^9 m^3 s^{-2} and U_o = 62636860.85 m^2 s^{-2} [**20**]. The Earth's geocentric gravitational constant and the gravity potential on the geoid were set equal to the values GM = 398600.4415×10^9 m^3 s^{-2} and W_o = 62636856.00 m^2 s^{-2}, respectively. The GRS80 value R = 6371008.771 m was adopted for the mean Earth radius, while the normal gravity γ on the reference ellipsoid was computed at each test point through Somigliana's formula [**10**].

Based on the previous conventional choices, the zero-degree term from Eq. (3) yields an almost constant value ($N_o \approx$ -0.442 m) throughout the test network, which has been added to the geoid undulations obtained from the SHCs of each geopotential model.

Table 2. *Statistics of height datasets over the Hellenic test network (units in m)*

	Max	*Min*	*Mean*	σ
h	2562.753	24.950	545.676	442.418
H	2518.889	0.088	510.084	442.077
$N^{GPS} = h\text{-}H$	43.864	19.481	35.592	5.758
N (EGM08)	44.374	19.663	35.968	5.800
N (EIGEN-GL05C)	43.938	19.571	36.039	5.824
N (EIGEN-GL04C)	44.104	19.303	35.874	5.878
N (GGM03C)	43.893	19.386	35.908	5.796
N (EGM96)	44.007	19.687	36.037	5.753

Note that the numerical computations for the spherical harmonic synthesis of the GGM geoid undulations have been performed in the *zero-tide* system (with respect to a geometrically fixed reference ellipsoid - GRS80) using the 'harmonic_synth_v02' software program that is freely provided by the EGM08 development team; see [**11**].

Statistics of height data sets

The statistics of the individual datasets that will be used in our evaluation tests are given in Table 2. The statistics for the GGM geoid undulations refer to the values computed from Eq. (2) at the 1542 GPS/levelling benchmarks using the *full* spectral resolution of each model.

From the *mean values* given in the above table, it is evident the existence of a large discrepancy (> 28 cm) between the reference surface of the Hellenic vertical datum and the equipotential surface that is specified by the IERS conventional value W_o = 62636856.00 $m^2 s^{-2}$ [**19**] and realized by the various GGMs over the Hellenic region. Some additional comments on this spatial offset will be given in subsequent sections of the paper.

GEOID HEIGHT EVALUATION AFTER A CONSTANT BIAS FIT

Several GGM evaluation tests were performed based on the point values for the ellipsoidal and orthometric heights in the Hellenic test network. The statistics of the differences between the GPS-based and the GGM-based geoid undulations are given in Table 3. In all cases, the values shown in this table refer to the statistics after a least-squares *constant bias fit* was applied to the original misclosures *h-H-N* at the 1542 GPS/levelling benchmarks.

From the results in Table 3, it is clear that EGM08 offers a remarkable improvement for the agreement among ellipsoidal, orthometric and geoidal heights throughout the Hellenic mainland. Compared to all other GGMs, the standard deviation of the EGM08 residuals N^{GPS}-N decreases by a factor of 3 (or more). The improvement achieved with the new model is also visible in its 30′ limited-resolution version (n_{max}=360) which matches the GPS/H geoid within ±37 cm, while all other GGMs of similar resolution do not perform better than ±41 cm. The major contribution, however, comes from the ultra-high frequency band of EGM08 (360 < n ≤ 2190) which enhances the consistency between GGM and GPS/H geoid undulations at ±14 cm (1σ level).

Table 3. *Statistics of the residuals N^{GPS}–N, after a least-squares constant bias fit, at the 1542 GPS/levelling benchmarks (units in m)*

	Max	*Min*	σ	*Bias*
EGM08 *(n_{max}=2190)*	0.542	-0.437	0.142	-0.377
EGM08 *(n_{max}=360)*	1.476	-1.287	0.370	-0.334
EIGEN-GL05C	1.997	-1.050	0.461	-0.448
EIGEN-GL04C	1.773	-1.174	0.453	-0.283
GGM03C	1.541	-1.057	0.413	-0.316
EGM96	1.577	-1.063	0.423	-0.446

The differences in the estimated bias obtained from each model (see last column in Table 3) indicate the existence of systematic offsets among the GGM geoids over Greece, which are likely caused by long/medium-wavelength errors in their original SHCs. Furthermore, the magnitude of the estimated bias between N^{GPS} and N confirms that there is a significant deviation between the equipotential surface corresponding to the IERS conventional value W_o = 62636856.00 m^2s^{-2} (and realized by the various GGMs over the Hellenic region) and the HVRF reference surface that is realized through the GPS/H geoidal heights N^{GPS} at the test points. For example, based on our current results, the HVRF reference surface appears to be located 38 cm below the EGM08/W_o/GRS80 geoid.

In Table 4 we also see the percentage of the GPS/levelling benchmarks whose adjusted residuals *h-H-N* (after the constant bias fit) fall within some typical geoid accuracy levels. Evidently, the agreement between EGM08 and GPS/H geoid heights is better than 10 cm for more than half of the 1542 test points, whereas for other GGMs the same consistency level is only reached at 20% (or less) of the test points. Moreover, almost 85% of the test points give an agreement between the EGM08 geoid and the GPS/levelling data that is better than 20 cm, compared to 37.5% (or less) for the other global models that were tested

Table 4. *Percentages of the 1542 test points whose adjusted residuals $N^{GPS}–N$ are smaller than some typical geoid accuracy levels (after a least-squares constant bias fit)*

	< 2 cm	*< 5 cm*	*< 10 cm*	*< 15 cm*	*< 20 cm*
EGM08 *(n_{max}=2190)*	13.3 %	29.8 %	53.5 %	73.0 %	84.6 %
EGM08 *(n_{max}=360)*	4.5 %	11.6 %	22.8 %	32.7 %	43.7 %
EIGEN-GL05C	4.1 %	10.5 %	20.3 %	28.6 %	36.2 %
EIGEN-GL04C	3.6 %	9.3 %	17.7 %	27.5 %	36.0 %
GGM03C	3.6 %	9.3 %	19.1 %	28.7 %	37.5 %
EGM96	4.3 %	9.8 %	17.5 %	27.7 %	35.5 %

The horizontal spatial variations of the EGM08 residuals N^{GPS}-N did not reveal any particular systematic pattern within the test network. Both their latitude-dependent and longitude-dependent scatter plots are free of any sizeable north/south or east/west tilts over the Hellenic mainland. In other geopotential models, however, some strong *regional tilts* and *oscillations* can be identified in their corresponding residuals N^{GPS}-N, mainly due to significant GGM omission errors involved in the recovery of the geoid height signal (see Figures 2 and 3).

Our results have confirmed that EGM08 performs exceedingly better than the other models over the mountainous parts of the Hellenic test network. A strong indication can be seen in the scatter plots of the residuals N^{GPS}-N (after the least-squares constant bias fit) with respect to the orthometric heights of the GPS/levelling benchmarks, see Figure 4. These plots reveal a *height-dependent bias* between the GGM and GPS/H geoid undulations, which is considerably reduced in the case of EGM08. Evidently, the higher frequency content of the new model gives a better approximation for the terrain-dependent gravity field features over the Hellenic mainland, a fact that is visible from the comparative analysis of the scatter plots in Figure 4.

GEOID HEIGHT EVALUATION USING DIFFERENT PARAMETRIC MODELS

An additional test series was carried out using a number of different parametric models for the least-squares adjustment of the differences $N^{GPS} – N$. The motivation for these experiments was to investigate the performance of various linear models that are frequently used in geoid evaluation studies with heterogeneous height data, and to assess their feasibility in modeling the systematic discrepancies between the GPS/H and GGM geoid surfaces over the Hellenic mainland. Although these tests have been implemented with all geopotential models that were selected for our study, only the results from EGM96, EIGEN-GL05C and EGM08 will be presented herein for economy reasons.

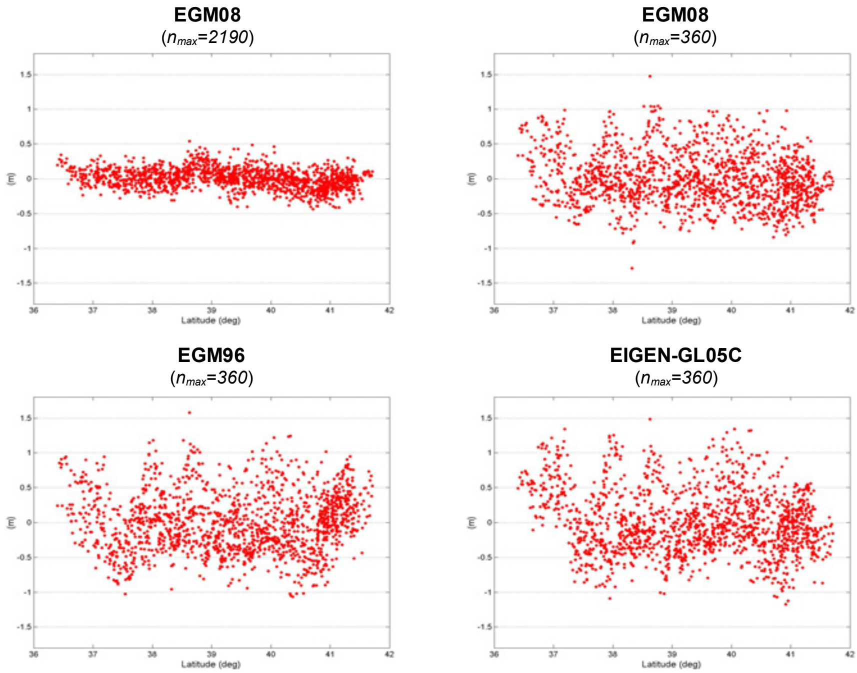

Fig. 2. Latitude-dependent variations of the residuals N^{GPS}–N *(after a least-squares constant bias fit)* at the 1542 GPS/levelling benchmarks

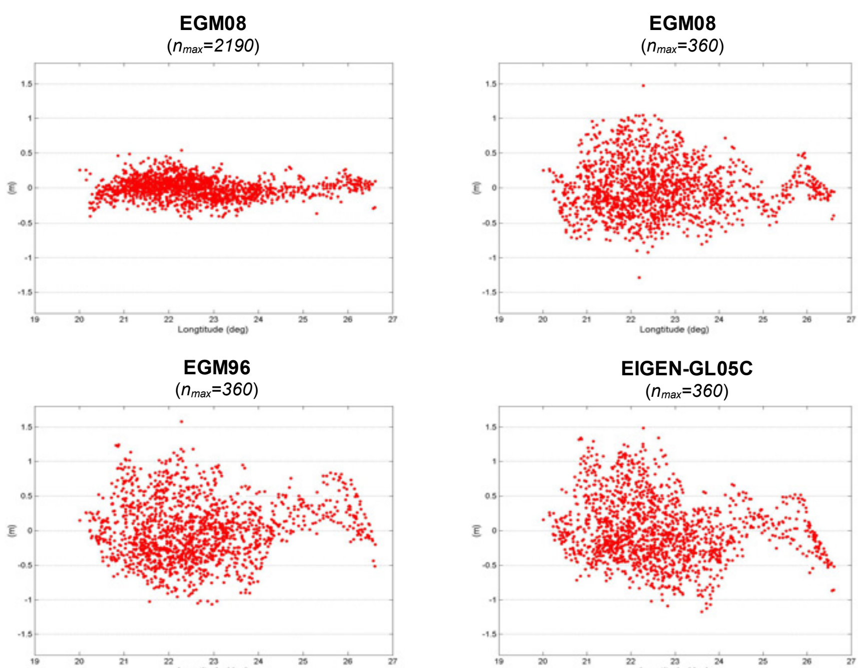

Fig. 3. Longitude-dependent variations of the residuals N^{GPS}–N *(after a least-squares constant bias fit)* at the 1542 GPS/levelling benchmarks

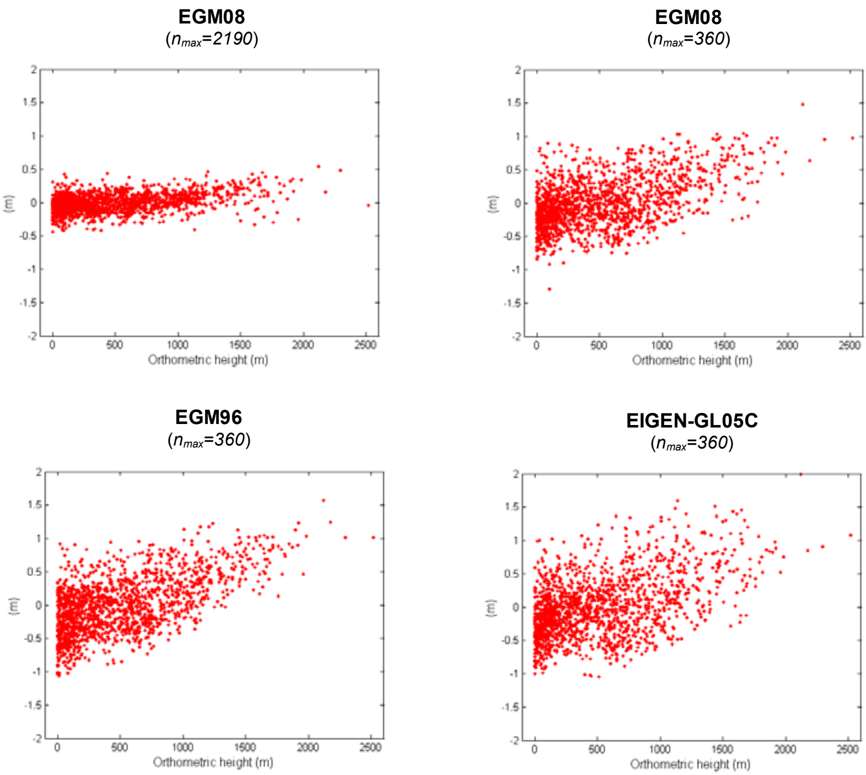

Fig. 4. Height-dependent variations of the residuals N^{GPS}–N *(after a least-squares constant bias fit)* at the 1542 GPS/levelling benchmarks

The alternative parametric models that have been fitted to the geoid height misclosures have the general linear form $h_i - H_i - N_i = \mathbf{a}_i^T \mathbf{x} + v_i$ (see, e.g., [**15**]) and they are presented in Table 5.

Table 5. *Different parametric models used for the least-squares fit between N^{GPS} and the GGM-based geoid undulations N*

	Functional form of 'observation equation'
Model 1 *(bias-only model)*	$h_i - H_i - N_i = \mu + v_i$
Model 2 *(bias & tilt model)*	$h_i - H_i - N_i = \mu + a(\varphi_i - \varphi_o) + b(\lambda_i - \lambda_o)\cos\varphi_i + v_i$
Model 3 *('4-parameter model')*	$h_i - H_i - N_i = \mu + a\cos\varphi_i\cos\lambda_i + b\cos\varphi_i\sin\lambda_i + c\sin\varphi_i + v_i$
Model 4	$h_i - H_i - N_i = \mu + \delta s_H H_i + v_i$
Model 5	$h_i - H_i - N_i = \mu + \delta s_N N_i + v_i$
Model 6	$h_i - H_i - N_i = \mu + \delta s_H H_i + \delta s_N N_i + v_i$

Model 1 employs a single-bias parametric term and it has already been used for the evaluation tests of the previous section. *Model 2* incorporates two additional terms

representing the north-south and east-west components of an average spatial tilt between the GGM and GPS/H geoid surfaces over the network area. *Model 3* is the usual '4-parameter model' which geometrically corresponds to a 3D spatial shift and a scale change of the GGM's reference frame with respect to the underlying frame of the GPS heights (or vice versa). Obviously, the four parameters in *model 3* not only will be affected by datum inconsistencies among the height data, but they will also absorb other systematic errors that may exist in all three data types.

Models 4, 5 and 6 represent height-dependent corrector surfaces that constrain the relation among ellipsoidal, orthometric and geoidal heights in terms of the generalized equation

$$h - (1+\delta s_H)H - (1+\delta s_N)N = \mu \qquad (4)$$

The above equation takes into consideration the fact that the spatial scale of the observed GPS heights does not necessarily conform with the spatial scale induced by the GGM geoid undulations and/or the inherent scale of the orthometric heights determined from terrestrial levelling techniques. Moreover, the GGM geoid undulations and/or the local orthometric heights are often affected by errors that are systematically correlated with the Earth's topography, a fact that can also justify the use of *model 4* or *model 6* for the optimal fit between N^{GPS} and N. Note that the presence of the bias parameter μ in Eq. (4) signifies the spatial offset that usually exists between a local vertical datum and the equipotential surface realized by a GGM-based geoid. Although the adoption of a constant offset is only a first-order approximation for such a systematic effect (as different equipotential surfaces of Earth's gravity field are not exactly parallel), it is considered a sufficient modeling choice for our current study. In fact, the inclusion of its spatial variability within the parametric model requires the knowledge of surface gravity values at the test points, which are not available in our case. The primary role of the parameter μ in Eq. (4) is thus to describe the *mean offset* between the equipotential surface of the local vertical datum and the GGM geoid over the test area.

The statistics of the residuals $\{v_i\}$ and the corresponding estimates of the bias parameter μ, after the least-squares fit of the previous models, are given in Tables 6, 7 and 8 for the case of EGM96, EIGEN-GL05C and EGM08, respectively.

Table 6. *Statistics of the differences N^{GPS}–N for the EGM96 geoid undulations, after the least-squares fit of various parametric models at the 1542 GPS/levelling benchmarks (units in m)*

	Max	*Min*	σ	*Bias* (μ)
Model 1	1.577	-1.063	0.423	-0.446
Model 2	1.587	-1.073	0.422	-0.445
Model 3	1.681	-1.097	0.411	303.983
Model 4	1.198	-0.847	0.341	-0.735
Model 5	1.572	-1.053	0.423	-0.381
Model 6	1.176	-0.861	0.341	-0.656

Table 7. *Statistics of the differences N^{GPS}–N for the EIGEN-GL05C geoid undulations, after the least-squares fit of various parametric models at the 1542 GPS/levelling benchmarks (units in m)*

	Max	*Min*	*σ*	*Bias (μ)*
Model 1	1.997	-1.050	0.461	-0.448
Model 2	1.951	-1.160	0.452	-0.448
Model 3	1.852	-1.096	0.441	-319.447
Model 4	1.294	-1.051	0.408	-0.695
Model 5	1.949	-1.135	0.453	0.068
Model 6	1.260	-1.140	0.399	-0.159

Table 8. *Statistics of the differences N^{GPS}–N for the EGM08 geoid undulations, after the least-squares fit of various parametric models at the 1542 GPS/levelling benchmarks (units in m)*

	Max	*Min*	*σ*	*Bias (μ)*
Model 1	0.542	-0.437	0.142	-0.377
Model 2	0.521	-0.398	0.137	-0.377
Model 3	0.522	-0.398	0.137	3.479
Model 4	0.480	-0.476	0.131	-0.440
Model 5	0.528	-0.442	0.135	-0.109
Model 6	0.474	-0.421	0.123	-0.160

From the above results, it can be concluded that the ‘standard’ parametric models which are used for the combined adjustment of GPS, geoidal and orthometric heights (*model 2* and *model 3*) do not offer any significant improvement in the overall fitting between the GGM geoids and the GPS/levelling data over the Hellenic mainland (i.e. compared to the performance of the single-bias *model 1*).

On the other hand, the height-dependent *model 6* enhances the statistical fit of the EGM96, EIGEN-GL05C and EGM08 geoid undulations with the GPS/levelling data by 8 cm, 6 cm and 2 cm respectively (i.e. compared to the performance of the single-bias *model 1*). The improvement obtained from the least-squares fit of this model (and also from *model 4*) should be attributed to the elimination of the linear correlation trend that was previously identified between the misclosures *h-H-N* and the orthometric heights at the GPS/levelling benchmarks (see Figure 4).

ESTIMATION OF MEAN SPATIAL OFFSET BETWEEN GPS/H & GGM GEOIDS

All parametric models that were considered in the previous section include a common term in the form of a *constant bias* between N^{GPS} and *N*. However, the estimates of this bias parameter μ, as obtained from the least-squares fit of each model, exhibit significant variations among each other for all tested GGMs (see last column in Tables 6, 7 and 8).

Specifically, the estimated bias between N^{GPS} and *N* from the usual ‘4-parameter model’ (*model 3*) appears to be highly inconsistent with respect to the corresponding bias values derived from the other parametric models. In the case of EGM08 geoid heights, for example, the estimate of μ from this particular parametric model differs by almost 4 m compared to the resulting estimates of μ from the other models, whereas for the EIGEN-GL05C geoid heights these differences increase to more than 300 m!

Such a result is not surprising since the key role of the bias parameter μ in *model 3* is not to represent the average vertical offset between the GPS/H and GGM geoid surfaces over the test network, as it happens for example in *model 1*. In fact, the three additional terms in *model 3* are the ones that largely absorb the systematic part of the differences $N^{GPS}-N$ in the form of a 3D spatial shift, leaving to the bias parameter μ the auxiliary role of a differential scale change factor.

To better understand the previous remark, let us recall the linearized transformation for geoid undulations between two parallel Earth-fixed reference frames [**16**]

$$N'_i - N_i = (\mathrm{a}w_i + N_i)\delta s + t_x \cos\varphi_i \cos\lambda_i + t_y \cos\varphi_i \sin\lambda_i + t_z \sin\varphi_i \tag{5}$$

where the term 'a' is the semi-major axis of the associated reference ellipsoid, t_x, t_y, t_z and δs are the translation parameters and the differential scale change between the underlying frames, and w_i corresponds to the unitless term $(1-e^2 \sin^2 \varphi_i)^{1/2}$ that is approximately equal to 1 (e.g. the squared eccentricity of the GRS80 reference ellipsoid is $e^2 \approx 0.0067$). The last formula conveys, in the language of geodetic datum transformation, the geometric aspects of the '4-parameter model' which is often employed for the optimal fit between GPS, geoidal and leveled height data. Based on Eq. (5), it is seen that the bias parameter μ in *model 3* emulates a scaling correction to the (GPS/H or GGM) geoid heights, which is applied *in conjunction* with an additional spatial offset ($a \rightarrow t_x$, $b \rightarrow t_y$, $c \rightarrow t_z$) between the two different geoid realizations.

The estimates of the bias μ from the other parametric models show smaller differences (up to dm-level) among their values for each tested GGM. The inclusion of a constant spatial tilt between N^{GPS} and N over Greece does not actually affect the initial estimate of μ that is obtained from the bias-only *model 1*. On the other hand, the use of height-dependent corrector terms (*models 4*, *5* and *6*) affects considerably the final estimate of μ, as it can be verified from the results in Tables 6, 7 and 8.

All in all, the determination of the mean spatial offset between a local vertical datum with an associated unknown W_o value (e.g. HVRF in our case) and a GGM-based geoid seems to depend strongly on the particular parametric model that is used for adjusting heterogeneous height data over a network of GPS/levelling benchmarks. The '4-parameter model' that is frequently used for such an adjustment task is generally unable to provide a realistic estimate for this vertical offset. Taking into account the arguments towards the use of the generalized constraint in Eq. (4) (i.e. topography-correlated errors in the H and/or N values, spatial scale inconsistencies among the different types of height data) and the results presented in Tables 6, 7 and 8, it should also be underlined that the single-bias *model 1* or the bias-tilt *model 2* may not always be the safest choice for estimating the average shift between a GGM and a GPS/H geoid over a regional area.

Evaluation of Relative Geoid Heights

A third set of evaluation results was obtained through the comparison of GGM and GPS/H *geoid slopes* over the Hellenic test network of 1542 benchmarks. For all baselines formed within this network, the following differences of relative geoid undulations were computed

$$\Delta N_{ij}^{GPS} - \Delta N_{ij} = (h_j - H_j - h_i + H_i) - (N_j - N_i) \tag{6}$$

Depending on the baseline length, the residuals from Eq. (6) were grouped into various classes and their statistics were then derived within each class. Given the actual coverage and spatial density of the control points in our test network, the length of the baselines considered for these computations varied from 2 km up to 600 km (in spherical approximation). Note that the computation of the relative geoid differences was performed *after* the implementation of an initial least-squares fit between the pointwise GGM and GPS/H geoid heights using *model 6*.

The statistics of the differences between GGM and GPS/H geoid slopes are given in the following tables for some characteristic baseline classes (0-5 km, 5-10 km, 10-50 km, 50-100 km). The selected classes cover spatial wavelengths that exceed the inherent resolution for most of the tested GGMs (n_{max} = 360, half-wavelength ≈ 55 km), yet they hold a key role for the evaluation of geopotential models and their ability to contribute to a unified vertical datum realization over regional areas.

As seen from the results in Tables 9 through 12, the full-resolution EGM08 model performs consistently better than all other GGMs over all baseline classes. The improvement becomes more pronounced as the baseline length increases, indicating the significant contribution of EGM08 high-degree harmonics (n > 360) for the slope representation of the Hellenic geoid over baselines 10-100 km. The standard deviation of the EGM08 residuals $\Delta N^{GPS} - \Delta N$ lies between 10-16 cm, resulting to an improvement by 29% for baselines <5 km, by 39% for baselines 5-10 km, by 60% for baselines 10-50 km, and by 65% for baselines 50-100 km over the performance of the other GGMs.

Table 9. *Statistics of the differences between GGM and GPS/H relative geoid heights for baselines < 5 km (number of baselines: 289, units in m)*

	Max	*Min*	σ	*Mean*
EGM08 *(n_{max}=2190)*	0.644	-0.469	0.106	0.005
EGM08 *(n_{max}=360)*	0.651	-0.479	0.133	-0.002
EIGEN-GL05C	0.656	-0.478	0.136	-0.002
EIGEN-GL04C	0.650	-0.452	0.145	-0.002
GGM03C	0.614	-0.509	0.140	-0.002
EGM96	0.645	-0.476	0.138	-0.002

Table 10. *Statistics of the differences between GGM and GPS/H relative geoid heights for baselines 5-10 km (number of baselines: 2119, units in m)*

	Max	*Min*	σ	*Mean*
EGM08 *(n_{max}=2190)*	0.453	-0.613	0.113	0.000
EGM08 *(n_{max}=360)*	0.633	-0.890	0.178	-0.007
EIGEN-GL05C	0.575	-0.801	0.180	-0.003
EIGEN-GL04C	0.657	-0.834	0.180	-0.004
GGM03C	0.623	-0.856	0.178	-0.004
EGM96	0.623	-0.848	0.176	-0.001

With the exception of the EGM08 results, a significant bias exists in the geoid slope residuals $\Delta N^{GPS} - \Delta N$ over baselines 50-100 km (see last column in Table 12). The main reason is probably the systematic character of GGM omission errors in the recovered geoid height signal, which generates an apparent scale difference with respect to the GPS/H relative geoid heights within this particular baseline class.

Table 11. *Statistics of the differences between GGM and GPS/H relative geoid heights for baselines 10-50 km (number of baselines: 56575, units in m)*

	Max	*Min*	σ	*Mean*
EGM08 *(n_{max}=2190)*	0.811	-0.772	0.147	-0.000
EGM08 *(n_{max}=360)*	1.925	-1.545	0.392	-0.008
EIGEN-GL05C	1.696	-1.868	0.428	-0.006
EIGEN-GL04C	1.547	-1.435	0.381	-0.014
GGM03C	1.702	-1.441	0.388	-0.010
EGM96	1.603	-1.545	0.384	-0.008

Table 12. *Statistics of the differences between GGM and GPS/H relative geoid heights for baselines 50-100 km (number of baselines: 135970, units in m)*

	Max	*Min*	σ	*Mean*
EGM08 *(n_{max}=2190)*	0.810	-0.798	0.164	-0.002
EGM08 *(n_{max}=360)*	1.824	-1.590	0.433	-0.009
EIGEN-GL05C	2.034	-2.134	0.543	-0.024
EIGEN-GL04C	1.684	-1.672	0.467	-0.033
GGM03C	1.865	-1.860	0.452	-0.025
EGM96	1.717	-1.714	0.451	-0.021

EMPIRICAL ASSESSMENT OF RELATIVE GGM/GPS-AIDED ORTHOMETRIC HEIGHT DETERMINATION

For the purpose of assessing the accuracy in GPS levelling when the required geoid undulations in Greece are obtained from the tested geopotential models, we have studied the agreement of orthometric height differences ΔH_{ij} that are derived: *(a)* directly from the known orthometric heights at the GPS/levelling benchmarks and *(b)* indirectly from the observed ellipsoidal heights and the GGM geoid undulations. As a pre-processing step, a single bias fit has only been applied to the original misclosures *h-H-N* at the 1542 control points.

The standard deviation of the residual height differences $\Delta H^{GGM/GPS}-\Delta H$, for baselines up to 50 km, yields the behaviour shown in Figure 5. The results in this figure refer only to two models (EGM08, EGM96) since the empirical estimates of $\sigma_{\Delta H}$ from the other GGMs follow closely the behaviour of EGM96.

Using the empirical error model $\sigma_{\Delta H} = \sigma_o L^{1/2}$ that is commonly employed in vertical positioning studies, the 'external accuracy' of EGM08-based relative orthometric heights can be approximated by a factor $\sigma_o \approx$ 3-5 cm per square-root km (for baselines L < 20 km) and $\sigma_o \approx$ 2 cm per square-root km (for longer baselines up to L = 50 km); see Figure 5. On the other hand, the 'external accuracy' of EGM96-based relative orthometric heights is described by a factor $\sigma_o \approx$ 6-7 cm per square-root km for all baselines up to 50 km.

The absolute values of the actual differences $\Delta H^{GGM/GPS}-\Delta H$ over the Hellenic test network are plotted in Figure 6 (for baselines 0-20 km) as a function of their corresponding baseline length. The improvement obtained from the EGM08 model is clearly visible, as 75% of the baselines exhibit a consistency level with the available

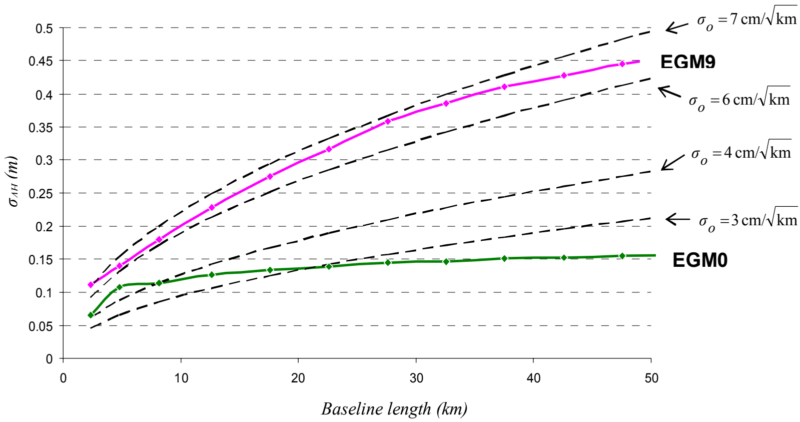

Fig. 5. Standard deviation of the residual height differences $\Delta H^{GGM/GPS} - \Delta H$ as a function of baseline length. The dashed curves correspond to the statistical error model $\sigma_{\Delta H} = \sigma_o L^{1/2}$ for some typical values of the accuracy factor σ_o

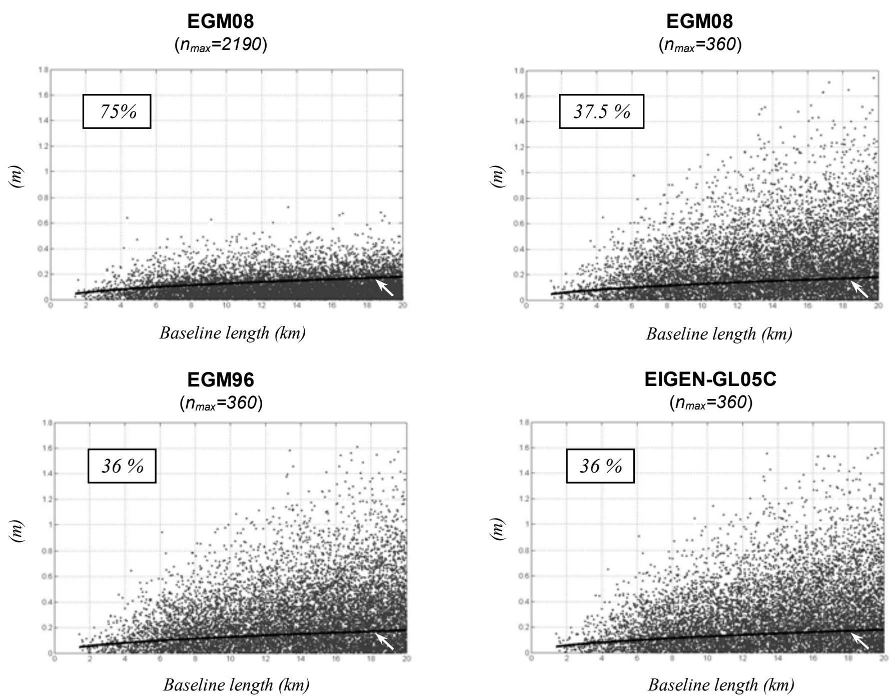

Fig. 6. Scatter plots of the absolute values of the residuals $\Delta H^{GGM/GPS} - \Delta H$ for baselines 0-20 km (number of baselines: 10419). The solid black curve represents the statistical error model $\sigma_{\Delta H} = \sigma_o L^{1/2}$ with $\sigma_o = 4$ cm per square-root km. The percentage of baselines whose residual height differences fall within the corresponding interval ($-\sigma_{\Delta H}$, $\sigma_{\Delta H}$) are noted in each graph

orthometric height data that is better than 4 cm per square-root km, compared to 36% (or less) in the case of the other GGMs.

It should be noted that the orthometric heights in our test network are not error-free, and thus the current assessment for the external accuracy of $\Delta H^{GGM/GPS}$ does not reflect only the GGM commission/omission errors, but it is affected also by errors in the orthometric (and the ellipsoidal) height data.

CONCLUSIONS

1. The results of our GGM evaluation tests unveiled the (expected) superiority of EGM08 over all existing combined geopotential models for the area of mainland Greece. The new model outperforms all other GGMs at the 1542 GPS/levelling benchmarks and it improves the statistical fit with the Hellenic GPS/H geoid by approximately 27 cm (or more)! After a single bias fit, the consistency among ellipsoidal, orthometric and EGM08-based geoid heights amounts to ±14 cm (1σ level), reflecting mainly the regional effects of the model's commission errors and other systematic distortions in the HVRF orthometric heights at the test points.
2. The old EGM96 model remains better than the latest EIGEN-type combined models of similar resolution, in terms of the achieved statistical fit with the GPS/levelling data at the 1542 Hellenic test points. This result should probably be attributed to the incorporation of additional local gravity data over Greece in the development of EGM96.
3. A correlation trend between the residuals N^{GPS}-N and the orthometric heights of the test points has been identified in all GGMs that were considered in this study. Such a result suggests the presence of a height-dependent bias between the GGM and GPS/H geoid undulations, which is primarily caused by GGM approximation errors of the terrain-dependent gravity field signal over the Hellenic mainland. To a lesser extent, this height-dependent bias should also be attributed to an inherent scale difference between the GPS heights and the HVRF Helmert orthometric heights, which is due to the different measurement procedures and other modeling assumptions that were involved in their determination. The EGM08 model showed the smallest height-dependent bias in its evaluation results (approximately 0.12 mm/m compared to 0.57 mm/m in the case of EGM96, see Figure 4), thus confirming that its ultra-high frequency content gives a significantly improved approximation for the terrain-dependent gravity field over the Hellenic mainland.
4. In terms of relative geoid accuracy, the EGM08 model showed a rather stable performance for the standard deviation of the slope residuals $\Delta N^{GPS} - \Delta N$ over all baseline classes that were considered in this study. Compared to other GGMs whose relative geoid accuracy decreases rapidly for baselines ranging from 10 km to 100 km (i.e. empirically estimated values of $\sigma_{\Delta N}$ reach more than 50 cm), the EGM08 model gives a more balanced behavior with the corresponding estimates of $\sigma_{\Delta N}$ never exceeding 20 cm even for baselines up to 600 km. For short baselines (up to 10 km) the improvement in the relative geoid undulation accuracy from EGM08 is also significant, yielding an ΔN consistency level with the GPS/levelling data in the order of 10 cm (1σ level).
5. Although the stand-alone performance of EGM08 cannot yet satisfy mm-level accuracy requirements for vertical positioning, it provides a major step towards cm-level determination of orthometric height differences via GPS levelling techniques. However, a more detailed analysis with least-squares (or other types of) interpolation techniques and spatial corrector surfaces for modeling the differences $N^{GPS} - N$ or $\Delta N^{GPS} - \Delta N$ is still required to achieve true cm-level conversion between EGM08/GPS and HVRF orthometric heights over the entire Hellenic mainland.

ACKNOWLEDGEMENTS

The GPS and levelling data used for this study were kindly provided by Ktimatologio S.A under an ongoing research collaboration with the Department of Geodesy and Surveying, Aristotle University of Thessaloniki, in the frame of the HEPOS project. The authors would like to acknowledge Will Featherstone for his useful comments during the review of this paper.

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