The role of a conventional transformation scheme for vertical reference frames

C. Kotsakis
Department of Geodesy and Surveying
Aristotle University of Thessaloniki, Thessaloniki 541 24, Greece
Email: kotsaki@topo.auth.gr

Abstract. A conventional transformation between different realizations of a vertical reference system is an important tool for geodetic studies on precise vertical positioning and physical height determination. Its main role is the evaluation of the consistency for co-located vertical reference frames (VRFs) on the basis of some fundamental 'datum perturbation' parameters. Our scope herein is to discuss a number of key issues related to the formulation of such a VRF transformation model and to present a few examples from its practical implementation in the comparison of various existing vertical frames over Europe.

Keywords. Vertical reference frames, physical height transformation.

1 Introduction

The comparison of terrestrial reference frames (TRFs) that are established by different observation techniques and/or optimal estimation strategies is a common task which is often implemented in geodetic studies, constituting either a research goal in itself or an auxiliary step for other applications that depend on precise geometric positioning. Such a comparison is typically based on the *linearized similarity transformation* (e.g., Leick and van Gelder 1975), a useful tool that supports the evaluation of Earth-fixed TRFs on the basis of some datum-perturbation parameters that are inherently associated with the definition of geodetic terrestrial reference systems (Altamimi et al. 2007). Following a least-squares fitting of this transformation model over a network of common points, a set of estimated parameters can be obtained that quantify the origin, orientation and scale consistency of the underlying TRFs in terms of their relative translation/rotation/scale variation. The aforementioned scheme provides a geodetically meaningful framework for comparing, transforming and combining Euclidean spatial reference frames, and also for assisting their quality assessment through a suitable de-trending of their systematic discrepancies in order to identify any localized distortions in their respective coordinate sets.

To a large extent, a similar situation occurs also in geodetic applications related to the establishment of *vertical reference systems* for physical height determination. Several realizations of a vertical reference system (VRS) may be available over a regional or even continental network, originating from separate leveling campaigns, alternative data combination schemes and different adjustment strategies. As an example, let us consider a set of national leveling benchmarks, within some EU country, that is part of the United European Leveling Network (UELN). At least three different vertical reference frames (VRFs) co-exist in this regional leveling network, whose physical heights are respectively obtained from the EVRF00 and EVRF07 continental solutions (Ihde and Augath 2001; Sacher et al. 2008), and by the (usually older) national adjustment of the primary height network in the underlying country. If, in addition, GPS data are available at the particular UELN stations, then more VRFs could emerge through the synergetic use of gravimetric geoid/quasi-geoid models that enable the conversion of observed geometric heights to physical heights.

An objective comparison among different VRFs needs to be rigorously based on a conventional transformation model that quantifies the inconsistencies in the realization of a vertical reference system from co-located physical height datasets. The adopted model must resemble the role of the linearized similarity transformation that is used in TRF studies, while its associated parameters should reflect the vertical datum disturbances implied by the corresponding VRFs. Eventually, the utmost role of the underlying height transformation is to be used for generating a combined optimal VRF solution from individual realizations that are jointly merged into a unified vertical frame by postulating appropriate minimum constraints to the datum-related parameters of the height transformation model.

The aim of this paper is to discuss some general aspects about the formulation of a conventional height transformation model for vertical frame evaluation studies, and to present a few examples from its practical use in the comparison of various existing VRFs over Europe.

2 Height transformation schemes in practice

Various transformation algorithms for physical heights exist in geodetic practice, mostly in support of gravity field modeling and vertical positioning with heterogeneous data. Typical examples include the reduction of physical heights to a conventional permanent tide system and/or to a reference time epoch due to temporal variations caused by various geodynamical effects (Mäkinen and Ihde 2009; Jekeli 2000), the conversion from normal to orthometric heights (and vice versa), and the determination of apparent height variations due to a known geopotential offset in the zero-height level of the underlying vertical datum.

Moreover, a number of modeling schemes have appeared in the geodetic literature for the optimal fitting of co-located height datasets and the inference of hidden systematic disturbances between them. The treatment of these problems relies on the inverse implementation of a height transformation model that is adopted on the basis of (mostly) empirical criteria. A well-known example is the combined adjustment of ellipsoidal, geoid/quasi-geoid, and leveled heights over a terrestrial control network, which represents a common procedure that has been applied under different objectives in numerous geodetic studies. In the context of our discussion herein, such an adjustment task shall be perceived in terms of a generalized transformation scheme for physical heights:

$$H_i' - H_i = \mathbf{a}_i^{\mathrm{T}} \mathbf{x} + s_i + v_i \tag{1}$$

where H_i and H_i' denote the orthometric (or normal) heights obtained from levelling measurements and GPS/geoid (or quasi-geoid) data, respectively. Their systematic discrepancies are modelled by a low-order parametric model and (optionally) a spatially-correlated zero-mean signal, whereas v_i contains the remaining random errors in the height data. The estimated values of the unknown parameters $\mathbf{x}$ and the predicted values of the stochastic signals s_i are obtained from a least-squares (LS) inversion of Eq. (1) over a number of control stations, using some a-priori information for the data noise level and the signal covariance function (Kotsakis and Sideris 1999).

Several choices have been used in practice for the parametric component $\mathbf{a}_i^{\mathrm{T}}\mathbf{x}$ in Eq. (1), none of which has ever assumed the role of a geodetically meaningful evaluator of the systematic differences between the underlying VRFs; that is, between the levelling-based frame $\{H_i\}$ and the GPS/geoid-based frame $\{H_i'\}$. In most cases, the suitability of the adopted model is judged by the reduction of the sample variance of the adjusted height errors $\{v_i\}$ within the test network, and not by the physical or geometrical meaning (if any) of its parameters. In fact, the estimated values of $\mathbf{x}$ have never been of any actual importance in geodetic studies, other than offering a more or less arbitrary parametric description for the spatial trend of the height differences $H_i'–H_i$.

It is worth noting that the use of the well-known *4-parameter model*:

$$\mathbf{a}_i^{\mathrm{T}} \mathbf{x} = x_o + x_1 \cos\varphi_i \cos\lambda_i + x_2 \cos\varphi_i \sin\lambda_i + \\ + x_3 \sin\varphi_i \tag{2}$$

may be viewed, to some extent, as an attempt to infer 'datum perturbations' between the physical height frames $\{H_i\}$ and $\{H_i'\}$. Such a viewpoint relies on the equivalent form of Eq. (1)

$$N_i' - N_i = \mathbf{a}_i^{\mathrm{T}} \mathbf{x} + s_i + v_i \tag{3}$$

where N_i and N_i' denote the corresponding geoid or quasi-geoid undulations from a gravimetric model and GPS/levelling data, respectively. If the 4-parameter model is used into Eq. (1), then the systematic part of the differences $H_i'–H_i$ is essentially described, in view of Eq. (3), through *a 3-D spatial shift (x_1, x_2, x_3) and a scale change (x_o) between the associated zero-height reference surfaces of the physical heights*. This fictitious perspective may also be evoked for the comparison of vertical frames that are obtained exclusively from terrestrial levelling, without the external aid of GPS heights and gravimetric geoids/quasi-geoids.

The aforementioned 4-parameter model was often used in older studies as a basic tool for estimating geodetic datum differences from heterogeneous height data; especially for assessing the geocentricity of TRFs based on Doppler-derived and gravimetrically-derived geoid undulations and also for determining the Earth's optimal equatorial radius from geometric and physical heights (e.g., Schaab and Groten 1979; Grappo 1980; Soler and van Gelder 1987). These tasks require a global data distribution, otherwise the 3-D translation parameters (x_1, x_2, x_3) become highly correlated with the zero 'scaling' term (x_o), and their adjusted values may be entirely unrealistic from a physical point of view. For that reason, the LS inversion of Eq. (1) will not always produce a geodetically meaningful solution for the individual components of the 4-parameter model - not even for the estimated height bias x_o; for some examples, see Kotsakis and Katsambalos (2010). Moreover, the conceptual drawback of this model for VRF evaluation studies is that it compares the zero-height surfaces between two vertical frames with respect to a (fictitious) geocentric reference system, *without* considering the most important element in vertical datum realization: a geopotential reference value W_o and its possible variation between alternative VRFs.

The estimation of the (usually unknown) zero-height level W_o that is inherently linked to any vertical frame can be carried out through various strategies based on 'external' geopotential information and space geodetic measurements at a number of leveling benchmarks (e.g., Burša et al. 2001; Ardalan et al. 2002). In this way, any VRF is comparable to another, not necessarily co-located, VRF′ in terms of the estimated geopotential difference (δW_o) of their zero-height levels. However, such a value is affected by the errors in the adopted geopotential model and thus it may give a misleading assessment of the zero-height consistency between the tested VRFs.

Furthermore, a comprehensive comparison of vertical frames should take into account the spatial scale variation due to systematic differences in their associated measurement techniques and modeling assumptions. In fact, one should not forget that the fundamental height constraint h–H–N=0, or its equivalent differential form Δh–ΔH–ΔN=0, requires not only the 'origin consistency' among the heterogeneous height types, but also their reciprocal vertical scale uniformity.

3 Basic formulation of a conventional VRF transformation

An objective assessment of the consistency between VRFs requires a conventional model describing their systematic discrepancies over a common group of control stations. The parameters **x** of such a model:

$$H_i' - H_i = f(\mathbf{x}) + v_i \tag{4}$$

should quantify the (actual and/or apparent) vertical datum perturbations induced by the physical height datasets $\{H_i\}$ and $\{H_i'\}$, while the remaining residuals, after a LS adjustment of Eq. (4), indicate the relative accuracy level of the corresponding vertical frames. Further analysis of the adjusted height differences is useful for identifying systematic distortions and other spatially correlated errors within the tested VRFs, which cannot be absorbed by the transformation parameters **x**.

Note that a VRF is a realization of a 1-D terrestrial coordinate system with respect to an equipotential surface of Earth's gravity field. The latter defines a conventional zero-height level relative to which vertical positions (geopotential numbers and their equivalent physical heights) can be obtained by various geodetic techniques and terrain modeling hypotheses. Hence, the key role of Eq. (4) is to appraise the variation of the reference equipotential surface and the vertical metric scale, which both signify the fundamental datum constituents for vertical positioning within each of the tested frames.

Two essential parameters should be incorporated in $f(\mathbf{x})$, namely a VRF *translation parameter* in the form of a geopotential disturbance δW_o, and a VRF *scale-change parameter* in the form of a unitless factor δs reflecting the scale difference between the corresponding height frames. In case of dynamic VRFs, the time derivatives of the above parameters need also to be considered when transforming physical heights between different epochs and/or vertical velocities from a VRF to another VRF′. In contrast to the Helmert-type transformation scheme that is used in geometric Cartesian TRFs, there are not rotational terms within the VRF transformation model $f(\mathbf{x})$ since the frame orientation aspect is not a geodetically meaningful characteristic of vertical reference systems.

3.1 General remarks

The notion of the 'scale' in a vertical reference system is often linked to the geopotential value W_o that is adopted for defining absolute vertical coordinates (geopotential numbers) and their equivalent physical heights on the Earth's surface. Specifically, the VRS scale is explicitly related to an equipotential surface realized by the combination of a mean sea surface topography model and a global gravity field model, in accordance with the classic Gauss-Listing definition of the geoid (Ihde 2007). This is mainly a simplified approach to quantify the *average size* of the reference surface for vertical positioning, since the ratio of the geocentric gravitational constant to the adopted reference geopotential level:

$$R = GM / W_o \tag{5}$$

yields the mean radius of the geoid, which itself defines a physical metric for the geocentric spatial position of terrestrial points with zero heights! Obviously, any change of W_o induces an apparent offset to the terrestrial physical heights, which can be perceived as an indirect 'scaling effect' due to the changed spatial dimension of their zero-height reference surface (with respect to a fixed geopotential model).

The previous viewpoint aims at the standardization of the Earth's global scale in terms of the physical parameters GM and W_o, and it is not related to the notion of a scale variation between different realizations of a VRS. In fact, a change of W_o is related to a transformation from a conventional height 'origin' to another one, whereas the scope of a VRF scale change is to account for the systematic discrepancy of the vertical metric scale realized by alternative heighting techniques and datasets when determining physical height differences. Both types of VRF perturbation (origin and scale) are feasible and they may co-exist in the joint analysis of vertical frames.

3.2 The effect of δW_o

Changing the zero-height level of a VRF means that a new vertical frame with a different equipotential surface will be used as a reference for physical heights. Such a transformation is described through a single parameter (δW_o) reflecting the geopotential disturbance of the zero-height equipotential surface with respect to a conventional representation $W(\cdot)$ of the Earth's gravity field. The effect on the VRF geopotential numbers is a simple offset equal to δW_o, while for the VRF orthometric or normal heights it takes the form of a nonlinear and spatially inhomogeneous variation according to the following power series expansions:

$$H_i' - H_i \;=\; \frac{\delta W_o}{g_i} - \frac{\partial g}{\partial H}\frac{\delta W_o^2}{2g_i^3} + \ldots \tag{6}$$

for the case of *orthometric* heights, or

$$H_i' - H_i \;=\; \frac{\delta W_o}{\gamma_i} - \frac{\partial \gamma}{\partial H}\frac{\delta W_o^2}{2\gamma_i^3} + \ldots \tag{7}$$

for the case of *normal* heights. The terms g_i and $\partial g/\partial H$ denote the actual gravity and its vertical gradient on the geoid, or more precisely on the equipotential surface associated with the initial orthometric height. Also, γ_i and $\partial\gamma/\partial H$ denote the normal gravity and its vertical gradient on the reference ellipsoid which is associated with the initial normal height.

For practical purposes, both Eqs. (6) and (7) can be replaced by the simplified linearized formula:

$$H_i' - H_i = \frac{\delta W_o}{\gamma_i} \tag{8}$$

since their second, and higher, order terms have a negligible contribution (< 1 mm) for reasonably low values of δW_o (up to 1-2 gpu, or equivalently up to 10-20 $m^2 s^{-2}$). Moreover, the geoidal gravity g_i in Eq. (6) may be safely substituted by the normal gravity γ_i on the reference ellipsoid, causing an approximation error into the transformed orthometric height that is below the mm level, even for gravity anomaly values up to 500 mGal.

3.3 The effect of δs

In contrast to geometric Cartesian TRFs, the assessment of a systematic scale difference between VRFs is not a straightforward issue. The effect of a scale change on the physical heights depends on the way we (choose to) handle the Earth's gravity field and its equipotential surfaces under a uniform spatial re-scaling. The underlying problem is similar to the TRF similarity transformation of GPS heights with respect to a reference ellipsoid, where the latter may or may not 'follow' the spatial scale variation that is imposed by the TRF scale change (Soler and van Gelder 1987; Kotsakis 2008).

Starting from the fundamental differential formula (Heiskanen and Moritz 1967, p. 50)

$$dW = -g \; dH \tag{9}$$

where g denotes the magnitude of the gravity vector, the following relationship can be obtained:

$$dH = \frac{g_x}{g}dx + \frac{g_y}{g}dy + \frac{g_z}{g}dz = -\frac{gradW \cdot \mathbf{dr}}{g} \tag{10}$$

which gives the vertical (physical height) metric in terms of a weighted combination of the Euclidean metric components with respect to an Earth-fixed spatial coordinate system. The associated weights are the normalized geopotential gradients and they represent the influence of the Earth's gravity field on the physical height scale.

Assuming that the geopotential signal and its gradient vector remain invariant under a uniform scale change $\mathbf{dr}'=(1+\delta s)\mathbf{dr}$, then the resulting effect on the physical heights over the Earth's surface is expressed through a simple linear re-scaling:

$$H_i' = (1+\delta s)H_i \tag{11}$$

The above formula provides the basis for assessing the scale difference between VRFs relative to a fixed reference surface; note that zero-height points are preserved by the scaling transformation of Eq. (11). In essence, the differential factor δs absorbs the (linear part of) topographically-correlated discrepancies between $\{H_i\}$ and $\{H_i'\}$, which cause an apparent scale variation between their corresponding VRFs.

4 Least-squares adjustment of the VRF transformation model

Based on the discussion given in Sect. 3, a conventional VRF transformation model can be formulated in terms of the linearized expression:

$$H_i' - H_i = \frac{\delta W_o}{\gamma_i} + \delta s\, H_i \tag{12}$$

where the meaning of each term has already been explained in previous paragraphs. Essentially, the above model represents the 1D-equivalent of the similarity transformation for vertical positions (physical heights) from a VRF to another VRF′.

The LS adjustment of Eq. (12) over a network of m control points leads to the following system of normal equations (NEQs):

$$\begin{bmatrix} \mathbf{q}^{\mathrm{T}}\mathbf{P}\mathbf{q} & \mathbf{q}^{\mathrm{T}}\mathbf{P}\mathbf{d} \\ \mathbf{d}^{\mathrm{T}}\mathbf{P}\mathbf{q} & \mathbf{d}^{\mathrm{T}}\mathbf{P}\mathbf{d} \end{bmatrix} \begin{bmatrix} \delta\hat{W}_o \\ \delta\hat{s} \end{bmatrix} = \begin{bmatrix} \mathbf{q}^{\mathrm{T}}\mathbf{P}(\mathbf{d}'-\mathbf{d}) \\ \mathbf{d}^{\mathrm{T}}\mathbf{P}(\mathbf{d}'-\mathbf{d}) \end{bmatrix} \tag{13}$$

where the vectors $\mathbf{d}$ and $\mathbf{d}'$ contain the known physical heights (orthometric or normal) with respect to different vertical frames, i.e.,

$$\mathbf{d} = \begin{bmatrix} H_1 & \cdots & H_m \end{bmatrix}^{\mathrm{T}} \quad \mathbf{d}' = \begin{bmatrix} H_1' & \cdots & H_m' \end{bmatrix}^{\mathrm{T}}$$

while $\mathbf{P}$ is a weight matrix for their differences, and the auxiliary vector $\mathbf{q}$ is defined as: $\mathbf{q}(i) = 1/\gamma_i$.

The previous NEQs system is always invertible provided that $\mathbf{q}$ and $\mathbf{d}$ are not co-linear with each other. Given that the elements of the auxiliary vector $\mathbf{q}$ retain an almost constant value (their relative deviation does not exceed 10^{-4} even in large-scale continental networks), the inversion of Eq. (13) is practically guaranteed as long as the m control points do not have the same height level!

The correlation coefficient between the VRF transformation parameters is always negative and it is given by the general expression:

$$\rho_{\delta\hat{W}_o,\delta\hat{s}} = -\frac{\mathbf{q}^{\mathrm{T}}\mathbf{P}\mathbf{d}}{(\mathbf{d}^{\mathrm{T}}\mathbf{P}\mathbf{d})^{1/2}(\mathbf{q}^{\mathrm{T}}\mathbf{P}\mathbf{q})^{1/2}} \simeq -\frac{mean[\mathbf{d}]}{rms[\mathbf{d}]} \tag{14}$$

A useful algebraic relationship for the optimal estimates obtained from the inversion of Eq. (13), as a function of their correlation coefficient, is:

$$\delta\hat{W}_o = \frac{\mathbf{q}^{\mathrm{T}}\mathbf{P}(\mathbf{d}'-\mathbf{d})}{\mathbf{q}^{\mathrm{T}}\mathbf{P}\mathbf{q}} + \rho_{\delta\hat{W}_o,\delta\hat{s}} \frac{(\mathbf{d}^{\mathrm{T}}\mathbf{P}\mathbf{d})^{1/2}}{(\mathbf{q}^{\mathrm{T}}\mathbf{P}\mathbf{q})^{1/2}} \delta\hat{s} \tag{15}$$

The separability of the VRF transformation parameters depends on the vertical network configuration. In the context of the joint estimation of δW_o and δs, an optimal vertical network geometry is not related to a homogeneous coverage over the Earth's surface, but to the *height variability* among its control stations. Specifically, the dispersion of the data vector $\mathbf{d}$ must be sufficiently large (with respect to the average height of the control stations) in order for the correlation coefficient in Eq. (14) to retain a reasonably low value.

Let us give a few examples from the LS inversion of the transformation model in Eq. (12) for a number of VRFs in Europe. The first example employs the EVRF00 and EVRF07 normal heights at the 13 UELN fiducial stations that were used for the primary definition of the zero-height level in the official EVRF07 solution (Sacher et al. 2008). Although the zero-height levels of these two frames were a-priori aligned at the particular stations through a single constraint within the EVRF07 adjustment (see Sacher et al. 2008), our results in Table 1 show a small (mm-level) offset between their corresponding reference surfaces. This is caused by the inherent correlation between the estimated parameters $\delta\hat{W}_o$ and $\delta\hat{s}$ (ρ = -0.7 in this case), representing an unavoidable 'leakage' effect that occurs in most adjustment problems with coordinate transformation models. Nevertheless, the values of the transformation parameters between EVRF00 and EVRF07 seem to be statistically insignificant, within the limits of their statistical precision, over the particular 13-station UELN continental network.

Table 1. Transformation parameters between EVRF00 and EVRF07 based on the normal heights at 13 UELN stations over Europe. The initial weight matrix **P** was set equal to a unit matrix, and the a-posteriori variance factor of unit weight was estimated at σ_o = 9 mm.

d	**d′**	$\delta\hat{W}_o$ (gpu)	$\delta\hat{s}$ (ppm)
EVRF00	EVRF07	0.002 ± 0.004	-25.5 ± 27.7

The second example uses several VRFs that are realized over a network of 20 Swiss leveling benchmarks which are part of the EUVN-DA network (Marti 2010). The tested frames were compared on the basis of Eq. (12) using normal heights from: the EVRF00 and EVRF07 continental solutions, the combination of GPS heights with the European gravimetric geoid model EGG08, the official Swiss national height system LN02, and the LHN95 rigorous adjustment of the Swiss national height network. The estimated transformation parameters are given in Table 2, whereas the standard deviation of the height residuals (before and after the VRF transformation) are listed in Table 3.

Table 2. Transformation parameters between different VRFs in Switzerland based on 20 EUVN-DA Swiss stations.

d	**d′**	$\delta\hat{W}_o$ (gpu)	$\delta\hat{s}$ (ppm)
EVRF00	EVRF07	0.025 ± 0.001	2.9 ± 0.8
GPS/EGG08	EVRF07	0.044 ± 0.012	-76.6 ± 10.7
LN02	EVRF07	-0.251 ± 0.030	35.7 ± 26.8
LHN95	EVRF07	-0.060 ± 0.026	-220.7 ± 22.9

Table 3. Standard deviation (in cm) of the height residuals as obtained before and after the adjustment of Eq. (12) at 20 EUVN-DA Swiss stations.

d	**d′**	σ (before)	σ (after)
EVRF00	EVRF07	0.3	0.2
GPS/EGG08	EVRF07	5.2	2.6
LN02	EVRF07	6.9	6.6
LHN95	EVRF07	14.0	5.6

Some notable highlights of the previous results are: the considerable scale difference between LHN95 and EVRF07 and the significant origin discrepancy between LN02 and EVRF07, the superiority of the GPS/EGG08 height frame (compared to the Swiss national VRFs) regarding its agreement with the official EVRF07 heights, and finally the sub-cm consistency between EVRF00 and EVRF07 at the particular 20 EUVN-DA Swiss stations, even before the implementation of the height transformation model of Eq. (12).

5 Epilogue

A preparatory discussion on the use of a conventional transformation model for evaluating and comparing VRFs has been presented in this paper. Our analysis has been restricted only to a static (time-independent) height transformation setting, yet its generalization for cases of dynamic vertical frames is also necessary, especially in view of the following key tasks: (i) the assessment of systematic discrepancies in vertical velocity models obtained by different geodetic techniques and modeling assumptions, and (ii) the optimal combination of individual time-dependent VRF realizations over a global or continental control network.

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