Weighted vs. unweighted MCs for the datum definition in regional networks

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Abstract.

Minimum constraints (MCs) is a standard tool for the datum definition in geodetic network adjustment and they are regularly used for the alignment of regional GNSS networks to successive realizations of the International Terrestrial Reference System. Their implementation has been restricted to an unweighted setting without using any weighting for the reference stations that participate in the datum definition process. The aim of this paper is to discuss the optimal choice problem for the weight matrix of the reference stations within the context of MC network adjustment, and to expose its relevance for practical applications having particularly in mind network densification problems. Numerical examples are presented from the alignment of two regional GNSS networks to ITRF2008 using weighted and unweighted MCs. The results obtained from our analysis offer a preliminary view of the expected improvement in the estimation accuracy of MC solutions due to the optimal weighting of the used reference stations.

Keywords. Minimum constraints, network adjustment, reference station weighting, datum definition.

1 Introduction

The use of minimum constraints (MCs) in geodetic network adjustment is a well known tool of fundamental importance for the realization and densification of reference coordinate frames. Although the formal meaning of the term "minimum constraints" is associated with any set of sufficient and non-distorting datum conditions whose number is equal to the rank defect of the network's observational model (e.g. Sillard and Boucher 1999), herein we adapt to its usage that is currently followed in most geodetic studies. Therefore, we refer to MCs in the sense described by Altamimi et al. (2002) which closely corresponds to the so-called inner constraints that were introduced by Meissl (1969) and Blaha (1971) for the optimum datum definition in geodetic networks. These datum constraints are applied over a set of reference stations that are included in the network adjustment, and they result in the nullification of the (non-estimable) frame transformation parameters between the adjusted coordinates and the prior coordinates at the reference stations. Their implementation for the integration of regional GNSS networks to the International Terrestrial Reference Frame (ITRF) was suggested by Altamimi (2003) and since then several IAG regional frame sub-commissions have been promoting this strategy for: (i) the alignment of their weekly combined solutions to successive ITRF realizations and (ii) the densification of their permanent reference networks by the national mapping agencies in different countries (e.g. Bruyninx et al. 2013).

To the authors' knowledge the use of MCs is always utilized without the aid of a weight matrix for the reference stations, although this option has been sporadically mentioned in some studies (e.g. Angermann et al. 2004, Heinkelmann et al. 2007). Recently, a more general MC framework was presented by Kotsakis (2013, 2015) which incorporates a weighting scheme for the reference stations based on certain optimality criteria for the adjusted network coordinates. It is well known that a network solution by the classic (unweighted) MCs is already optimal in the sense of minimizing the propagated data noise to the estimated coordinates of the reference stations. Such a solution, however, does not provide optimal control (of the propagated data noise) on the estimated coordinates of other network stations, and it also ignores the random errors of the reference coordinates and their propagated effect to the final network solution – the latter represents what we shall briefly refer to as datum noise effect. In the aforementioned studies it was shown that both of these limitations can be handled through a weight matrix for the reference stations which is computed by straightforward closed-form analytic formulae. Generally speaking, this weighting tool allows us to obtain a minimally constrained solution in the same frame of the reference stations, with optimal accuracy over the entire network, by considering the total effect of the data and datum noise on all estimated coordinates.

The aim of this paper is to present, for the first time, numerical results from the implementation of the weighted MCs in the alignment of regional GNSS networks to the ITRF2008 frame. Our tests are performed to weekly solutions of different networks with continental and national size, namely the SIRGAS reference network and a Hellenic reference network. The presented comparisons between the unweighted and weighted MC solutions refer to the weekly rms of their coordinate differences and the average station accuracy that is obtained in each case for every week.

2 Weighted MCs in network adjustment

In this section we give an overview of the optimally weighted MCs in geodetic network adjustment. For the sake of simplicity the notation herein is slightly different from the one used in the original presentation by Kotsakis (2013) which should be consulted for a more detailed description and related theoretical proofs.

We start with a *singular system* of normal equations (NEQ)

$$\underbrace{\begin{bmatrix} \mathbf{N}_{\mathbf{x}} & \mathbf{N}_{\mathbf{x}\bar{\mathbf{x}}} \\ \mathbf{N}_{\bar{\mathbf{x}}\mathbf{x}} & \mathbf{N}_{\bar{\mathbf{x}}} \end{bmatrix}}_{\mathbf{N}} \underbrace{\begin{bmatrix} \mathbf{x}-\mathbf{x}^{o} \\ \bar{\mathbf{x}}-\bar{\mathbf{x}}^{o} \end{bmatrix}}_{\mathbf{X}-\mathbf{X}^{o}} = \underbrace{\begin{bmatrix} \mathbf{u}_{\mathbf{x}} \\ \mathbf{u}_{\bar{\mathbf{x}}} \end{bmatrix}}_{\mathbf{u}} \tag{1}$$

which is deduced from the least-squares adjustment of a geodetic network in the usual linearized context. The partitioning of the above system indicates the network separation into two parts: (i) a set of reference stations (with unknown coordinates $\mathbf{x}$ and known approximate coordinates $\mathbf{x}^{o}$) that will be used in the datum definition process and (ii) a set of non-reference stations (with unknown coordinates $\bar{\mathbf{x}}$ and known approximate coordinates $\bar{\mathbf{x}}^{o}$). It is considered that the rank defect of Eq. (1) is caused by the datum deficiency of the observables and that any nuisance parameters have been eliminated beforehand without causing any additional singularity problems.

Based on Eq. (1) we seek a network solution to be expressed in the same frame of the reference stations using the minimum required information for the datum definition. This requires an auxiliary (and consistent) set of linear constraints that do not distort the geometrical information of the original observations. Such a set can be generally expressed as

$$\underbrace{[\mathbf{Q} \quad \mathbf{0}]}_{\mathbf{H}} \underbrace{\begin{bmatrix} \mathbf{x}-\mathbf{x}^{o} \\ \bar{\mathbf{x}}-\bar{\mathbf{x}}^{o} \end{bmatrix}}_{\mathbf{X}-\mathbf{X}^{o}} = \underbrace{\mathbf{Q}(\mathbf{x}^{ref}-\mathbf{x}^{o})}_{\mathbf{c}} \tag{2}$$

or equivalently

$$\mathbf{Q}(\mathbf{x}-\mathbf{x}^{ref}) = \mathbf{0} \tag{3}$$

where $\mathbf{x}^{ref}$ is the known coordinate vector of the reference stations in the desired frame, which does not necessarily coincide with the approximate coordinates that were used in the linearization process. The constraint matrix $\mathbf{Q}$ should satisfy some general algebraic properties that are imposed by the MC theory in rank-deficient linear models (see e.g. Koch 1999) but it remains otherwise unspecified. In principle, there are infinite options for the selection of this matrix - herein it will be uniquely determined on the basis of an optimal criterion for the estimated coordinates in the desired frame of the adjusted network.

Considering Eq. (2), the minimally constrained solution of Eq. (1) has the general form

$$\hat{\mathbf{X}} = \mathbf{X}^o + (\mathbf{N} + \mathbf{H}^T\mathbf{H})^{-1}(\mathbf{u} + \mathbf{H}^T\mathbf{c}) \tag{4}$$

while its covariance (CV) matrix consists of two independent components

$$\boldsymbol{\Sigma}_{\hat{\mathbf{X}}} = \boldsymbol{\Sigma}_{\hat{\mathbf{X}}}^{obs} + \boldsymbol{\Sigma}_{\hat{\mathbf{X}}}^{mc} \tag{5}$$

The first component quantifies the data noise effect in the estimated coordinates and the second component represents the datum noise effect due to random errors in the known coordinates $\mathbf{x}^{ref}$ of the reference stations. Detailed analytic expressions for both of these components can be found in Kotsakis (2013, 2015).

The sought constraint matrix $\mathbf{Q}$ is hidden within the matrix $\mathbf{H}$ (see Eq. (2)) and it imposes the datum definition based on the prior information of the reference stations. It is reasonable to choose this matrix such that the error variances of the estimated coordinates over the entire network are minimized. In this way, the frame of the reference stations is “transferred” in an optimal way through Eq. (3) into the adjusted network or, conversely, the adjusted network is optimally “aligned” to the frame of the reference stations.

The aforementioned requirement can be expressed in terms of the minimization principle

$$\min_{\mathbf{Q}} \; tr\left(\boldsymbol{\Sigma}_{\hat{\mathbf{X}}}^{obs} + \boldsymbol{\Sigma}_{\hat{\mathbf{X}}}^{mc}\right) \tag{6}$$

which leads to the following optimal form of the constraint matrix (for the proof see Kotsakis 2013)

$$\boxed{\mathbf{Q} = \mathbf{E}_{\mathbf{x}} \left(\boldsymbol{\Sigma} + \boldsymbol{\Sigma}_{\mathbf{x}}^{ref}\right)^{-1}} \tag{7}$$

where $\mathbf{E}_{\mathbf{x}}$ is the usual MC matrix that refers to the reference stations and the non-estimable frame parameters of the underlying network (i.e. the classic inner-constraint matrix).

The matrix $\boldsymbol{\Sigma}_{\mathbf{x}}^{ref}$ is the prior CV matrix of the known coordinates of the reference stations, and the matrix $\boldsymbol{\Sigma}$ is given by the equation (ibid.)

$$\left(\mathbf{N} + \mathbf{E}^T\mathbf{E}\right)^{-1} = \left[\begin{array}{c|c} \boldsymbol{\Sigma} & \# \\ \hline \# & \# \end{array}\right] \tag{8}$$

where $\mathbf{E}$ is the usual MC matrix for the entire network, i.e. $\mathbf{E} = \left[\mathbf{E}_{\mathbf{x}} \;\; \mathbf{E}_{\overline{\mathbf{x}}}\right]$. Note that the matrix partitioning in the last equation conforms to the NEQ partition of Eq. (1).

The fundamental result of Eq. (7) shows that a weighted type of MCs should be applied to the reference stations in order to obtain an optimal solution in the desired frame. Consequently, the weighted MCs do not represent just an additional option for the datum definition in network adjustment problems but they are, in fact, the optimum scheme under the criterion of Eq. (6) among any other choice of minimum constraints for the underlying network. The weight matrix depends on the components $\mathbf{\Sigma}$ and $\mathbf{\Sigma}_{\mathbf{x}}^{ref}$, each of which controls the influence of the reference stations into the datum definition with regard to the data and datum noise effect, respectively. The first component is dictated by the network's own characteristics as per Eq. (8), whereas the second component depends on the prior accuracy of the reference coordinates in the desired frame; for more details see Kotsakis (2013).

3 Numerical tests

Two case studies are presented to compare the performance of weighted and unweighted MCs for the constrained adjustment of regional GNSS networks. The first example refers to the alignment of 51 weekly solutions of the SIRGAS reference network (Sánchez et al. 2013) to the ITRF2008 frame (Altamimi et al. 2011). The second example employs a Hellenic reference network that was recently used for studying the Greek horizontal velocity field (Chatzinikos 2013) and it examines its alignment, also, to the ITRF2008 frame over a sample of 300 weekly solutions. The weighted/unweighted MCs are applied to the weekly unconstrained NEQs of each network according to the formulation of the previous section. The prior coordinates of the reference stations ($\mathbf{x}^{ref}$) and their full CV matrix ($\mathbf{\Sigma}_{\mathbf{x}}^{ref}$) were extracted from the official ITRF2008 sinex file by reducing them to the mean epoch of every week with the use of the reference stations velocities and their associated CV matrix.

It is noted that the unconstrained NEQs originating from the GNSS data processing in the context of the weekly network adjustments are not strictly singular. In order to conform with the theoretical setting of MCs (as implied, for example, in the proof of Eq. (7) in Kotsakis 2013), a preliminary correction was applied to remove the (weak) frame origin information from the original NEQs of each network. This step was implemented according to the covariance projection formulae and the analytic pseudoinverse computation given in Pope (1973); relevant algorithms are also given in Sillard and Boucher (2001). In this way we are able to work with truly rank-deficient NEQs, and thus exploit the genuine properties of MCs without distorting the geometrical content of the network observations.

3.1 SIRGAS Network

The entire network consists of 274 stations, 18 of which were used as reference stations for its weekly alignment to ITRF2008 based on the no-net-translation (NNT) condition (this is similar to the frame alignment methodology used by the SIRGAS analysis center). The geographical distribution of all network stations is shown in Fig. 1. Both weighted and unweighted MC solutions were computed on a weekly basis for a one-year period (2013.5-2014.5) starting from GPS week 1750.

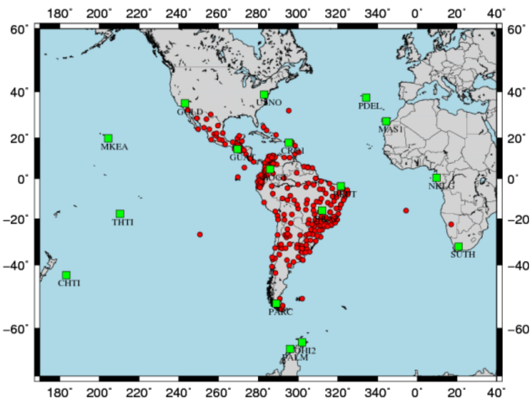

Fig. 1 The SIRGAS network (green squares: reference stations, red circles: non-reference stations).

As expected, our results revealed that the traces of the weekly CV matrices $\mathbf{\Sigma}_{\hat{\mathbf{X}}}$ are always smaller in the weighted MC solutions thus implying better accuracy for the estimated coordinates over the entire network. The average improvement of the estimation accuracy in each Cartesian coordinate component for every week is depicted in Fig. 2. The most significant improvement occurs in the X and Z components whose mean error variances (over all network stations) are reduced up to the level of 31% and 23%, respectively. Note that more than half of the weekly solutions showed an average decrease by more than 10% in the error variances of the estimated X and Z coordinates in the SIRGAS network.

A representative example of the accuracy improvement in the estimated coordinates at each network station, for a particular GPS week, is shown in Fig. 3. Overall, the weighted MC solution leads to coordinate accuracies that are better (in average sense over all stations) by 30%, 7% and 24% in the X, Y and Z component, respectively. In a significant number of stations the accuracy improvement exceeds the 50% level for their estimated X and Z coordinates. However, seven out of the eighteen reference stations showed worse accuracy by almost 10% in their estimated coordinates that were determined by the weighted MC solution (see left side of the plots in Fig. 3). This is not surprising and it represents an unavoidable "tradeoff" so that we can have better estimation accuracy for all the rest of the network stations (remember that the unweighted MC solution gives optimally estimated coordinates only at the reference stations while the weighted MC solution has been optimized over the entire network – see also the related discussions in the previous sections).

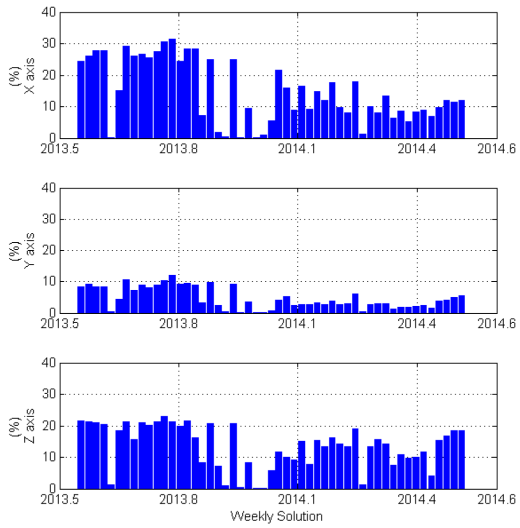

Fig. 2 Average percentage improvement of the estimation accuracy in the ITRF2008 weekly coordinates by the optimally weighted MCs in the SIRGAS network.

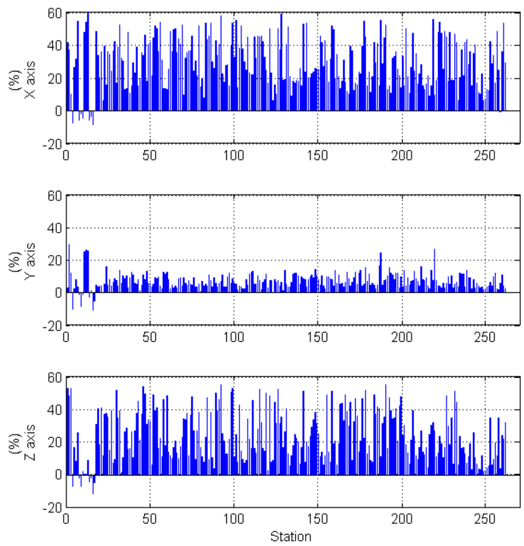

Fig. 3 Percentage improvement of the estimation accuracy in the ITRF2008 weekly coordinates *for each station* by the optimally weighted MCs in the SIRGAS network (GPS week: 1752).

Finally the coordinate differences between the unweighted and weighted MC solutions, in terms of their weekly rms for each Cartesian component, are depicted in Fig. 4. Since both solutions are minimally constrained based on the NNT condition (with or without weighting for the reference stations), this rms reflects the apparent shifts between the unweighted/weighted MC solutions of the SIRGAS network along each coordinate axis. As it can be seen from Fig. 4, their values range from 0 to 4 mm throughout the considered period.

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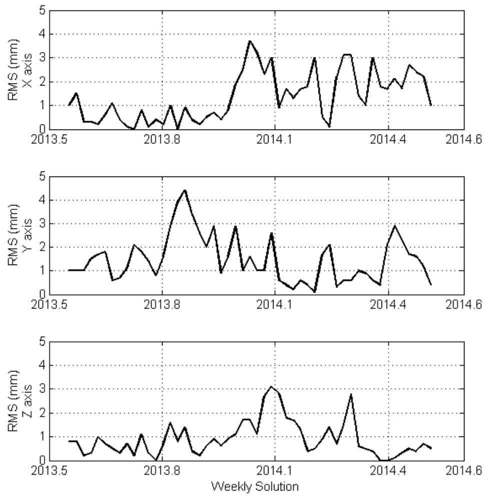

Fig. 4 Weekly rms of the coordinate differences between the weighted and unweighted MC solutions in the SIRGAS network.

3.2 Hellenic Network

The second example refers to a GNSS network consisting of 68 Greek permanent stations that belong to the Leica SmartNet network and also 16 EUREF reference stations which were used for the alignment to ITRF2008 based on the NNT condition (see Fig. 5). Three hundred weekly solutions, using both unweighted and weighted MCs, were computed in this network for a six-year period (2007-2013) starting from GPS week 1408.

Similarly to the previous example, the weighted MC solutions always improve the average accuracy (mean error variance over all network stations) of the weekly estimated coordinates. The percentage levels of this accuracy improvement for each week are shown in Fig. 6. For the Y and Z coordinates, the average reduction of their error variances throughout the six-year period is 11% and 5%, reaching up to 22% and 10% in some weeks. On the other hand, the accuracy of the X coordinates seems to be unaffected by the use of the weighted MCs since their mean error variances are not reduced by more than 2-3% in the considered period.

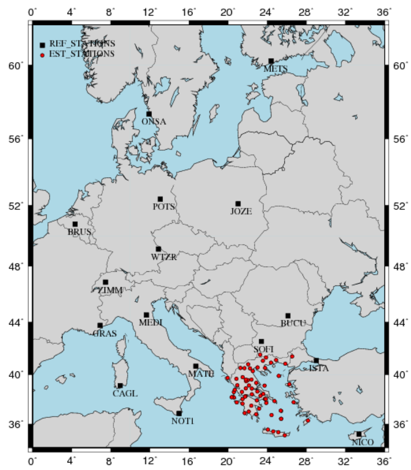

Fig. 5 The Hellenic network (black squares: reference stations, red circles: non-reference stations).

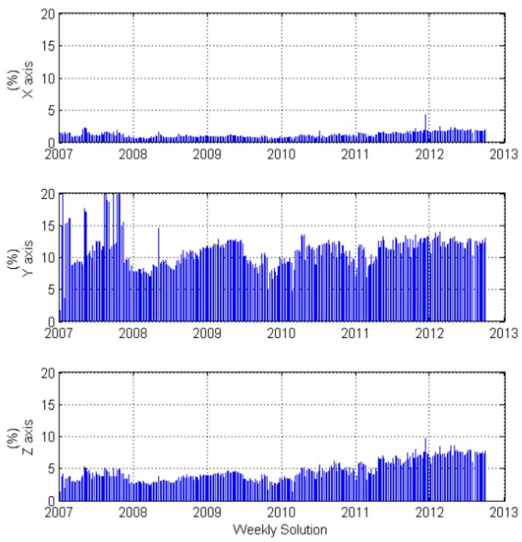

Fig. 6 Average percentage improvement of the estimation accuracy in the ITRF2008 weekly coordinates by the optimally weighted MCs in the Hellenic network.

A representative example of the accuracy improvement in the estimated coordinates at each network station, for a particular GPS week, is shown in Fig. 7. As in the previous test with the SIRGAS network, we see again that some of the 16 reference stations have worse accuracy in their weekly estimated coordinates (especially in the Y component) obtained by the weighted MC solution. We repeat here that this is a tradeoff implied by the optimality properties of the unweighted and weighted MCs, and it essentially secures the improvement of the coordinate estimation accuracy for all the rest of the Hellenic network stations.

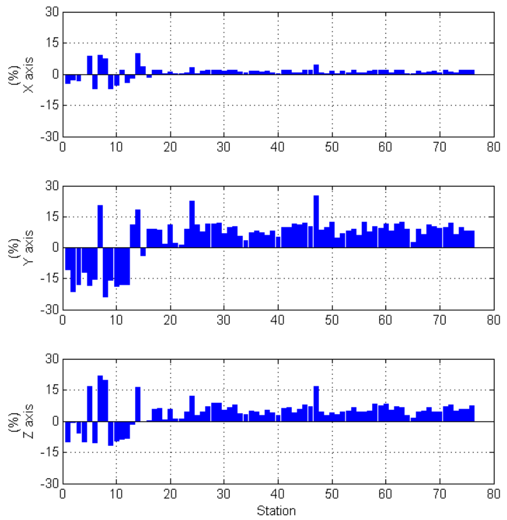

Fig. 7 Percentage improvement of the estimation accuracy in the ITRF2008 weekly coordinates *for each station* by the optimally weighted MCs in the Hellenic network (GPS week: 1669).

The weekly rms of the coordinate differences between the weighted and unweighted MC solutions in the Hellenic network is shown in Fig. 8. The largest differences occur mainly in the Z component and they reach up to 4 mm, whereas for the other two components they remain below 2 mm in most weekly solutions.

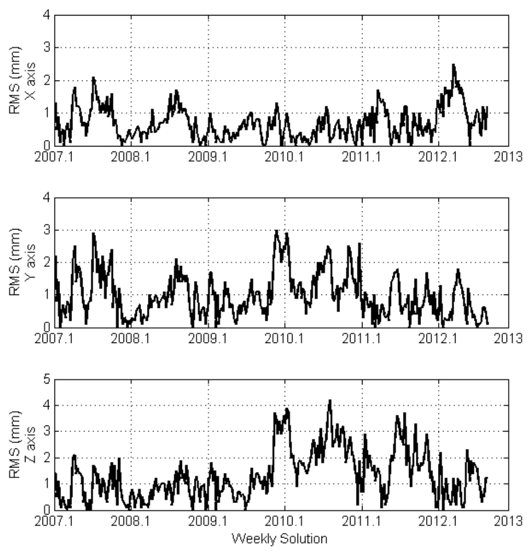

Fig. 8 Weekly rms of the coordinate differences between the weighted and unweighted MC solutions in the Hellenic network.

4 Conclusions

The use of a weight matrix for the optimal handling of reference stations in MC network adjustment has been studied in this paper. The theoretical background of the weighting methodology was briefly presented along with some first numerical results from its implementation with real data. Our tests were performed in two regional GNSS networks using weighted and unweighted MCs for their weekly alignment to the ITRF2008 frame. Expectedly, a significant improvement in the estimation accuracy of the constrained weekly solutions was achieved in both networks by the optimally weighted MCs. The percentage reduction of the error variances of the estimated coordinates varied both spatially (over different stations) and temporally (over different weeks), especially in the SIRGAS network, and it was in most cases well above the 15% level with maximum up to 50%. The differences of the estimated weekly coordinates in ITRF2008 by the two adjustment approaches remain the same over all network stations – this is expected since the datum definition in both cases is based on the NNT condition either with or without weighting for the reference stations – and they vary among different weeks. In general, the weekly shifts between the unweighted and weighted MC solutions range within 0-4 mm for all coordinate components as shown in Figs. 4 and 8. Considering the formal accuracy of the GNSS weekly solutions, such differences can become rather significant in view of their utilization for geodynamical studies in Earth monitoring networks.

It is noted that, in current geodetic practice, the known (prior) coordinates of the reference stations refer to a secular frame, such as the ITRF, without accounting for nonlinear effects in their temporal evolution. The aim of MCs is often to align a series of network solutions to such a secular frame in order to investigate unmodeled seasonal signals of geophysical interest in the resulting coordinate time series. The weighting scheme presented in this paper is “blind” to such unmodeled signals while its rationale is to guarantee that the adjusted network is integrated into the secular frame without any geometrical distortion and in a way that the estimated coordinates of all stations have the best accuracy (with respect to that secular frame). Whether or not our weighting methodology provides any actual advantages to better identify unmodeled effects within the estimated coordinate time series of ITRF-aligned networks is of course an interesting question that requires separate investigation.

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