# A conventional approach for comparing vertical reference frames

Research Article

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#### Abstract:

A conventional transformation model between different realizations of a vertical reference system is an important tool for geodetic studies related to precise vertical positioning and physical height determination. Its fundamental role is the evaluation of the consistency for colocated vertical reference frames that are obtained from different observation techniques, data sources or optimal estimation strategies in terms of an appropriate set of "vertical datum perturbation" parameters. Our scope herein is to discuss a number of key issues related to the formulation of such a transformation model and to present some simple examples from its practical implementation in the comparison of existing vertical frames over Europe.

#### **Keywords:**

Conventional height transformation • physical heights • vertical datums • vertical reference frames (a) Versita sp. z o.o.

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#### 1. Introduction

The comparison of terrestrial reference frames (TRFs) that are obtained by different observation techniques, modeling assumptions and optimal estimation strategies is a common geodetic problem constituting either a research goal in itself or an auxiliary task for other geodetic applications. Such a comparison relies on the use of the linearized similarity transformation, also known as Helmert transformation (e.g. Leick and van Gelder 1975), which supports the evaluation of TRFs on the basis of datum-perturbation parameters that are associated with the theoretical definition of a terrestrial reference system (Altamimi et al. 2007). Based on the least squares adjustment of this model over a network of common stations, a set of estimated parameters is obtained that quantify the origin, orientation and metric consistency of two TRFs in terms of their relative translation, rotation and scale variation. The aforementioned scheme provides a geodetically meaningful framework for comparing and transforming Euclidean spatial reference frames, and also for assisting their quality assessment through a suitable de-trending of their systematic differences in order to identify any localized distortions in their respective coordinate sets.

A similar situation as the one described above occurs also in geodetic studies related to the establishment of vertical reference frames (VRFs) for physical height determination. Different realizations of a vertical reference system (VRS) may be available over a regional or even continental network, originating from separate leveling campaigns, alternative data sources and modeling strategies. As an example, consider a set of national leveling benchmarks that is part of the United European Leveling Network (UELN): three vertical frames co-exist in such a leveling network whose physical heights are respectively obtained from the EVRF00 and EVRF07 continental solutions (Ihde and Augath 2001, Sacher et al. 2008) and also by the (usually older) national adjustment of the primary height network in the underlying country. If, in addition, Global Positioning System (GPS) data are available at the particular stations, then more VRFs could emerge through the synergetic use of gravimetric geoid models that enable the conversion of observed geometric heights to physical heights.



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An objective comparison of different VRFs needs to be based on a conventional model that is able to map the differences of colocated physical heights to a set of geodetically meaningful parameters. The adopted model must resemble the role of the Helmert transformation while its associated parameters should reflect the vertical datum disturbance implied by the corresponding height datasets. Eventually, the utmost role of such a model is to be used for generating a combined optimal VRF solution from multiple realizations that are jointly merged into a unified vertical frame by postulating appropriate minimum constraints to the datum-related parameters of the underlying height transformation model.

The aim of this paper is to discuss some general aspects about the formulation of a conventional height transformation model and to present a few examples from its practical use in the comparison of existing VRFs over Europe.

#### 2. Height transformation schemes in practice

Several transformation schemes for physical heights are commonly used in geodetic practice. Typical examples include the reduction of physical heights to a conventional permanent tide system and/or to a reference time epoch due to temporal variations caused by various geodynamical effects (Ekman 1989; Mäkinen and Ihde 2009), the conversion from normal to orthometric heights and vice versa (Flury and Rummel 2009; Sjöberg 2010), and the determination of apparent height variations due to a geopotential offset in the zero-height level of the underlying vertical datum (Jekeli 2000). Also a number of empirical transformation schemes have been employed for the analysis of co-located heterogeneous heights and the inference of systematic differences between them. A classic example is the combined adjustment of ellipsoidal, geoid and leveled height data which can be perceived as a parameter estimation problem in a generalized height transformation model:

$$H'_{i} - H_{i} = \mathbf{a}_{i}^{\mathsf{T}} \mathbf{x} + s_{i} + v_{i} \quad i = 1, 2, \dots, m$$
 (1)

The terms  $H_i$  and  $H'_i$  correspond to the orthometric heights obtained from leveling measurements and GPS/geoid data, respectively. Their systematic differences are usually modeled by a loworder parametric component  $\mathbf{a}_i^T \mathbf{x}$  and (optionally) a spatially correlated zero-mean signal  $s_i$ , whereas  $v_i$  contains the remaining random errors in the height data. The estimated values of the model parameters and the predicted values of the stochastic signals can be jointly obtained from the least squares (LS) inversion of Eq. (1) using some a-priori information for the data noise level and the signal covariance function; for more details see Kotsakis and Sideris (1999).

Numerous modelling options have been followed in practice for the parametric term  $\mathbf{a}_i^T \mathbf{x}$  in Eq. (1), none of which has ever served as a "geodetically meaningful" transformation model between the underlying VRFs - that is, between the leveling-based frame  $\{H_i\}$ and the GPS/geoid-based frame  $H'_i$ . In most cases, the suitability of the adopted model is judged by the reduction of the sample variance of the height residuals and not by the physical or geometrical



meaning (if any) of its associated parameters. In fact, the estimated values of x have never been of any actual importance in geodetic studies, other than offering a more or less arbitrary parametric description for the overall trend of the height differences  $H'_i - H_i$  in support of GPS-based leveling techniques within an existing vertical datum.

It is worth noting that the use of the well-known *4-parameter model* (Heiskanen and Moritz 1967, ch. 5)

$$\mathbf{a}_i^{\mathsf{T}} \mathbf{x} = x_0 + \Delta x \cos \varphi_i \cos \lambda_i + \Delta y \cos \varphi_i \sin \lambda_i + \Delta z \sin \varphi_i \quad (2)$$

may be viewed, to some extent, as an attempt to infer hidden "datum disturbances" between the height frames  $\{H_i\}$  and  $\{H'_i\}$ . Such a viewpoint relies on the equivalent form of Eq. (1)

$$N'_i - N_i = \mathbf{a}_i^{\mathsf{I}} \mathbf{x} + s_i + v_i \quad i = 1, 2, ..., m$$
 (3)

where  $N_i$  and  $N'_i$  denote the geoid undulations obtained from a gravimetric model and GPS/leveling data, respectively. In view of Eq. (3), the use of the 4-parameter model with Eq. (1) implies that the systematic part of the differences  $H'_i - H_i$  is essentially described through a 3D spatial shift ( $\Delta x$ ,  $\Delta y$ ,  $\Delta z$ ) and an apparent scale change ( $x_0$ ) between the corresponding reference surfaces of the physical heights (see Kotsakis 2008).

The aforementioned 4-parameter model was regularly used in older studies for estimating geodetic datum differences from heterogeneous height data; especially for assessing the geocentricity of TRFs based on Doppler-derived and gravimetrically-derived geoid undulations and also for determining the Earth's optimal equatorial radius from geometric and physical heights (e.g. Schaab and Groten 1979, Grappo 1980, Soler and van Gelder 1987). These tasks require a global height data distribution otherwise the translation parameters  $\Delta x$ ,  $\Delta y$ ,  $\Delta z$  and the scaling term  $x_0$  become highly correlated, and their adjusted values may be totally unrealistic from a physical point of view. This is the reason that the LS inversion of Eq. (1) will not always produce a geodetically meaningful solution for the individual parameters of the 4-parameter model (not even for the estimated "height bias"  $x_0$ ) when applied over a regional test network; for some numerical examples see Kotsakis and Katsambalos (2010). Moreover, a theoretical drawback of this model for VRF comparison studies is that it neglects one of the key parameters for the definition and realization of vertical datums: a geopotential reference value  $W_0$  and, more importantly, its actual and/or apparent variation between different VRFs.

As a closing remark, we should note that the comparison of colocated vertical frames needs to consider their scale variation due to systematic differences originating from the measurement techniques and data modeling options that were used for the determination of the physical heights in each frame. In fact, one should not forget that the fundamental theoretical constraint h - H - N = 0requires not only the "origin consistency" among the heterogeneous height types, but also their reciprocal vertical scale uniformity.

## 3. Formulation of a conventional VRF transformation model

## 3.1. General considerations

The assessment of the consistency between different VRFs over a network of common stations requires a conventional transformation model of the general form

$$H'_i - H_i = f(\mathbf{x}) + v_i \quad i = 1, 2, ..., m$$
 (4)

The parameters x of this model should quantify the vertical datum perturbations induced by the height datasets  $\{H_i\}$  and  $\{H'_i\}$ while the remaining residuals reflect the internal accuracy of the corresponding frames. The spatial analysis of the estimated height residuals is also useful for identifying local distortions and other spatially correlated errors within the tested VRFs which cannot be absorbed by the model parameters x.

In general, a VRF is a realization of a 1D terrestrial coordinate system with respect to an equipotential surface of Earth's gravity field. The latter provides a conventional zero-height level relative to which vertical positions can be obtained by various geodetic techniques and terrain modeling hypotheses. The key role of Eq. (4) is thus to appraise the variation of the reference equipotential surface and the vertical metric scale, which both signify the fundamental constituents of any physical height frame (Schwarz and Sideris 1993).

Evidently two basic parameters must be incorporated into the previous model, that is a vertical translation parameter in the form of a geopotential disturbance  $\delta W_0$ , and a vertical scale change parameter in the form of a unitless factor  $\delta s$  reflecting the metric difference between the height frames. In contrast to the Helmert transformation model there are no rotational terms within the transformation model of Eq. (4) since the frame orientation aspect is not an inherent characteristic of physical height systems.

Note that the notion of *scale* in a vertical reference system is often linked to the geopotential value  $W_0$  which is adopted for defining absolute vertical coordinates (geopotential numbers and their equivalent physical heights) on the Earth's surface. In the geodetic literature the VRS scale is actually related to an equipotential surface realized by the combination of a mean sea surface topography model and a global gravity field model, in accordance with the classic Gauss-Listing definition of the geoid (lhde 2007). This is a common approach to quantify the average size of the reference surface used for vertical positioning, since the ratio of Earth's gravitational constant to the adopted reference geopotential level yields the *mean radius of the geoid*, i.e.

$$R = \frac{GM}{W_0} \tag{5}$$

which itself corresponds to a "metric" for the geocentric positions of terrestrial points with zero heights! Obviously, any change of  $W_0$  induces an apparent offset to the terrestrial physical heights

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which is perceived as an indirect scaling effect due to the changed dimension of the zero-height reference surface.

The aforementioned perspective aims at the standardization of Earth's global scale through the physical parameters GM and  $W_0$ , and it has nothing to do with the notion of a scale variation between different VRFs. In fact, a change of  $W_0$  implies a transformation from a zero-height origin to another one, whereas the scope of a VRF scale change is to account for the systematic discrepancy of the vertical scale that is realized by different techniques and/or datasets when determining physical height differences. Both of these types of datum variation (origin and scale) are feasible and they may co-exist in the joint analysis of different vertical frames.

## 3.2. The effect of $\delta W_0$

Changing the origin of a VRF means that a different equipotential surface will be used as a zero-height reference for physical heights. Such a transformation can be described through a single parameter ( $\delta W_0$ ) reflecting the change of the equipotential reference surface with respect to a conventional representation of the terrestrial gravity field. The effect on the VRF's geopotential numbers is a constant offset equal to  $\delta W_0$  while for the orthometric heights it takes the form of a nonlinear variation (due to the non-parallelism of the equipotential surfaces) according to the power series expansion:

$$H'_{i} - H_{i} = \frac{\delta W_{0}}{g_{i}} - \frac{\partial g}{\partial H} \frac{\delta W_{0}^{2}}{2g_{i}^{3}} + \dots$$
(6)

where  $g_i$  and  $\partial g/\partial H$  denote the actual gravity and its vertical gradient on the geoid, or more precisely on the equipotential surface associated with the initial orthometric height  $H_i$ . For the proof of Eq. (6) one needs to consider the Taylor series expansion of the gravity potential at an arbitrary point P' on the equipotential surface  $W = W_0 + \delta W_0$ , i.e.

$$W(P)' = W(P) + \frac{\partial W}{\partial H} \Big|_{P} \delta H + \frac{1}{2!} \frac{\partial^{2} W}{\partial H^{2}} \Big|_{P} \delta H^{2} + \dots$$
(7)

where the point P is taken along the vertical direction and located on the equipotential surface  $W = W_0$  (see Fig. 1). The last formula can be equivalently expressed in the following form (we omit for simplicity the symbol of the evaluation point in the partial derivative terms)

$$\delta W_0 = \frac{\partial W}{\partial H} \left( H' - H \right) + \frac{1}{2!} \frac{\partial^2 W}{\partial H^2} \left( H' - H \right)^2 + \dots \quad (8)$$

since the vertical offset  $\delta H$  between the two equipotential surfaces corresponds exactly to the orthometric height change on the Earth's surface. Using the well-known formulae for the inversion of a convergent power series (Harris and Stoker 1998, p. 540) and taking into account that  $\partial W/\partial H = -g$  and  $\partial^2 W/\partial H^2 = \partial g/\partial H$ , we finally obtain the series expression given in Eq. (6). For prac-



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tical purposes Eq. (6) can be replaced by the simplified linearized formula:

$$H'_{i} - H_{i} = \frac{\delta W_{0}}{\gamma_{i}} \tag{9}$$

since the higher order terms have a negligible contribution (< 1 mm) for reasonable values of  $\delta W_0$  (up to 1-2 gpu or equivalently up to 10-20 m<sup>2</sup>s<sup>-2</sup>). Note that the geoidal gravity  $g_i$  may be safely replaced by the normal gravit  $\gamma_i$  on the reference ellipsoid, causing an approximation error to the transformed orthometric height below the mm level even for gravity anomaly values  $\Delta g_i = g_i - \gamma_i$  up to 500 mGal.

## 3.3. The effect of $\delta s$

In contrast to geometric Cartesian TRFs, the assessment of a uniform scale difference between VRFs is not a straightforward issue. The effect of a scale change factor on physical heights depends on the way we (choose to) handle the Earth's gravity field and its equipotential surfaces under the influence of a spatial re-scaling. The underlying problem is similar to the similarity transformation of GPS heights with respect to a reference ellipsoid, where the latter may or may not follow the scale variation that is imposed by the TRF transformation model (Soler and van Gelder 1987, Kotsakis 2008).

Starting from the fundamental differential formula (Heiskanen and Moritz 1967, p. 50)

$$dW = -g dH \tag{10}$$

where *g* denotes the magnitude of the gravity vector, the following relationship can be obtained:

$$dH = -\frac{1}{g} \left[ \frac{\partial W}{\partial x} dx + \frac{\partial W}{\partial y} dy + \frac{\partial W}{\partial z} dz \right] = -\frac{\nabla W \cdot \mathbf{dr}}{g}$$
(11)

which gives the (physical) height metric as a weighted combination of the Euclidean metric components with respect to a terrestrial coordinate system. The associated weights correspond to the normalized geopotential gradients and they represent the influence of the Earth's gravity field on the physical height scale.

Now, we may postulate that a spatial scale change as dictated by the metric transformation  $d\mathbf{r}' = (1+\delta s) d\mathbf{r}$  does not affect the



magnitude of the Earth's gravity field, and it thus causes a similar change to the physical height metric in accordance to Eq. (11). Essentially this implies that the equipotential surfaces W = const will undergo a uniform geometric re-scaling so that the resulting effect on the physical (orthometric) heights over the Earth's surface is expressed through the simple formula:

$$H_i' = (1 + \delta s)H_i \tag{12}$$

or equivalently

$$H_i' - H_i = \delta s H_i \tag{13}$$

The above expression can be used for the assessment of the systematic scale discrepancy between VRFs relative to a common reference surface – note that zero-height points are preserved by the transformation of Eq. (12) or (13). The unitless factor  $\delta s$  absorbs the (linear part of) topographically correlated differences of the height datasets  $\{H_i\}$  and  $\{H'_i\}$  which inflict an apparent scale difference in their corresponding VRFs.

## 4. LS adjustment of the VRF transformation model

Based on the discussion of the previous sections, a conventional and geodetically meaningful comparison between VRFs may be applied in terms of the linearized transformation model:

$$H'_{i} - H_{i} = \frac{\delta W_{0}}{\gamma_{i}} + \delta s H_{i} + v_{i} \quad i = 1, 2, ..., m$$
(14)

where the meaning of each term has already been explained in previous paragraphs. Loosely speaking, the above model represents the 1D-equivalent of the similarity transformation for orthometric heights from a vertical frame to another vertical frame; an analogous expression may also be used for the case of normal heights. The LS adjustment of Eq. (14) over a network of *m* stations leads to the following system of normal equations (NEQs):

$$\begin{bmatrix} \mathbf{q}^{\mathsf{T}} \mathbf{P} \mathbf{q} & \mathbf{q}^{\mathsf{T}} \mathbf{P} \mathbf{d} \\ \mathbf{d}^{\mathsf{T}} \mathbf{P} \mathbf{q} & \mathbf{d}^{\mathsf{T}} \mathbf{P} \mathbf{d} \end{bmatrix} \begin{bmatrix} \delta \hat{\mathcal{W}}_{0} \\ \delta \hat{s} \end{bmatrix} = \begin{bmatrix} \mathbf{q}^{\mathsf{T}} \mathbf{P} (\mathbf{d}' - \mathbf{d}) \\ \mathbf{d}^{\mathsf{T}} \mathbf{P} (\mathbf{d}' - \mathbf{d}) \end{bmatrix}$$
(15)

where the vectors **d** and **d'** contain the known heights from two different vertical frames  $\mathbf{d} = \begin{bmatrix} H_1 & \cdots & H_m \end{bmatrix}^T$  and  $\mathbf{d'} = \begin{bmatrix} H'_1 & \cdots & H'_m \end{bmatrix}^T$  while **P** is a weight matrix for their differences and the auxiliary vector **q** is defined as  $\mathbf{q}(\mathbf{i}) = 1/\gamma_i$ . The above NEQ system is always invertible provided that **q** and **d** are not co-linear vectors with each other. Given that the elements of **q** retain an almost constant value (i.e. their relative deviation does not exceed  $10^{-4}$  even in large continental networks), the inversion of Eq. (15) is practically guaranteed as long as the *m* stations do not have the same height level!

The correlation coefficient between the estimated VRF transformation parameters is always negative and it can be expressed as (the proof is straightforward considering the analytic form of the inverse of the  $2 \times 2$  NEQ matrix)

$$\rho_{\delta \hat{W}_0, \delta \hat{s}} = -\frac{\mathbf{q}^{\mathsf{T}} \mathbf{P} \mathbf{d}}{\sqrt{\mathbf{d}^{\mathsf{T}} \mathbf{P} \mathbf{d}} \sqrt{\mathbf{q}^{\mathsf{T}} \mathbf{P} \mathbf{q}}}$$
(16)

and, if the weight matrix has the simple form  $P=1/\sigma^2 I$ , it can be further simplified as follows:

$$\rho_{\delta \hat{W}_{0},\delta \tilde{s}} = -\frac{\sum_{i=1}^{m} \frac{H_{i}}{\gamma_{i}}}{\left[\sum_{i=1}^{m} H_{i}^{2}\right]^{1/2} \left[\sum_{i=1}^{m} \frac{1}{\gamma_{i}^{2}}\right]^{1/2}} \\ \simeq -\frac{1}{\tilde{\gamma}} \frac{\sum_{i=1}^{m} H_{i}}{\left[\sum_{i=1}^{m} H_{i}^{2}\right]^{1/2} \frac{1}{\tilde{\gamma}} \sqrt{m}} \\ = -\frac{\left[\sum_{i=1}^{m} H_{i}^{2}/m\right]^{1/2}}{\left[\sum_{i=1}^{m} H_{i}^{2}/m\right]^{1/2}} = -\frac{\text{mean}[\mathbf{d}]}{\text{rms}[\mathbf{d}]}$$
(17)

A useful algebraic relationship for the optimal estimates obtained from the inversion of Eq. (15) in terms of their correlation coefficient is:

$$\delta \hat{W}_{0} = \frac{\mathbf{q}^{\mathsf{T}} \mathbf{P}(\mathbf{d}' - \mathbf{d})}{\mathbf{q}^{\mathsf{T}} \mathbf{P} \mathbf{q}} + \rho_{\delta \hat{W}_{0}, \delta \hat{s}} \sqrt{\frac{\mathbf{d}^{\mathsf{T}} \mathbf{P} \mathbf{d}}{\mathbf{q}^{\mathsf{T}} \mathbf{P} \mathbf{q}}} \delta \hat{s} \qquad (18)$$

The "separability" of the VRF transformation parameters depends strongly on the vertical network configuration. As a matter of fact, in the context of the joint estimation of  $\delta W_0$  and  $\delta s$ , an optimal network geometry is not related to a homogeneous coverage over the Earth's surface but to a high height variability among its control stations. Obviously the dispersion of the data vector **d** must be sufficiently large in order for the correlation coefficient in Eq. (16) to retain a reasonably low value.

#### 5. Examples

Let us now give a few examples from the LS inversion of the transformation model in Eq. (14) for a number of VRFs in Europe. The first example employs the EVRF00 and EVRF07 normal heights at the 13 UELN fiducial stations which were used for the primary definition of the zero-height level in the official EVRF07 solution (Sacher et al. 2008). Although the zero-height levels of these two frames were a-priori aligned at the particular stations through a single constraint that was implemented in the EVRF07 adjustment (ibid.), our results in Table 1 reveal a small (mm-level) offset between their reference surfaces. This is due to the inherent correlation between the estimated parameters  $\delta \hat{W}_0$  and  $\delta \hat{s}$  ( $\rho = -0.7$ in this case) which causes an unavoidable leakage effect in their corresponding values; see Eq. (18). Nevertheless, the estimated transformation parameters between EVRF00 and EVRF07 seem to be statistically insignificant, within the limits of their statistical precision, over the 13 UELN fiducial stations.

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Table 1. Estimated transformation parameters between EVRF00 and<br/>EVRF07 from the normal heights at 13 UELN fiducial sta-<br/>tions over Europe. The initial weight matrix  $\mathbf{P}$  was set equal<br/>to a unit matrix and the a-posteriori variance factor of unit<br/>weight was estimated at  $\sigma_0 = 9$  mm.

d	ď	$\delta \hat{W}_0$ (gpu)	δŝ (ppm)
EVRF00	EVRF07	0.002 ± 0.004	-25.5 ± 27.7

 
 Table
 2. Estimated transformation parameters between VRFs in Switzerland based on 20 EUVN-DA Swiss stations.

d	ď	$\delta \hat{W}_0$ (gpu)	$\delta \hat{s}$ (ppm)
EVRF00	EVRF07	$0.025 \pm 0.001$	$2.9 \pm 0.8$
GPS/EGG08	EVRF07	$0.044 \pm 0.012$	$-76.6 \pm 10.7$
LN02	EVRF07	$-0.251\pm0.030$	$35.7 \pm 26.8$
LHN95	EVRF07	$-0.060\pm0.026$	$-220.7 \pm 22.9$

The second example uses several different VRFs realized over a network of 20 Swiss leveling benchmarks which belong to the EUVN-DA network (Marti 2010, EVRS 2012). The vertical frames were compared on the basis of Eq. (14) using physical heights from: (i) the EVRF00 and EVRF07 continental adjustment solutions, (ii) the combination of GPS heights with the European gravimetric geoid model EGG08, (iii) the Swiss official height system LN02, and (iv) the LHN95 rigorous adjustment of the Swiss national height network. The estimated transformation parameters are given in Table 2, whereas the standard deviation of the height residuals (before and after the VRF transformation) are listed in Table 3.

Some notable highlights of the computed results in the Swiss test network are: the considerable scale difference between LHN95 and EVRF07 and the significant origin discrepancy between LN02 and EVRF07, the superiority of the GPS/EGG08 height frame compared to the Swiss national VRFs regarding its agreement with the EVRF07 frame, and finally the sub-cm consistency between EVRF00 and EVRF07 at the particular 20 EUVN-DA Swiss stations, even before the implementation of the height transformation model. The negligible reduction in the standard deviation of the height differences between LN02 and EVRF07 (see Table 3) indicates the presence of strong local distortions in the original LN02 heights which cannot be absorbed by the estimated VRF transformation parameters. On the other hand, the comparison of LHN95 and EVRF07 reveals that

 Table 3. Standard deviation (in cm) of the normal height residuals before and after the adjustment of Eq. (14) at 20 EUVN-DA Swiss stations.

d	ď	$\sigma$ (before)	$\sigma(after)$
EVRF00	EVRF07	0.3	0.2
GPS/EGG08	EVRF07	5.2	2.6
LN02	EVRF07	6.9	6.6
LHN95	EVRF07	14.0	5.6



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a large part of their original height differences is due to an apparent vertical scale variation that exists between the underlying frames.

## 6. Epilogue

A preparatory discussion on the use of a conventional transformation model for evaluating and comparing VRFs has been presented in this paper. Our approach has been restricted only to a static (time-independent) setting, yet its generalization for dynamic vertical frames is also necessary in view of important problems such as (i) the consistency assessment of vertical velocity models obtained by different geodetic techniques and/or modeling assumptions, and (ii) the optimal combination of multiple time-dependent VRF realizations over a global or continental control network.

## References

Altamimi Z., Collilieux X., Legrand J., Garayt B. and Boucher C., 2007, ITRF2005: A new release of the International Terrestrial Reference Frame based on time series of station positions and Earth orientation parameters. J Geoph Res, 112, B09401, DOI: 10.1029/2007JB004949.

Ekman M., 1989, Impacts of geodynamic phenomena on systems for height and gravity. Bull Geod, 63: 281-296.

EVRS, 2012, European Vertical Reference Network Densification Action (EUVN-DA project), http://www.bkg. bund.de/nn\_164794/geodIS/EVRS/EN/Projects/ 02EUVN-DA/.

Flury J. and Rummel R., 2009, On the geoid-quasigeoid separation in mountainous areas. J Geod, 83: 829-847.

Grappo G.A., 1980, Determination of the Earth's mean equatorial radius and center of mass from Doppler-derived and gravimetric geoid heights. Manusc Geod, 5: 201-216.

Harris J.W. and Stocker H., 1998, Handbook of mathematics and computational science. Springer-Verlag, Berlin Heidelberg New York.

Heiskanen W. and Moritz H., 1967, Physical geodesy. WH Freeman, San Francisco.

Ihde J. and Augath W., 2001, The European Vertical Reference system (EVRS), its relation to a World Height System and to ITRS. IAG Symposia, vol 125, pp 78-83, Springer-Verlag, Berlin Heidelberg New York. Ihde J., 2007, Conventions for the definition and realization of a conventional VRS. Report of the IAG Inter-Commission Project on Vertical Reference Frames (web access at http://iag.ign.fr/index.php?id=48).

Jekeli C., 2000, Heights, the Geopotential and Vertical Datums. Department of Geodetic Science, The Ohio State University, OSU Report No. 459, Columbus, Ohio.

Kotsakis C., 2008, Transforming ellipsoidal heights and geoid undulations between different geodetic reference frames. J Geod, 82, 249-260.

Kotsakis C. and Sideris M.G., 1999, On the adjustment of combined GPS/geoid/leveling networks. J Geod., 73, 412-421.

Kotsakis C. and Katsambalos K., 2010, Quality analysis of global geopotential models at 1542 GPS/levelling benchmarks over the Hellenic mainland. Surv Rev, 42, 318, 327-344.

Leick A. and van Gelder B.H.W., 1975, On similarity transformations and geodetic network distortions based on Doppler satellite observations. Department of Geodetic Science, The Ohio State University, OSU Report No. 235, Columbus, Ohio.

Mäkinen J. and Ihde J., 2009, The permanent tide in height systems. IAG Symposia, vol 133, pp 81-87, Springer-Verlag, Berlin Heidelberg New York.

Marti U., 2010, private communication.

Sacher M., Ihde J., Liebsch G. and Mäkinen J., 2008, EVRF07 as Realization of the European Vertical Reference System. Presented at the Annual EUREF Symposium, Brussels, June 18-21, 2008.

Schaab H. and Groten E., 1979, Comparison of geocentric origins of global systems from uniformly distributed data. Bull Geod, 53, 11-17.

Schwarz K.P. and Sideris M.G., 1993, Heights and GPS. GPS World, 4, 2, 50-56.

Sjöberg L.E., 2010, A strict formula for geoid-to-quasigeoid separation. J Geod., 84, 699-702.

Soler T. and van Gelder B.H.W., 1987, On differential scale changes and the satellite Doppler system z-shift. Geophys J R Astr Soc, 91, 639-656.

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