Aliasing Effects in Terrain Correction Computation Using Constant and Lateral Density Variation

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Abstract

Terrain correction (TC) needs to be treated in a precise and rigorous way in every gravity reduction technique within the context of precise geoid determination, especially for mountainous areas. Since the small grid spacing in the modern digital terrain models (DTMs) can represent the local features of rugged terrain very precisely, such high-resolution DTMs should be used, if available, in the numerical computation of TC according to the Newtonian attraction integral formula. Two areas within Canada are selected to study the effects of using different DTM grid spacing on TC computations. The DTM resolution levels used for this test are 15″, 30″, 45″, 1′ and 2′. Firstly, the computations are applied using constant crust density ($2.67\,\mathrm{g/cm^3}$) and the algorithm for the mass-prism (MP) topographic model, which is evaluated by fast Fourier transform technique in planar approximation. Secondly, we use lateral crust density variations in the TC computation for the MP model through the incorporation of the available digital density models (DDMs) with 30″, 1′ and 2′ grid resolution. The comparison of the results using different DTM and DDM grid resolutions is carried out. Finally, the terrain effect (direct, indirect and total) on the geoid is studied using constant crust density with different DTM resolution levels and the direct effect on geoid is studied using lateral density variation with DTM and DDM of different grid resolutions.

Keywords. Terrain correction, geoid undulation, digital terrain model, digital density model

1 Introduction

The TC is a key auxiliary quantity in gravity reductions, which are used in solving the geodetic boundary value problem of physical geodesy and in geophysics. It contains the high frequency part of the gravity signal representing the irregular part of the topography, which deviates from the Bouguer plate. Helmert's second method of condensation is mostly used in practice as the mass reduction technique in the classical solution of the geodetic boundary value problem. Faye anomaly (or Helmert anomaly), which consists of free-air anomaly plus TC, represents the boundary values in the Helmert Stokes approach since TC alone is the difference between the attraction of the topography and the attraction of the condensed topography in planar approximation; see Moritz (1968), Wichiencharoen (1982) and Sideris (1990). In Molodensky's problem, which is regarded as modern boundary value problem, TC can replace the g_1 term under the assumption that the gravity anomalies are linearly dependent on the heights (Moritz, 1980).

Various computational approaches have been developed based on the conventional methods, which usually evaluate the TC integral using a model of rectangular prisms with flat tops (Nagy, 1966) or even with inclined tops (Blais and Ferland, 1984). TC computation based on these formulas is very time-consuming, but rigorous. Recently, Biagi et al. (2001) have given a new formulation for residual terrain correction (RTC) and Strykowski et al. (2001) have introduced a polynomial model for TC computation. The TC computation can be performed very fast in the frequency domain by means of FFT, having the TC convolution integral expanded in the form of Taylor series; see for example, Sideris (1984), Forsberg (1984), Tziavos et al. (1988), Harison and Dickinson (1989), Sideris (1990), Li and Sideris (1994), Li et al. (2000).

There are different resolutions of DTM available these days throughout the world. TC computation using FFT technique is one of the most efficient tools to handle the large amounts of height data efficiently. The convergence condition, that the distance between computation and running point should be larger than the difference between their heights, can be regarded as a major problem in the application of FFT to the series expansion of the TC integral, especially in rugged areas. Divergence of the series is observed with densely sampled height data in rough terrain; for example see Martinec et al. (1996) and Tziavos et al. (1996). A combination method, based on the evaluation of the numerical integration method in the intermediate zone around the computation point and the use of FFT in the rest of area, has been used to tackle the convergence problem by Tsoulis (1998) and Tziavos et al. (1998).

The knowledge of actual crust density is required in each gravity reduction method (including TC) in order to effectively remove all the masses above the geoid. Constant density is often used in practice instead of actual crust density because of lack of actual bedrock density information. However, two-dimensional DDMs are becoming available these days in some countries though a three-dimensional model is required to represent a real topographical density distribution. These density models should be incorporated in the TC computation. This has been studied by Tziavos et al. (1996), Huang et al. (2000), and Tziavos and Featherstone (2000). The effect on RTC of lateral density variation and approximations made on density modelling has been shown by Biagi et al. (2001).

This paper mainly focuses on investigating the importance of using actual density information and the use of various grid resolutions of DTM for TC computation in flat and rough areas, within the context of precise geoid determination. In this paper, the term aliasing represents the loss of detail information as terrain corrections are evaluated from a high-resolution DTM to a coarse one. In other words, the results from the densest DTM are taken as "control values" and the differences between these results and the results obtained by using sparser DTMs are considered to be the aliasing effects. It also studies terrain effects on geoid undulation for Helmert's second method of condensation in planar approximation. Numerical tests are carried out in two areas in Canada, one in the most rugged areas of Canadian Rockies and another one in a modest area of Saskatchewan (Canada).

2 Computational formulas

The TC integral at a point (i,j) is given by (Heiskanen and Moritz, 1967)

$$c(i,j) = G\iint_E \int_{h_{ij}}^{h_p} \frac{\rho(x,y,z)(h_{ij}-z)}{r^3(x_i-x, y_j-y, h_{ij}-z)}\,dxdydz, \tag{1}$$

where G is Newton's gravitational constant, ρ (x, y, z) is the topographical density at the running point, h_{ij} and h_p are the computation and running points respectively, and E denotes the integration area.

r (x, y, z) is the distance kernel defined as

$$r(x,y,z) = (x^2 + y^2 + z^2)^{1/2}. \tag{2}$$

Equation (1) can be written for a gridded digital topographic model as (Li and Sideris, 1993)

$$c(i,j) = G\sum_{n=0}^{N-1}\sum_{m=0}^{M-1}\int_{x_n-\Delta x/2}^{x_n+\Delta x/2}\int_{y_m-\Delta y/2}^{y_m+\Delta y/2}\int_{h_{ij}}^{h_{nm}} \frac{\rho(x,y,z)(h_{ij}-z)}{r^3(x_i-x, y_i-y, h_{ij}-z)}\,dxdydz \tag{3}$$

Integrating equation (1) with respect to z gives

$$c(i,j) = G\iint_E \left(\frac{\rho}{l}\left[1-\left[1+\left(\frac{\Delta h}{l}\right)^2\right]^{-1/2}\right]\right)dxdy \tag{4}$$

where $l^2 = (x_i - x)^2 + (y_i - y)^2$ and $\Delta h = h_{ij} - h_p$

The term $\left[1+\left(\frac{\Delta h}{l}\right)^2\right]^{-1/2}$, with the condition $\left(\frac{\Delta h}{l}\right)^2 \le 1$, can be expanded into a series

$$\left[1+\left(\frac{\Delta h}{l}\right)^2\right]^{-1/2} = 1 - \frac{1}{2}\left(\frac{\Delta h}{l}\right)^2 + \frac{1.3}{2.4}\left(\frac{\Delta h}{l}\right)^4 - \frac{1.3.5}{2.4.6}\left[\frac{\Delta h}{l}\right]^6 + \ldots\ldots\ldots\ldots\ldots\ldots \tag{5}$$

Inserting equation (5) into equation (4) gives a principal formula for the FFT evaluation of the TC integral. Keeping only up to two terms in binomial series expansion, the following formula is obtained:

$$c(i,j) = G\rho \iint_E \left(\rho\left\{\frac{\Delta h^2}{2l^3} - \frac{3\Delta h^4}{8l^5}\right\}\right) dxdy \tag{6}$$

2.1 Mass prism and mass line topographic models

A prism with a mean height of the topography represents the height within each cell in MP topographic representation. The mass of the prism is concentrated along its vertical axis representing topography as a line in mass line (ML) model. The unified two dimensional convolution formulas for equation (6) using MP and ML algorithms can be evaluated by means of FFT as (Li et al., 2000).

$$\begin{aligned}
T_S^0 &= d_{00}F^{-1}\{H^0K_{S\alpha}^0\} - d_{01}F^{-1}\{H^0K_{S\beta}^0\} - d_{02}F^{-1}\{H^1K_{s\beta}^0\} \\
T_S^1 &= d_{10}F^{-1}\{H^0K_{S\alpha}^1\} - d_{11}F^{-1}\{H^0K_{S\beta}^1\} - d_{12}F^{-1}\{H^1K_{S\beta}^1\} - d_{13}F^{-1}\{H^2K_{S\beta}^1\} - d_{14}F^{-1}\{H^3K_{S\beta}^1\} \\
T_S^2 &= d_{20}F^{-1}\{H^0K_{S\alpha}^2\} - d_{21}F^{-1}\{H^0K_{S\beta}^2\} - d_{22}F^{-1}\{H^1K_{S\beta}^2\} - d_{23}F^{-1}\{H^2K_{S\beta}^2\} - d_{24}F^{-1}\{H^3K_{S\beta}^2\} \\
&\quad - d_{25}F^{-1}\{H^4K_{S\beta}^2\} - d_{26}F^{-1}\{H^5K_{S\beta}^2\}
\end{aligned} \tag{7}$$

where $H^i = F\{\rho h^i\}$

$$K_{sw}^i = F\{K_s^i(x,y,w)\},\ s=x, y,\ w=\alpha, \beta,\ i = 0, 1,2 \tag{8}$$

The coefficients d_{ij} and the analytical formulas for kernel functions used in above equations are given in details by Li et.al. (2000). α and β are parameters used to speed up the convergence of the series and the optimal value for this parameter is given by one-half of the standard deviation of the heights. The TC formula using DDM and MP algorithm is also given by Tziavos et. al. (1996) provided that the grid size of DTM is same as DDM.

2.2 Terrain effect on geoid undulation

The total geoid undulation in remove-restore technique can be expressed as

$$N = N_{GM} + N_{\Delta g} + N_{ind} \tag{9}$$

where N_{GM} represents the low frequency component of the geoid obtained from geopotential model, $N_{\Delta g}$ represents the medium frequency component of the geoid obtained from Stokes' formula and N_{ind} is the indirect effect on geoid, which depends on the gravity reduction method used. The Stokes' formula for the determination of medium wavelength part of the geoid for Helmert's second method of condensation in planar approximation can be formulated as

$$N_{\Delta g} = \frac{R}{4\pi\gamma}\iint_\sigma (\Delta g + c)S(\psi)d\sigma = \frac{R}{4\pi\gamma}\iint_\sigma \Delta g S(\psi)d\sigma + \frac{R}{4\pi\gamma}\iint_\sigma cS(\psi)d\sigma \tag{10}$$

where ($\Delta g+c$) represents Faye anomalies and c is terrain correction, the negative value of which represents the difference between the attraction of the topography computed on the surface of the topography and the attraction due to the condensed masses computed on the geoid. The second term in equation (10) is direct terrain effect on geoid undulation. Indirect effect on gravity is not included in above formula. The indirect effect for this condensation scheme can be formulated in planar approximation as (Wichiencharoen, 1982)

$$N_{ind} = -\frac{\pi G\rho}{\gamma} h_{(i,j)}^2 - \frac{G\rho}{6\gamma} \iint \frac{h^3 - h_{(i,j)}^3}{r_0^3} dxdy \qquad (11)$$

where r_0 is the planar distance between computation and running point. The total terrain effect on geoid undulation for this reduction scheme can be expressed as

$$N_{Total} = \frac{R}{4\pi\gamma} \iint_{\sigma} cS(\psi) d\sigma - \frac{\pi G\rho}{\gamma} h_{(i,j)}^2 - \frac{G\rho}{6\gamma} \iint \frac{h^3 - h_{(i,j)}^3}{r_0^3} dxdy \qquad (12)$$

3 Numerical tests

Two tests areas, one in the Canadian Rockies bounded by latitude between 49ºN and 54ºN and longitude between 124º W and 114ºW, and the other in Saskatchewan, bounded by latitude 49ºN and 54ºN and longitude between 110º W and 100º are selected for this numerical investigation. The statistics of the DTMs are presented in the table 1 for both test areas for different grid resolutions. The original grid resolution available for this test is 3″ while 15″, 30″, 45″, 1′ and 2′ grid files are produced by selecting the point height values from the 3″ grid for the corresponding grid levels. The resolution of original DDM available for these test areas is 30″ while 1′ and 2′ grid resolutions of DDM are produced from the 30″ DDM picking up the point density values for the corresponding grid levels. Figure 1 represents the topography model of Canadian Rockies. Figure 2 shows large contrasts in topographic density of Canadian Rockies with maximum and minimum values of 2.98 (gm/cm^3) and 2.63 (gm/cm^3) respectively, whereas Saskatchewan has smooth geological structure of the topography with constant density of 2.56 (gm/cm^3) except in some areas in the southern part.

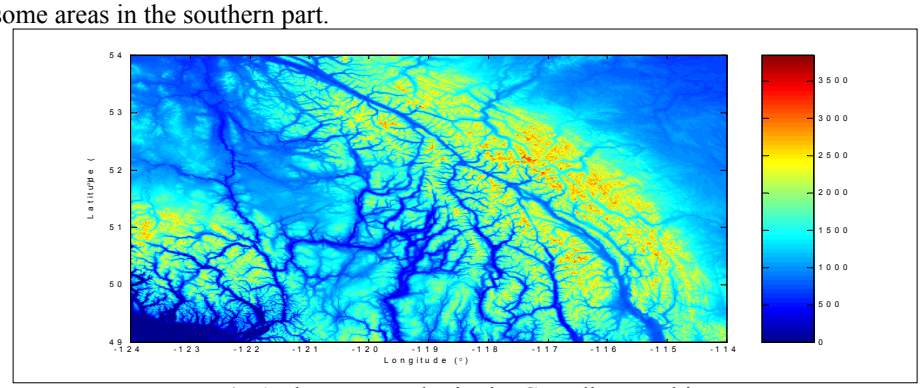

Fig.1 The topography in the Canadian Rockies

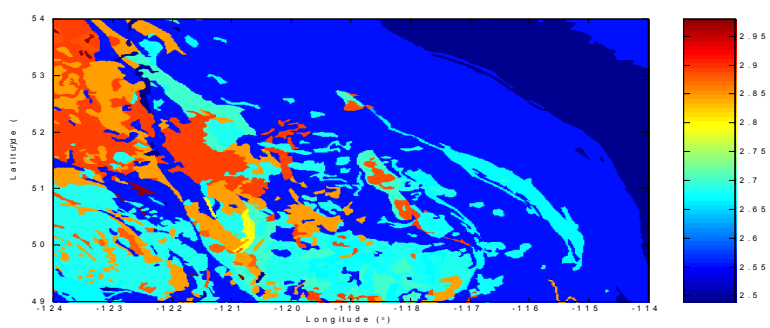

Fig. 2 The density model of Canadian Rockies

Table 1. Statistical characteristics of DTMs (m)

Test Area	Canadian Rockies					Saskatchewan				
Grid Resolution	**Max**	**Min**	**Mean**	**RMS**	**STD**	**Max**	**Min**	**Mean**	**RMS**	**STD**
15″×15″	3840	0	1355	1460	543	1385	244	581	603	159
30″×30″	3785	0	1355	1460	543	1381	244	581	603	159
45″×45″	3656	0	1354	1459	543	1380	244	581	603	159
1′×1′	3429	0	1354	1459	543	1379	244	581	603	159
2′×2′	3275	0	1353	1458	544	1379	244	581	603	159

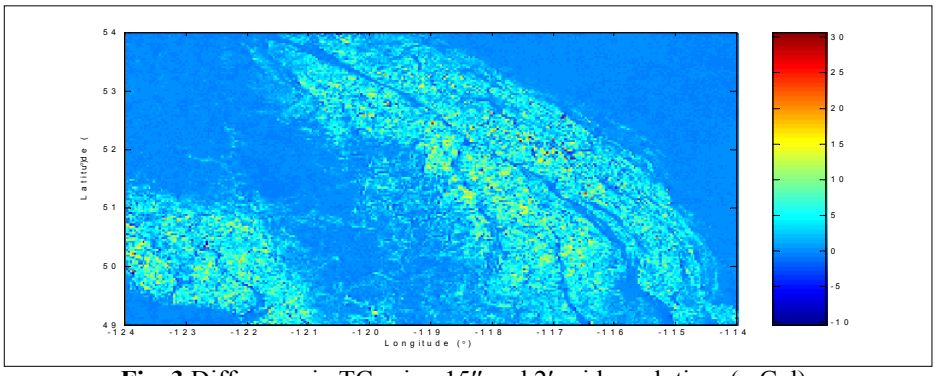

Fig. 3 Difference in TC using 15″ and 2′ grid resolution (mGal).

3.1 Aliasing effects on TC with constant density

The TC computation is carried out using MP model for different grid resolution of DTM in both test areas for up to third term in Taylor series expansion. The kernel function is computed over the whole area and 100% zero padding is performed around the matrices of heights and around the distance kernel in order to remove circular convolution effects. The third term in Canadian Rockies shows divergence of the series giving out unrealistic results whereas that term in Saskatchewan does not change the final result of TC showing the convergence of the series right after second term and the results are presented just up to second term in this paper. Results on different tests are presented just for Canadian Rockies in this paper. Table 2 summarizes the statistics of TC results using different grid resolution of DTM. Figure 3 shows the difference in TC using DTM grid resolution between 15″ and 2′, which shows the correlation of difference in TC using different DTM resolutions with topography. TC varies from 109 mGal to 42.7 mGal in maximum value and 9.9 mGal to 7.3 mGal in RMS in Canadian Rockies, while there is no considerable difference in the statistics of Saskatchewan using DTMs from 15″ grid resolution level up to 2′ grid level. Maximum and RMS values of TC decrease from 2.6 mGal and 0.1 mGal to 1.5 mGal and 0.1 mGal respectively for Saskatchewan.

Table 2. Terrain correction in Canadian Rockies (mGal) (C1-first term, C2-second term)

Grid resolution	Terms	Max	Min	Mean	RMS	STD
15″×15″’	C1	142.00	0.05	6.73	9.57	6.80
	C1+C2	108.76	0.05	7.06	9.89	6.93
30″×30″	C1	121.23	0.05	6.75	9.64	6.88
	C1+C2	100.06	0.05	6.96	9.77	6.86
45″×45″	C1	109.92	0.05	6.65	9.54	6.84
	C1+C2	83.38	0.05	6.71	9.44	6.64
1′×1′	C1	82.62	0.05	6.39	9.22	6.63
	C1+C2	59.83	0.05	6.36	8.98	6.34
2′×2′	C1	52.85	0.05	5.20	7.59	5.53
	C1+C2	42.73	0.05	5.09	7.31	5.24

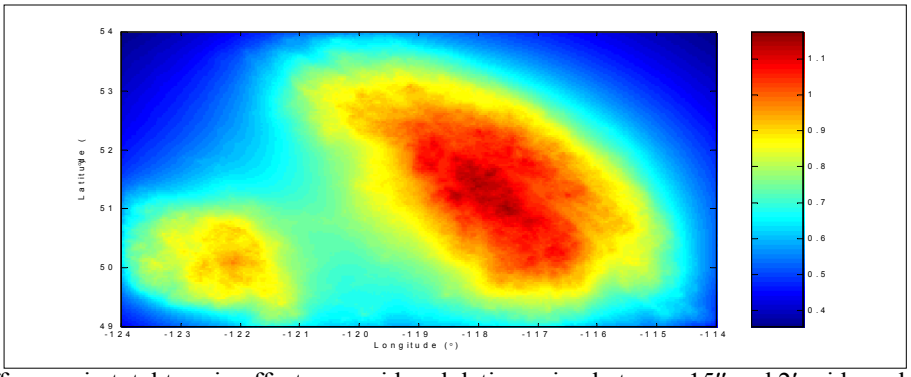

Fig. 4 Difference in total terrain effect on geoid undulation using between 15″ and 2′ grid resolutions (m)

3.2 Aliasing effects on geoid with constant density

The direct effect on geoid undulation is computed from Stokes' formula with the rigorous spherical kernel by the one-dimensional fast Fourier transform algorithm. Both terms, a regular and an irregular part, are computed for indirect effects on geoid. Total terrain effects on geoid undulation vary from 3.777 m to 2.663 m in maximum value and 2.497 m to 1.728 m in RMS in Canadian Rockies, whereas there is just a

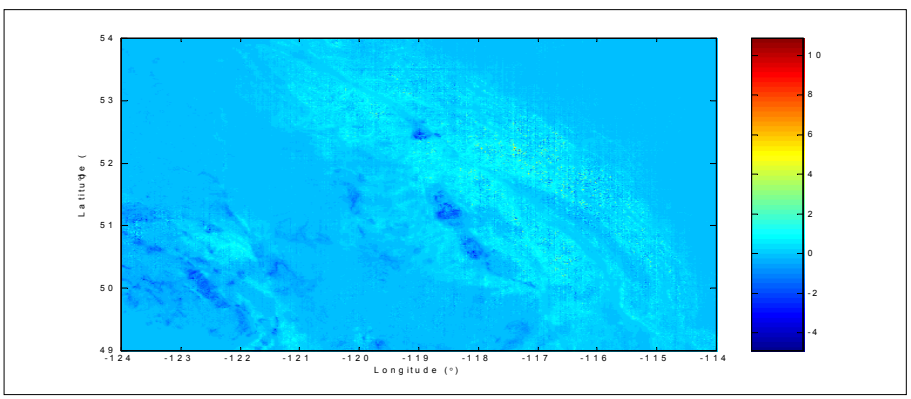

Fig. 5 Difference in TC using constant and variable density (mGal)

Table 3. Terrain effect on geoid undulation (m) (E-effect, D-direct effect, I-indirect effect, T-total terrain effect)

Grid Resolution	E	Max	Min	Mean	RMS	STD
15″×15″	D	3.866	1.207	2.536	2.614	0.634
	I	0.005	-0.470	-0.121	0.144	0.078
	T	3.777	1.188	2.418	2.497	0.623
30″×30″	D	3.779	1.179	2.482	2.556	0.612
	I	0.003	-0.470	-0.121	0.144	0.078
	T	3.691	1.160	2.362	2.439	0.609
45″×45″	D	3.619	1.128	2.376	2.448	0.588
	I	0.002	-0.463	-0.121	0.144	0.078
	T	3.535	1.104	2.257	2.324	0.554
1′×1′	D	3.410	1.066	2.247	2.314	0.556
	I	0.001	-0.468	-0.121	0.144	0.079
	T	3.337	1.042	2.127	2.191	0.524
2′×2′	D	2.729	0.852	1.795	1.850	0.445
	I	0.000	-0.457	-0.121	0.145	0.080
	T	2.663	0.827	1.677	1.728	0.416

change of 3-4mm in maximum value and RMS in Saskatchewan. The statistics of indirect effect are practically constant using different grid resolution levels for both test areas. Table 3 shows the direct, indirect and total terrain effect on geoid undulation for different DTM grid resolutions. Figure 4 shows the difference in total terrain effect on geoid undulation between using 15″ and 2′ grid resolution in Canadian Rockies. The maximum difference is at the top of the mountains.

3.3 Aliasing effects on TC using variable density

The TC is computed using DDM in both of the test areas using MP algorithm. The grid spacing of DTM and DDM used for this test are 30″, 1′ and 2′. Table 4 shows the statistics of difference in TC using constant density and gridded actual density information. Their effect on geoid undulation is given in table 5. There is a difference of 10.9 mGal to 1.8 mGal in maximum value with RMS from 0.4 mGal to 0.2 mGal using constant and actual density information for the grid spacing of DTM and DDM from 30″ up to 2′ arc minute in Canadian Rockies. There is no considerable difference in TC and their effects on geoid undulation using constant and actual density information in Saskatchewan. There is a maximum effect of 10 cm with an RMS of 4.6 cm on geoid undulation using constant and lateral density variation in Canadian Rockies. Figure 5 shows the difference in TC using constant and lateral density variation for 30″ grid resolution while figure 6 shows its effect on geoid undulation. Figure 7 and figure 8 show the change in RMS value in TC and direct effect of TC on geoid undulation using constant and variable density.

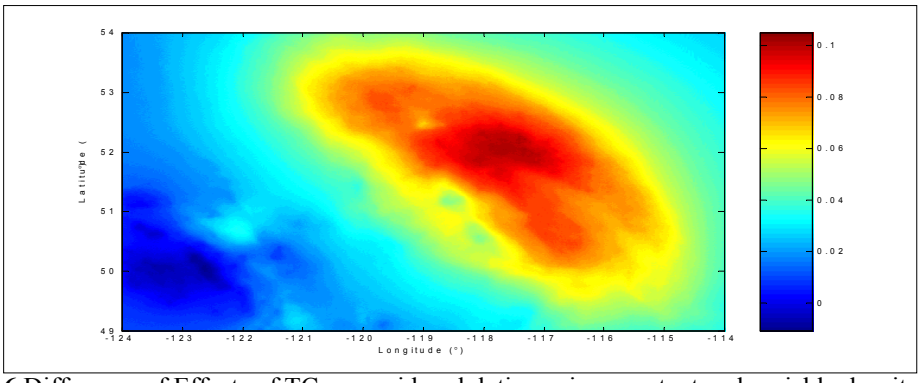

Fig. 6 Difference of Effects of TC on geoid undulation using constant and variable density (m)

Fig. 7 RMS value of TC using constant and variable density

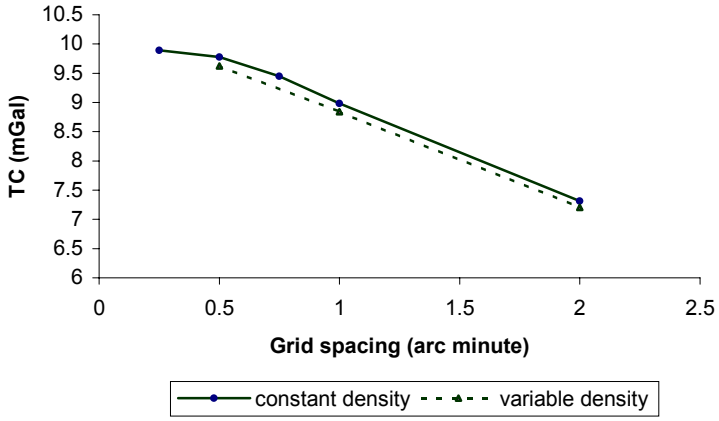

Fig. 8 RMS value of direct effect on geoid undulation using constant and variable density (m)

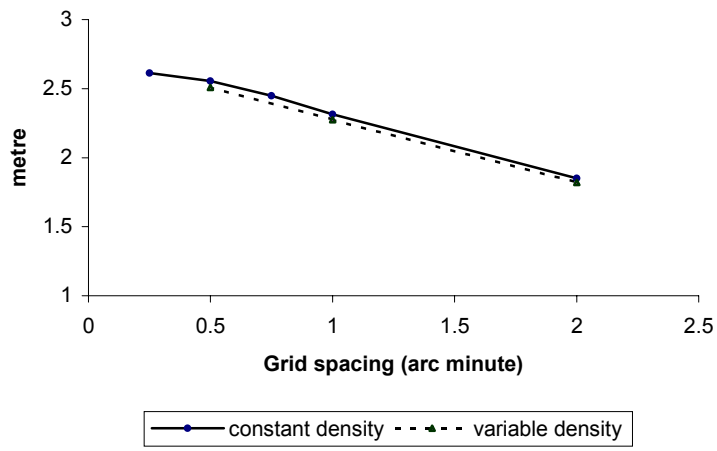

Table 4. The difference in TC using constant and variable density (mGal)

Grid Spacing	Max	Min	Mean	RMS	STD
30″×30″	10.87	-4.91	0.13	0.38	0.36
1′×1′	3.26	-2.94	0.11	0.29	0.27
2′×2′	1.77	-2.46	0.08	0.23	0.22

Table 5. Difference of Effects of TC on geoid undulation using constant and variable density (m)

Grid spacing	Max	Min	Mean	RMS	STD
30″×30″	0.105	-0.011	0.046	0.052	0.024
1′×1′	0.085	-0.012	0.038	0.043	0.021
2′×2′	0.068	-0.010	0.029	0.034	0.016

4 Conclusion

Fine DTM resolution should be used in precise geoid determination in rugged areas. Our results show that the difference in total terrain effect on geoid undulation using from 30″ grid spacing of DTM up to 2′ arc minute can vary from 3.777 m to 2.663 m in maximum value and 2.497 m to1.728 m in RMS in Canadian Rockies. These values can even go higher if 3″ grid resolution of DTM is used. The effect of second term of the TC Taylor series on geoid undulation is 16 cm in maximum value with RMS of 11 cm in Canadian Rockies. The use of high resolution DTM is not critical in the computation of terrain effect in non-mountainous regions. Divergence of the series was observed in the third term of Taylor series expansion. More studies regarding convergence problem should be carried out using FFT and high-resolution DTMs.

The actual crust density information, if available, should be used in precise geoid determination in high mountains. Our results show that the difference in TC effect on geoid undulation using constant density and horizontal density variation can go up to 10 cm in maximum value with an RMS of 5 cm. It would be wise to use at least the mean density of the area, if available, for terrain correction computation in mountains if DDM is not available. The knowledge of actual density information is not crucial in the TC computation of non-mountainous areas.

The difference in TC using different grid resolution of DTM and constant and lateral density variation is correlated with the topography and the maximum difference is observed in high mountains.

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