The need of a Local Reference Frame in Greece: the deficiency of ETRS89 and a new proposed strategy

D. Ampatzidis, C. Kotsakis, K. Katsambalos
Department of Geodesy and Surveying
Aristotle University of Thessaloniki
Univ. Box 440, Thessaloniki 541 24, Greece

Abstract. This study outlines a new strategy for the optimal implementation of a Local Reference Frame (LRF) realization in Greece. The main concept lies on the application of a Helmert-type similarity transformation to a given velocity field that will satisfy a minimum kinetic energy condition with respect to the new optimal LRF. The initial velocity set could be expressed either in 3D or 2D, and it may refer to any global or regional reference frame (e.g. ITRF or ETRF). Our approach is tested over a GPS control network in Greece that exhibits a rather strong and inhomogeneous ETRF-based velocity field, which is significantly reduced after the optimal transformation scheme that is proposed herein.

Keywords. Terrestrial reference frame, ETRS89, Hellenic area, velocity field, network kinetic energy.

1 Introduction

ETRS89 is a conventional terrestrial reference system that is used for GPS-based spatial positioning over the European continent (Gubler et al. 1992). Apart from serving the need of a uniform spatial reference framework for surveying and mapping applications in the European countries, the primary geodetic objective of ETRS89 is the definition, realization and maintenance of a stable 3D coordinate system with minimal horizontal velocities, thus allowing an easier handling of time-dependent positions in national geodetic networks.

To a large extent, the aforementioned objective is achieved by ensuring that ETRS89 follows the Eulerian rotational motion of the Eurasian plate through its alignment to a pre-determined set of appropriate Euler Pole Parameters (EPPs). In fact, one of the key aspects within the ETRS89 realization is the reduction of an ITRF-based velocity field to the so-called stable part of Europe, using the following linearized equation (Boucher and Altamimi 2008):

$$\mathbf{v}_i^{ETRFyy} = \mathbf{v}_i^{ITRFyy} + \dot{\mathbf{R}}\,\mathbf{x}_i^{ITRFyy} \qquad (1)$$

where the velocity vectors $\mathbf{v}_i^{ITRFyy}$ and $\mathbf{v}_i^{ETRFyy}$ refer to the same point i with respect to a global ITRF frame and an induced ETRS89/ETRF frame, respectively. The term $\mathbf{x}_i^{ITRFyy}$ corresponds to the 3D position vector of the respective point with respect to the underling ITRF frame (the epoch is, in principle, irrelevant), whereas $\dot{\mathbf{R}}$ is an 3×3 anti-symmetric matrix that contains the Cartesian components of the angular velocity vector of the Eurasian plate with respect to the same ITRF frame. Note that for the actual velocity computation in a particular ETRF frame, the angular velocity vector (EPPs) of the Eurasian plate is deduced either from a global geophysical model such as AM0-2 or NNR-NUVEL-1/1A or from a geodetically derived velocity field over a network of high-quality ITRF stations within the stable part of Europe. For more details see the well-known memo by Boucher and Altamimi (2008).

The magnitude of the ETRF-based horizontal velocities obtained through Eq. (1) over the central and northern parts of Europe does not exceed a few mm/yr, thus allowing ETRS89 to be used as a rather stable framework for geodetic positioning in these regions. On the other hand, at the south-eastern part of Europe (and particularly in Greece) the situation is completely different. Specifically, the horizontal velocities with respect to the stable part of Eurasia in Greece can reach up to several cm/yr (more than 3 cm/yr at the TUC2 EPN station in Crete) following also a spatially inhomogeneous behavior, a fact that has been corroborated by several studies (e.g. Clarke et al. 1998, Nyst and Thatcher 2004, Hollenstein et al. 2008); see Fig. 1. This is largely caused by the unique geodynamical setting of the Hellenic area that directly affects the behavior of the ETRS89/ETRF velocity field in Greece, which is in fact considerably stronger than in the central and northern Europe when viewed from an Eurasia-fixed reference frame. In terms of its geodynamical behavior, the Hellenic area seems actually to be divided into two major parts: northern Greece exhibits an apparent consistency (at the few mm/yr level) with the stable Eurasia, whereas the southern regions suffer from a clear S/W velocity trend at the level of several cm/yr.

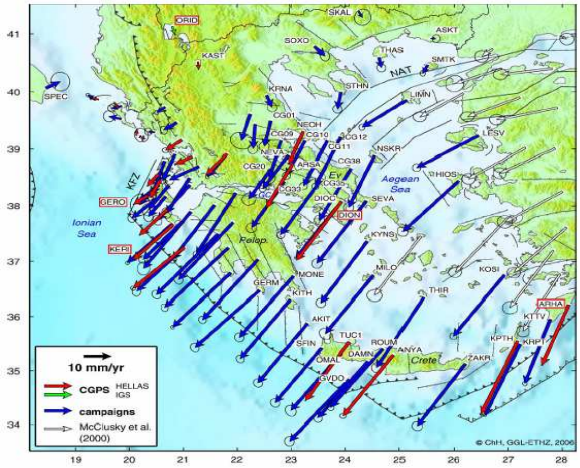

Fig. 1 The inhomogeneous velocity field in the Hellenic area with respect to an Eurasian-fixed reference frame (after Hollenstein et al. 2008).

Considering the previous remarks, an ETRF-based implementation of a national TRF in Greece (as described, for example, in Boucher and Altamimi 2008) will create a strong velocity field over most of the country, thus canceling out one of the key reasons for adopting ETRS89 as a standard geodetic reference system by the national mapping agencies around Europe. The aim of this paper is to present an alternative optimal scheme for implementing a national TRF in Greece. Our approach is based on the use of an initial ETRF (or ITRF) realization over a national network consisting of an estimated set of coordinates and velocities at some reference epoch, which can be then rotationally transformed as to ensure the 'weakest' possible velocity field throughout the entire part of the country.

2 A proposed strategy for an optimal LRF realization

2.1 Mathematical formulation

The main idea of the proposed scheme relies on the implementation of an optimized Helmert-type similarity transformation of a velocity field from a given ITRF or ETRF frame to a new local reference frame (LRF). The pointwise mathematical expression for such a velocity transformation is given by the linearized formula (Altamimi et al. 2002):

$$\mathbf{v}_i^{LRF} = \mathbf{v}_i^{TRF} + \mathbf{E}_i^{\mathrm{T}}\dot{\boldsymbol{\theta}} \tag{2}$$

where $\mathbf{v}_i^{TRF}$ and $\mathbf{v}_i^{LRF}$ denote the 3D velocity vectors of a terrestrial point i with respect to the initial TRF and the new LRF (to be later optimized), respectively. The transformation matrix $\mathbf{E}_i$ is formed by the well-known expression

$$\mathbf{E}_i^{\mathrm{T}} = \begin{bmatrix} 1 & 0 & 0 & x_i^{TRF} & 0 & -z_i^{TRF} & y_i^{TRF} \\ 0 & 1 & 0 & y_i^{TRF} & z_i^{TRF} & 0 & -x_i^{TRF} \\ 0 & 0 & 1 & z_i^{TRF} & -y_i^{TRF} & x_i^{TRF} & 0 \end{bmatrix}$$

and it is evaluated through the known coordinates of the particular point with respect to the initial TRF (the reference epoch of these coordinates is, in principle, irrelevant), whereas the vector $\dot{\boldsymbol{\theta}}$ contains the unknown rates of the 7 similarity transformation parameters between the TRF and the LRF, namely

$$\dot{\boldsymbol{\theta}} = \begin{bmatrix} \dot{t}_x & \dot{t}_y & \dot{t}_z & \dot{D} & \dot{r}_x & \dot{r}_y & \dot{r}_z \end{bmatrix}^{\mathrm{T}}$$

In the case of a spatial network of n points, the previous expression in Eq. (2) can be generalized in vector form as follows:

$$\mathbf{v}^{LRF} = \mathbf{v}^{TRF} + \mathbf{E}^{\mathrm{T}}\dot{\boldsymbol{\theta}} \tag{3}$$

where the vectors $\mathbf{v}^{LRF}$ and $\mathbf{v}^{TRF}$ contain the individual velocity components of all network stations, and the total transformation matrix $\mathbf{E}$ has the block structure

$$\mathbf{E}^{\mathrm{T}} = \begin{bmatrix} \mathbf{E}_1^{\mathrm{T}} \\ \vdots \\ \mathbf{E}_n^{\mathrm{T}} \end{bmatrix}.$$

The unknown transformation parameters $\dot{\boldsymbol{\theta}}$ shall be determined according to the following optimal criterion for the magnitude of the transformed velocities into the new frame:

$$\phi = (\mathbf{v}^{LRF})^{\mathrm{T}}(\mathbf{v}^{LRF}) = \min \tag{4}$$

The physical meaning of the above criterion is that it leads to local reference frame in which the total kinetic energy of the network stations becomes as small possible (compared to any other frame in the 'neighbourhood' of the initial TRF). The minimization of the above quantity is achieved by setting to zero its partial derivative with respect to the vector of the unknown transformation parameters, that is

$$\frac{\partial \phi}{\partial \dot{\boldsymbol{\theta}}} = \mathbf{0} \tag{5}$$

and, taking into the general equation (3), it leads to the optimal solution:

$$\dot{\boldsymbol{\theta}} = -(\mathbf{E}\mathbf{E}^{\mathrm{T}})^{-1}\mathbf{E}\,\mathbf{v}^{TRF} \tag{6}$$

Finally, the new optimized velocities with respect to the LRF can be now computed from the forward formula:

$$\mathbf{v}^{LRF} = \left(\mathbf{I} - \mathbf{E}^{\mathrm{T}}(\mathbf{E}\mathbf{E}^{\mathrm{T}})^{-1}\mathbf{E}\right)\mathbf{v}^{TRF} \tag{7}$$

2.2 Remarks

Our optimization procedure creates an LRF that differs from the initial TRF in terms of its temporal evolution law. The latter is actually dictated by the norm minimization of the velocity field $\mathbf{v}^{LRF}$ that is obtained from an existing velocity field $\mathbf{v}^{TRF}$ under a Helmert-type similarity transformation.

A critical issue for the implementation of this procedure is the choice of the transformation parameters $\dot{\boldsymbol{\theta}}$ that should participate within the optimization algorithm. In fact, a scale-rate transformation parameter should not be used, otherwise an artificial distortion would be introduced into the new LRF. Moreover, in small geographical areas (as the case of Greece) the use of shift-rate transformation parameters should also be avoided due to their high correlation with the three rotation-rate parameters. Thus, a rotational-only form of the general transformation model (2) or (3) seems a more appropriate choice for the implementation of the reference frame optimization over the Hellenic area. Note that, in this case, our strategy becomes equivalent to the determination of an optimal Euler rotation pole that will give the weakest possible residual velocity field throughout the working area.

The optimal LRF could be practically realized through an operational scheme consisting of the following basic steps. First, a conventional reference epoch t_0 needs to be selected, at which the TRF frame and the optimal LRF frame are supposed to coincide, i.e.

$$\mathbf{x}_i^{LRF}(t_{\mathrm{o}}) \equiv \mathbf{x}_i^{TRF}(t_{\mathrm{o}}) \tag{8}$$

At an arbitrary epoch t the position vector with respect to the LRF is

$$\mathbf{x}_i^{LRF}(t) = \mathbf{x}_i^{LRF}(t_{\mathrm{o}}) + (t - t_{\mathrm{o}})\,\mathbf{v}_i^{LRF} \tag{9}$$

If we take into account the relationship between the LRF and TRF velocities according to Eq. (7) and also the previous condition stated in Eq. (8), we have that

$$\mathbf{x}_i^{LRF}(t) = \mathbf{x}_i^{TRF}(t) + (t - t_{\mathrm{o}})\,\mathbf{E}_i^{\mathrm{T}}(\mathbf{E}_i\,\mathbf{E}_i^{\mathrm{T}})^{-1}\mathbf{E}_i\,\mathbf{v}_i^{TRF} \tag{10}$$

Based on the last formula, the realization of the optimal LRF can be fully achieved, at any time epoch t, in terms of position/velocity information that is available from the initial 'working frame' (ITRF or ETRF based) over the area of interest.

2.3 Exclusive use of 2D (horizontal) velocities

Alternatively, our optimization procedure could be also implemented in a 2D (horizontal-only) version by ignoring the vertical velocity component in the TRF frame. This may be approximately achieved if we multiply both sides of Eq. (2) with a suitable matrix of the following structure (Soler and Marshall 2002):

$$\mathbf{C}_i = \begin{bmatrix} -\sin\varphi_i\cos\lambda_i & -\sin\varphi_i\sin\lambda_i & \cos\varphi_i \\ -\sin\lambda_i & \cos\lambda_i & 0 \end{bmatrix} \tag{11}$$

which converts the initial 3D Cartesian geocentric velocities to their 2D horizontal counterparts (with respect to the local northing and easting directions at point i). In fact, the Helmert-type linearized similarity transformation for the horizontal velocities can be written in the general form:

$$\tilde{\mathbf{v}}_i^{LRF} = \tilde{\mathbf{v}}_i^{TRF} + \tilde{\mathbf{E}}_i^{\mathrm{T}}\dot{\boldsymbol{\theta}} \tag{12}$$

where

$$\tilde{\mathbf{v}}_i^{TRF} = \begin{bmatrix} v_{northing}^{TRF} & v_{easting}^{TRF} \end{bmatrix}_i \quad \text{and}$$

$$\tilde{\mathbf{v}}_i^{LRF} = \begin{bmatrix} v_{northing}^{LRF} & v_{easting}^{LRF} \end{bmatrix}_i$$

correspond to the horizontal velocity vectors with respect to the initial TRF and the optimal LRF, while the Helmert transformation matrix is now given as $\tilde{\mathbf{E}}_i^{\mathrm{T}} = \mathbf{C}_i\,\mathbf{E}_i^{\mathrm{T}}$. The LRF optimization procedure of the total horizontal velocity vector $\tilde{\mathbf{v}}^{LRF}$ over a given network can then follow along the same steps that were described in Sect. 2.1.

3. Numerical results

3.1 Test network

The numerical evaluation of our proposed strategy has been performed in a nationwide GPS control

network in Greece (see Fig. 2). The test network consists of 16 permanent reference stations: 11 of them are located within Greece, including 4 EPN Hellenic stations (AUTH, DUTH, NOA1, TUC2) and 7 additional stations that are part of the NOANET GPS network which is maintained by the Astronomical Observatory of Athens, while the rest 5 reference stations are distributed in Central Europe and they belong to the European Permanent Network (BRUS, GRAZ, PTBB, WTZR, ZIMM).

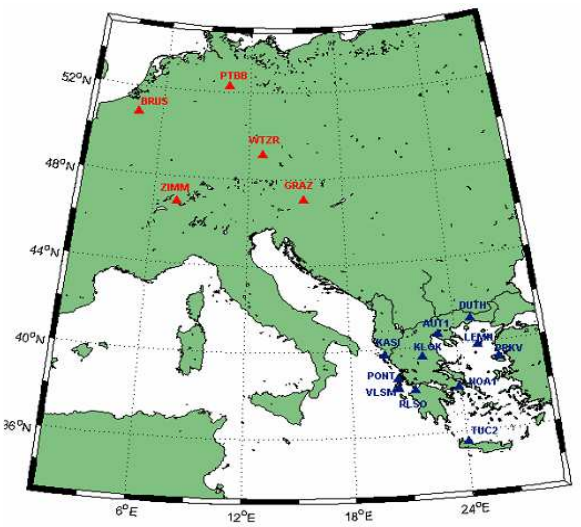

Fig. 2 The GPS test network used in our study. Red color: Central Europe EPN stations. Blue color: Hellenic stations.

An initial velocity set ($\mathbf{v}^{TRF}$) was estimated at the 11 Hellenic stations from GPS data over a three-year period (2007-2010) using the Bernese software package ver. 5 (Dach et al. 2007). The reference frame was fixed through tight constraints at the 5 EPN stations in Central Europe. Initially, station coordinates and velocities were computed with respect to ITRF 2005 and then were transformed to ETRF 2000 according to the guidelines described in Boucher and Altamimi (2008). The individual horizontal ad vertical components of the final ETRF 2000 velocity estimates are given in Table 1. Note that the rms error of these values is at the 0.5 mm/yr level.

The estimated ETRF 2000 horizontal velocities at the 11 Hellenic GPS reference stations exhibit an inhomogeneous behavior over the geographical area of Greece (see Table 1). Specifically, test stations that are located in the northern mainland part (AUT1, DUTH and KASI) show horizontal velocities smaller than 1 cm/yr. On the other hand, GPS stations located in the southern part (NOA1 and RLSO), Crete (TUC2) and northern Aegean region (LEMN and PRKV) show a rather different pattern, with larger velocity magnitudes that reach up to 3 cm/yr (see NOA1 and TUC2 stations).

Table 1. ETRF 2000 topocentric velocities (v_N, v_E, v_U: north, east and up velocity components, v_{hor}: total horizontal velocity) at the 11 Hellenic stations. Units in mm/yr.

Station	v_N	v_E	v_U	v_{hor}
AUT1	-6.4	0.5	-3.2	6.4
DUTH	-2.1	1.5	1.3	2.6
KASI	0.9	-3.4	-4.2	3.5
KLOK	-7.4	-4.2	-3.6	8.5
LEMN	-12.9	-16.5	3.4	20.9
NOA1	-25.1	-17.1	1.1	30.4
PONT	-5.2	-4.2	-3.3	6.7
PRKV	-12.9	-18.9	-0.3	22.9
RLSO	-18.5	-16.3	4.8	24.7
TUC2	-24.4	-15.7	-0.9	29.0
VLSM	-9.1	-6.4	-1.7	11.1

3.2 Optimal LRF velocity field

The results obtained from the optimized transformation of the ETRF 2000 velocities into the new LRF are shown in Table 2. Note that the horizontal components of the transformed velocities $\mathbf{v}^{LRF}$ which were obtained from Eq. (7) are only listed in this table. It is emphasized that the chosen transformation parameters that were used in the LRF optimization included only the 3 rotation rates, in accordance with the rationale that was discussed in Sect. 2.2.

Table 2. Transformed horizontal velocities in the optimal LRF at the 11 Hellenic stations. Units in mm/yr.

Station	v_N	v_E	v_{hor}
AUT1	-2.1	-7.9	8.2
DUTH	-12.9	-0.1	12.9
KASI	1.7	7.7	7.9
KLOK	-4.7	-1.6	5.0
LEMN	-6.1	-8.2	10.2
NOA1	-9.8	9.0	13.3
PONT	-4.1	4.7	6.2
PRKV	8.6	-5.9	10.4
RLSO	-0.9	-4.6	4.7
TUC2	4.0	5.9	7.1
VLSM	-4.3	0.9	4.4

Comparing the values in Tables 1 and 2, we see a reduction of the mean horizontal velocity of about 6 mm/yr (from 15 mm/yr in ETRF 2000 drops to 9 mm/yr in LRF). Moreover, the magnitude dispersion of the horizontal velocity vectors shows also a significant reduction, going from σ = 10.5 mm/yr (ETRF 2000) down to σ = 3.4 mm/yr (LRF).

3.3 Optimal LRF using 2D-only velocities

In this case, the implementation of the optimal LRF is based on the minimization of the 2D (horizontal) transformed velocities, according to the procedure that was described earlier in Sect .2.3.

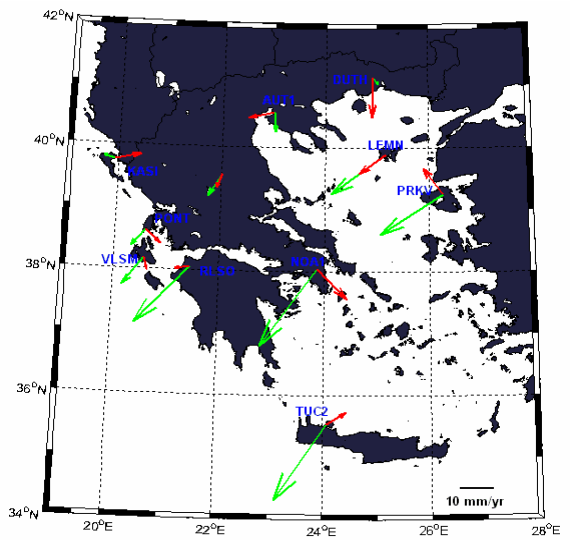

Fig. 3 Horizontal velocities at the 11 Hellenic stations with respect to ETRF 2000 (green color) and the optimal LRF (red color) in the case of the 3D optimization scenario.

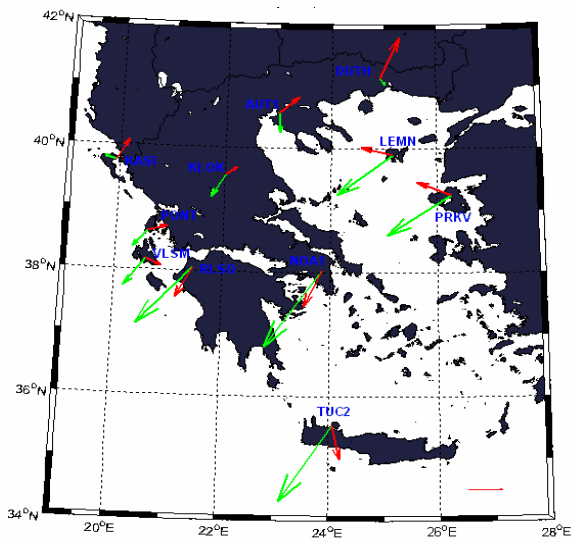

Fig. 4 Horizontal velocities at the 11 Hellenic stations with respect to ETRF 2000 (green color) and the optimal LRF (red color), in the case of the 2D optimization scenario. Note that the scale of the velocity vectors is the same as in Fig. 3.

Compared to the results from the 3D optimization scheme that were given in the previous section (see Table 2), the results from the 2D-only optimization (see Table 3) show better homogeneity in the orientation of the horizontal velocity vectors with respect to the new LRF frame. This is further illustrated in Figs. 3 and 4 where the horizontal components of $\mathbf{v}^{LRF}$ from each optimization scenario are plotted together with the initial ETRF 2000 horizontal velocity vectors.

Table 3. Horizontal velocities at the 11 Hellenic stations with respect to the new LRF (2D-only optimization). Units in mm/yr.

station	v_N	v_E	v_{hor}
AUT1	5.1	5.7	7.6
DUTH	12.9	5.6	14.1
KASI	6.7	3.5	7.6
KLOK	2.3	3.3	4.0
LEMN	2.5	-9.3	9.6
NOA1	-12.1	-5.6	13.3
PONT	1.8	5.6	5.9
PRKV	4.5	-9.9	10.9
RLSO	-9.8	-5.1	11.0
TUC2	-11	2	11.2
VLSM	-2.1	4.4	4.9

The estimated transformation parameters between the initial TRF (ETRF 2000) and the new LRF, for each one of the optimization scenarios, are given in Table 4. It is seen that the three rotation rates between the two frames are practically the same in both cases. This is actually an indication of the fact that the major part of the GPS-derived velocity field in Greece lies on its horizontal component, and thus the vertical velocity values do not affect the estimated transformation parameters $\dot{\boldsymbol{\theta}}$.

Finally, the statistics of the horizontal velocity vectors with respect to (i) the initial TRF used in our study (ETRF 2000), (ii) the new LRF according to the 3D optimization scenario, and (iii) the new LRF according to the 2D optimization scenario, are given in Table 5. These values show that our proposed strategy brings a significant reduction on the average magnitude and dispersion of the horizontal velocity field over the Hellenic area, with the 2D optimization scheme performing slightly better than the 3D optimization scheme.

Table 4. Estimated rotation-rate parameters between ETRF 2000 and the new LRF for each velocity optimization scenario. Units in mas/yr.

Rotation rate	using 3D velocity optimization	using 2D velocity optimization
$\dot{r}_x$	-3.264	-3.265
$\dot{r}_y$	-0.982	-0.983
$\dot{r}_z$	-3.101	-3.103

Table 5. Statistics of the horizontal velocities in the test network with respect to the three different reference frames. Units in mm/yr.

Reference Frame	mean	std	max	min
ETRF 2000	15.2	10.5	30.4	2.6
Optimal LRF (using 2D velocities)	8.2	3.2	13.3	4.4
Optimal LRF (using 3D velocities)	9.1	3.4	14.1	4.0

4 Conclusions

An optimal strategy for the realization of a local reference frame (e.g. over a particular country) has

been presented in this paper. The rationale of the proposed methodology is based on the minimization of the average magnitude of the LRF velocity field, a property that can be quite useful for the maintenance of national geodetic networks (and the dissemination of their up-to-date positional information to the users) in areas with strong and inhomogeneous geodynamical behavior such as Greece. The numerical results of our tests over most of the Hellenic area indicated that the horizontal velocity field in the optimized LRF will not exceed, on average, the 1 cm/yr level, and it will be significantly 'weaker' than the ETRF 2000 velocity field in terms of its associated rms value (see Table 5). Obviously, additional tests in a denser national GPS network are required in order to study in more detail the feasibility of our approach and its realistic performance in terms of the behavior of the transformed velocity vector ($\mathbf{v}^{LRF}$).

Acknowledgments

Dr. A. Ganas from the National Observatory of Athens (NOA) is acknowledged for kindly providing the GPS data from the NOANET network that were used in this study.

References

Altamimi Z., Sillard P., Boucher C. (2002) ITRF 2000: A new release of the International Terrestrial Reference Frame for Earth science applications, Journal of Geophysical Research, vol. 107, No. B10, pp. 2214 -2233.

Altamimi Z. (2004) Towards a Dense European Velocity Field. Report on the Symposium of the IAG Subcommission for Europe (EUREF), No. 13, Verlag des Bundesamtes fur Kartographieund Geodasie, Frankfurt am Main.

Boucher C., Altamimi Z. (2008) Memo: Specifications for reference frame fixing in the analysis of a EUREF GPS campaign. Available at http://etrs89.ensg.ign.fr/memo-V7.pdf.

Bruyninx C., Brockmann E., Schaer S. (2006) How to tie the EPN to ITRF 2005? Presented at EUREF TWG Meeting, November 6-8, Frankfurt, Germany.

Clarke P.J., Davies R.R., England P.C., Parsons B., Billiris H., Paradissis D., Veis G., Cross P.A., Denys P.H., Ashkenazi V., Bingley R., Kahle H-G., Muller M.V., Briole P. (1998) Crustal strain in central Greece from repeated GPS measurements in the interval 1989-1997, Geophysical Journal International, vol. 135, pp. 195-214.

Dach R., Hugentobler U., Fridez P., Meindl M. (2007) Bernese GPS Software Version 5.0, Astronomical Institute, University of Bern.

Gubler E., Poder K., Hornik H. (eds) (1992) Report on the Symposium of the IAG Subcommission European Reference Frame (EUREF), Florence, May 28-31, 1990. Verlag der Bayerischen Akademie der Wissenchaften, Munich, Germany.

Hollenstein C., Müller M.D. Geiger A., Kahle H-G. (2008) Crustal motion and deformation in Greece from a decade of GPS measurements, 1993–2003, Tectonophysics, vol. 449, pp. 17-40.

Legrand J., Altamimi Z., Olivier J. (2006) Interpolation of the European velocity field using least square collocation method. Presented at EUREF Symposium, Riga, Latvia, June 14-17, 2006.

Nyst M., Thatcher W. (2004), New constraints on the active tectonic deformation of the Aegean, Journal Geophysical Research, vol. 109, No. B11406, doi: 10.1029/2003JB002830.

Sillard P., Boucher C. (2001), Review of algebraic constraints in terrestrial reference frame datum definition, Journal of Geodesy, vol. 75, pp. 63–73.

Soler T., Marshall J. (2002) Rigorous transformation of variance-covariance matrices of GPS-derived coordinates and velocities. GPS Solutions, vol. 6, pp. 76-90.