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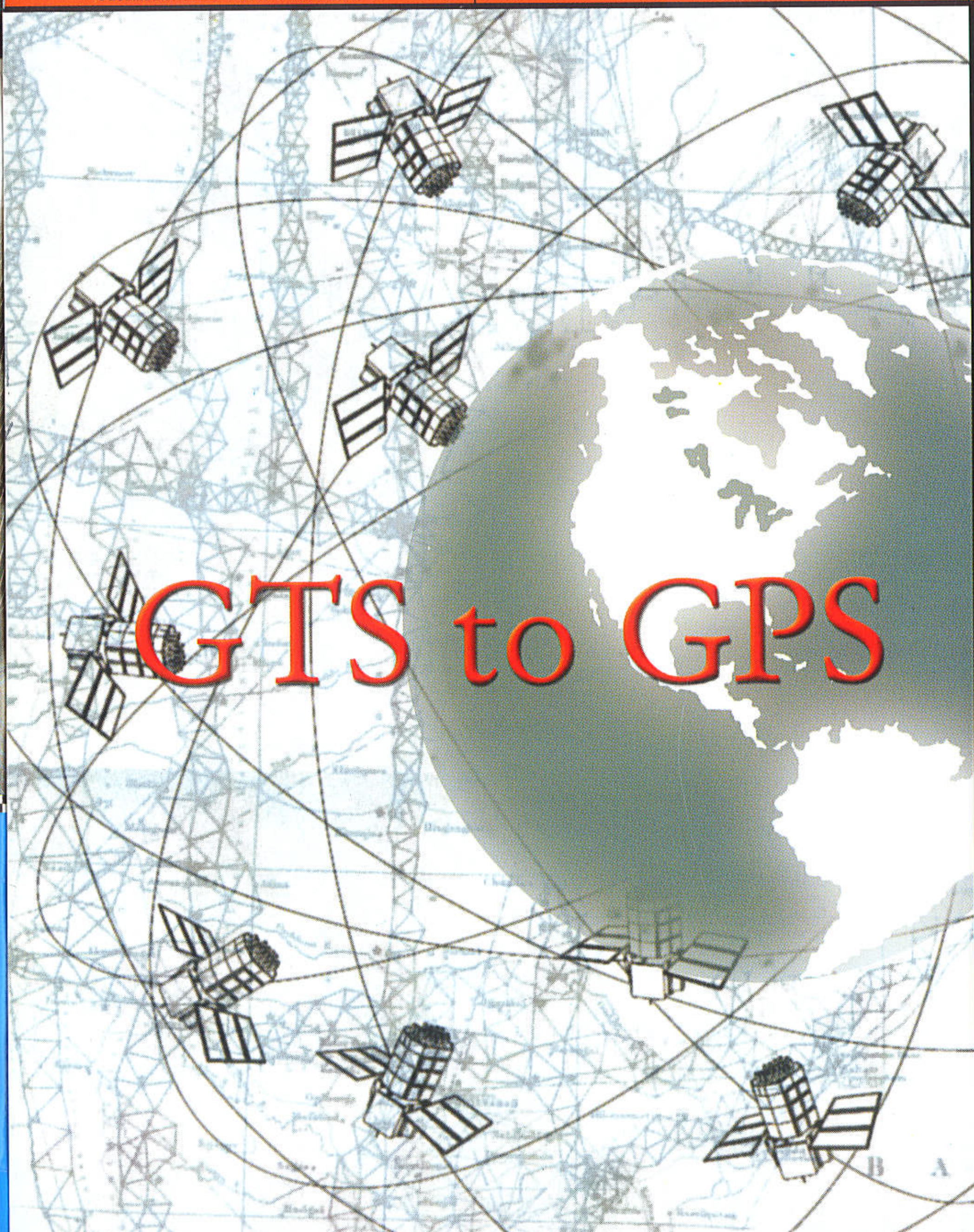

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Similarity transformations for GPS heighting

M N J P Vella | C Kotsakis

Orthometric heights traditionally are determined through optical methods involving the transfer of height difference from a datum point to the unknown point, where the orthometric height is required. This can sometimes be a very arduous task and now with the advent and proliferation of the use of the Global Positioning System, this task realistically seems more possible now than ever before

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WITH THE use of GPS and a regional gravimetric geoid model it is possible to transfer heights simply, as the following relationship demonstrates in an absolute sense:

$$H_A = h_A - N_A \quad (1)$$

or

$$h_A - H_A - N_A = 0$$

This relationship shows how the orthometric height H_A is related to the geometrical ellipsoidal height obtained from GPS measurements h_A and the physical geoid/ellipsoid separation N_A. The relationship, however, is not always appropriate due to the physical way in which GPS surveys are conducted. In general it is more suited to use the following relative case of Eq. (1).

$$\Delta H_{AB} = \Delta h_{AB} - \Delta N_{AB} \quad (2)$$

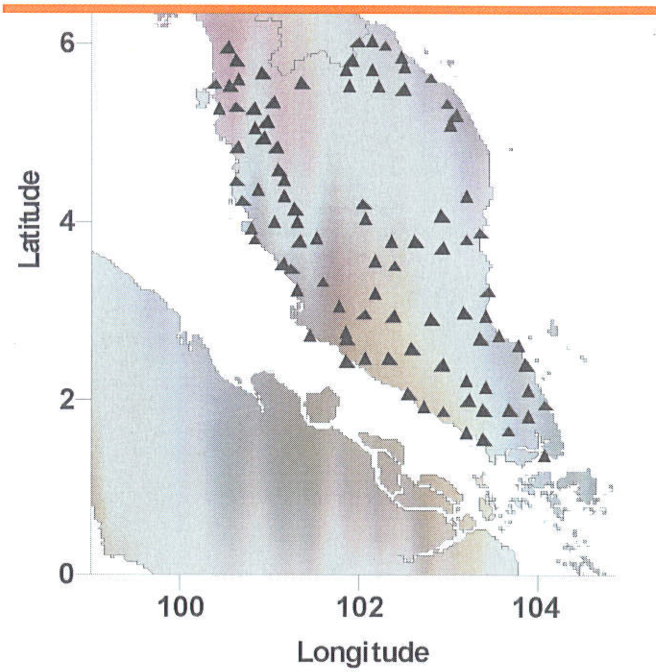
Eq. (2) shows the relationship with respect to relative differences for the orthometric heights H_A and H_B, the ellipsoid heights h_A and h_B and the physical geoid/ellipsoid separations N_A and N_B. This relationship allows differential GPS measurements to be used, which are known to be more precise, and in conjunction to this geoid models are now becoming more precise and are constantly pushing towards the 1cm level of accuracy. Having such accurate information, in the form of the geoid and ellipsoid heights enables geomatics engineers to apply this information in their respective fields thus realistically providing the opportunity for GPS to be used in GPS levelling and other applications.

Studies carried out on the geoid and GPS/Levelling, in different countries show that GPS and the geoid are now more than ever important tools [(Kotsakis and Sideris., 1999), (Mainville et al., 1997), (Zhong., 1997) and (Martensson, 2002)].

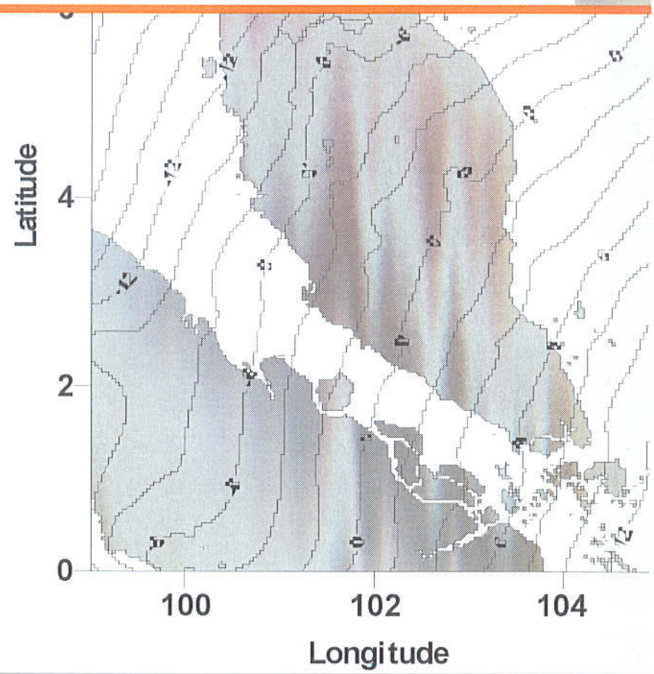
In Malaysia computing the geoid has been of prime interest in the past and geoid models have been computed for either the whole of peninsular Malaysia or a part thereof, (Vella., 2003). Peninsular Malaysia is a country traversed north and south by very rugged mountain ranges that have largely prevented access to the hinterland for conventional terrestrial gravity surveys. All previous attempts at computing the geoid in peninsular Malaysia have suffered from lack of data and non-homogeneity of the data distribution. However, the Department of Surveying and Mapping Malaysia (DSMM) has embarked upon a very ambitious project to collect new gravity data and update the existing database through the use and implementation of airborne gravity surveys. This provides the impetus for the current study in that the new data will provide new geoid models, which in turn will benefit from studies showing the most appropriate hybrid modelling technique to apply for modelling the bias between the vertical datum and the gravimetric geoid in peninsular Malaysia. Corrector surfaces need to be applied as the relationship in Eq. (1) is rarely satisfied. Reasons for this are described in (Kotsakis and Sideris., 1999).

EVALUATION OF CO-GEOID MODEL AND GPS DATA

DSMM provided data of peninsular Malaysia containing geodetic latitude, longitude, orthometric height and ellipsoidal height. The geoid heights were provided from an independent source namely (Vella., 2003) and EGM96 geopotential model coefficients by (Lemoine et al, 1997). Figure 1 shows the dis-



▲ Fig 1 Distribution of 95 GPS control stations, throughout Peninsular Malaysia



▲ Fig 2 Peninsular Malaysia Geoid 2003, co-geoid solution.

tribution of the GPS stations used in the comparison - a total of 95 in all.

Peninsular Malaysia Co-Geoid (PMG03)

Free-air gravity anomalies are usually found to be locally correlated with the elevation of observing points. The correlation between the observed height of the free-air anomaly and its corresponding value is derived using least squares. It is shown that a minimum height of 400 m yields the best correlation (R=0.886) between the free-air anomaly and the height and therefore this height of 400 m is used as a minimum height to interpolate data from the GTOPO30 global Digital Elevation Model (DEM). From the DEM all heights with a height equal to or greater than 400m are used to derive anomaly-height correlated values.

When these values are combined with the original gravity dataset and gridded, the gridding is better controlled, as results over peninsular Malaysia show. Using the 1D FFT and stokes integral with no modification the co-geoid is computed (Vella, 2003). The resulting free-air co-geoid model, (see Figure 2) is compared to 95 GPS points, giving a bias of 37.7 cm and a standard deviation of ±27.7 cm when compared at the GPS points, showing an improvement over EGM96. For ease of reference this co-geoid model will be referred to as PMG03 (Peninsular Malaysia Geoid 2003).

EGM96 Comparisons

The NASA Goddard Space Flight Center, the National Imagery and Mapping Agency (now called NGA) and the Ohio State University (OSU) have collaborated to produce EGM96, an improved 360° spherical harmonic model representing the earth's gravitational potential (Lemoine et al, 1997). The comparisons between the EGM96 model and the difference between the orthometric height and ellipsoidal height show that the mean bias is 46 cm with a standard deviation of ±33.5 cm at 95 GPS points.

SIMILARITY TRANSFORMATIONS

The basic model used is of a modified form of Eq. (1) which is as follows:

$$h_i - H_i - N_i = c_i^T x + v_i \qquad (3)$$

where h_i, H_i, and N_i are as previously described, x is an n x 1 vector of unknown parameters, a_i is an n x 1 vector of known coefficients, and v_i is the residual random noise term, (see e.g. Kotsakis and Sideris 1999). As stated, the introduction tests are conducted on three similarity transformation schemes and four polynomial schemes. All schemes are solved using parametric least squares techniques according to Eq. (3), but each with differing observation equations for obvious reasons.

It has been widely held that the four parameter model is best suited to this type of modelling. This, however, is not necessarily the case as the results will show. The four parameter (Eq. 4) model - from now on called CS4 - is an approximate similarity transformation model describing the geoid undulation transformation. This scheme is adequately discussed in Heiskanen and Moritz (1967, Sect. 5-9).

$$c_i^T x = \cos\varphi_i \cos\lambda_i \, x_1 + \cos\varphi_i \sin\lambda_i \, x_2 + \sin\varphi_i \, x_3 + x_4 \qquad (4)$$

The eight parameter scheme (CS8) (Kotsakis, 2002), which is a rigorous non-rigid similarity transformation model, is described as follows:

$$c_i^T x = \cos\varphi_i \cos\lambda_i \, x_1 + \cos\varphi_i \sin\lambda_i \, x_2 + \sin\varphi_i \, x_3 + \frac{\sin\varphi_i \cos\varphi_i \sin\lambda_i}{W_i} x_4 + \frac{\sin\varphi_i \cos\varphi_i \cos\lambda_i}{W_i} x_5 + (a W_i + h_i) x_6 + \frac{1 - f^2 \sin^2\varphi_i}{W_i} x_7 + \frac{\sin^2\varphi_i}{W_i} x_8 \qquad (5)$$

where the quantity W_i is given by the relationship:

$$W_i = \sqrt{1 - e^2 \sin^2 \varphi_i} \qquad (6)$$

and the quantities f, a and e in the above formulas correspond to the flattening, the semi-major axis and the first eccentricity, respectively of the reference ellipsoid (either the ellipsoid used for the GPS heights, or the ellipsoid used for the gravimetric geoid model).

TESTS CARRIED OUT ON CONTROL DATA

Before any schemes were tested all the GPS points were screened to make sure they all fell within three standard deviations of the mean of differences between NEGM96 derived using EGM96 and N_{h-H} derived using h and H. This was a quick and dirty data snooping technique and since there were no outliers there was no statistical reasoning to exclude any points although when compared the residual can be large. Table 1 shows the statistics of the comparisons from the residuals concerned with the above discussion. The larger of the residuals are confined to the east coast, since these residuals satisfy the standards set they are not excluded from the comparisons. Better data in the adjustment will give better-adjusted coefficients. However, there is the risk of excluding perfectly good data. For this study we have included all data. It is evident from Figure 3 that the largest residuals [(h - H) - N_{EGM96}] are along the east coast. This would suggest that either EGM96 or the vertical datum along the east coast is poorly defined, or it could just show there are problems in the adjustment of the vertical datum, this however is not investigated here. This could also be indicative of large discrepancies between the vertical datum (which is fixed at zero on the west coast) and sea surface topography SST.

COMPARISONS OF HYBRID MODELLING

Two types of comparisons are made: (1) All 95 GPS data points are used in the adjustment for the 4 parameter model. (2) All 95 GPS data points are used in the adjustment for the 8 parameter model.

RESULTS AND CONCLUSION

Comparisons are carried out using the following for original misclosures and adjusted residuals respectively in Eq (7) and Eq (8).

$$h_i - H_i - N_i = l_i \qquad (7)$$

and

$$V_i = l_i - C_i^T x \qquad (8)$$

In the following tables 'No CS' refers to 'no correction surface' is applied and the original misclosure statistics are listed using (Eq 7), where as for the correction surfaces Eq (8) is being employed and the statistics all represent those of the residuals from the adjustment. Table 2 shows the statistics for

Fig 3 Location of GPS points (95) showing residual contour plot [(h - H) - N_{EGM96}], contour interval 10cm

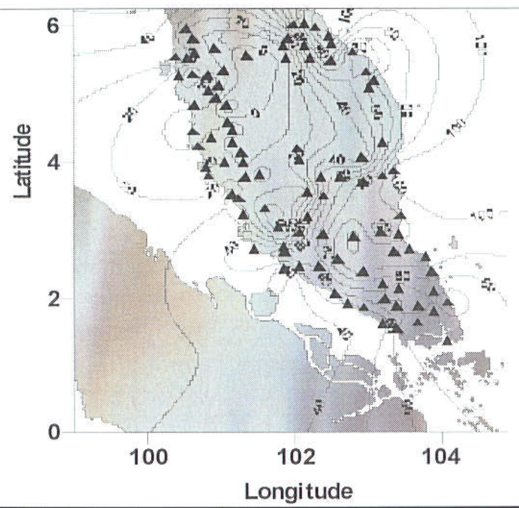
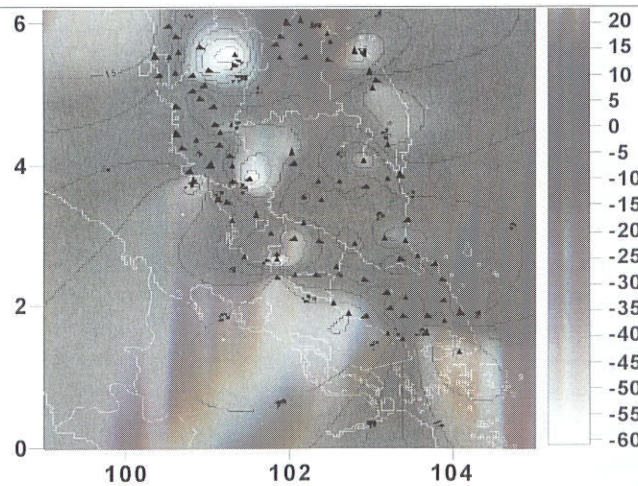


Fig 4 Distribution of 95 GPS points with the residuals from the CS8 comparison to GPS



PMG03 and the results of the adjustments. Figure 4 shows the national distribution of 95 GPS points. It is clearly seen that the hinterland is largely uncovered by any kind of geodetic information, be it gravity, GPS or levelling. When tests were carried out for the whole 95 GPS points, distributed nationally throughout peninsular Malaysia, it was evident as shown in the first column (No CS) of Table 2, that the fit of PMG03 to the national dataset is not too good. For the remaining columns a mean of 0 indicates that both the 4 and 8 parameter schemes are capable of removing any bias, however, the large standard deviations indicate there are still discrepancies between the three components, *h*, *H* and *N*. These discrepancies are most definitely due to the inadequacy of the co-geoid solution due to the lack of gravity coverage. Also as previously mentioned subsidence of levelling benchmarks and sea surface topography may also be taken into account. These errors tend to manifest in smaller areas and are assumed to be geographically correlated. One way to test this would be to increase the number of data points in these areas, however the likely outcome would still be similar due to the physical problems existing between the datums as previously mentioned. The geoid solution contributes to the overall success of the modelling as any deficiencies in the geoid will eventually show up in the modelling of the bias between the datums.

It could be argued that the GPS stations in the north-west and west could be eliminated, thus giving much more favourable results. However, this would then hide the discrepancies in the geoid solution. Hybrid modelling provides a method of improving the fit between the geoid model and the local vertical datum, at the same time providing, in general, external quality indicators of the geoid models ability to recover the geoid signal, which in this case is very important as the co-geoid model used was itself computed with a deficient gravity database. Due to the datum inconsistency between the co-geoid model PMG03 and the peninsular Malaysia vertical datum a new hybrid geoid surface has been computed. The different techniques used demonstrate the ability of each method to model the bias between the two datums. The transformation models used tried to compute parameters to transfer one surface to the other and tried not to fit the surfaces through elimination and smoothing of any residuals. It would be considered haste on the part of the Author to declare one method as being better than the other. It can be said the idea of using corrector surfaces is a safer way of eliminating the bias that might exist between different surfaces. This method does not smooth residuals and therefore shows which areas geographically might have potential problems with the data used as a check such as the GPS data used in the north-west of peninsular Malaysia. Therefore, by using corrector techniques it is possible to study the inadequacies in all the datasets used. The technique can be used as a method to derive a hybrid corrector surface for geoid. ■

TABLE 1: STATISTICS OF THE RESIDUALS FOR THE COMPARISON OF THE OUTLIER TEST CONDUCTED BETWEEN NEGM96 AND (H-H) IE: [(H - H) - NEGM96]. (ALL UNITS ARE IN CM).

No. Stns	95
Max	113.4
Min	-20.6
Mean	46.0
Stnd Dev ±	33.5

TABLE 2: STATISTICAL SUMMARY USING PMG03 FOR THE HYBRID MODELLING AT 95 GPS POINTS.

	No CS	CS4	CS8
# pts	95	95	95
Max	90.1	54.4	61.9
Min	-90.7	-115.6	-62.8
Mean	37.7	0.0	0.0
Std±	27.7	21.5	18.9

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