BGG

Since 1949

Bollettino di Geodesia e Scienze Affini
Bulletin of Geodesy and Geomatics

Heights and scale variation in their inherent reference frames

CHISTOPHER KOTSAKIS

ABSTRACT OF «*BOLLETTINO DI GEODESIA E SCIENZE AFFINI*
JOURNAL OF ISTITUTO GEOGRAFICO MILITARE
VOLUME LXIX - N. 1 - JANUARY/APRIL 2010

VOLUME LXIX - BOLLETTINO DI GEODESIA E SCIENZE AFFINI - N. 1, 2010

Heights and scale variation in their inherent reference frames

CHRISTOPHER KOTSAKIS
Department of Geodesy and Surveying, Aristotle University of Thessaloniki, Greece

Summary. - The scope of this paper is to explain the role of the spatial scale component in vertical geodetic positioning and physical height determination, and to identify its importance as a conventional datum-transformation parameter that could be used for the evaluation and connection of different realizations of a vertical reference system (VRS). The paper tackles a number of theoretical and practical aspects related to the notion of VRS scale, and it concludes with a list of relevant open questions that need to be answered and agreed upon by the geodetic community.

1. - INTRODUCTION

A *vertical reference system* (VRS) corresponds to a one-dimensional (1D) terrestrial coordinate system that is likely associated with an additional time dependency for its inherent vertical coordinates. From a geodetic perspective, the role of a VRS is to provide a coherent framework for vertical positioning that is embedded in the 3D Euclidean space and linked with one, or more, 2D reference surfaces with respect to which absolute heights (with certain geometrical and/or physical characteristics) can be defined and quantified over the Earth's solid and fluid parts.

A modern geodetic VRS should be tied to both principal facets of Earth's spatial representation: the geometric one in terms of 3D spatial positions (x, y, z) with respect to a crust-fixed Cartesian coordinate system that is supported by a conventional Earth reference ellipsoid, and the physical one based on the gravity potential signal $W(x, y, z)$ and its various equipotential surfaces generated by the terrestrial gravity field. Its primary realization, that is a *vertical reference frame* (VRF), consists of the estimated geopotential numbers at control points with known 3D spatial positions, and it requires the consistent synergy of its associated geometrical and physical components. Space geodetic measurements in conjunction with a high-resolution global geopotential model or a geoid/quasi-geoid model, and terrestrial leveling measurements along with local gravity data, provide the main types of height-like observables that may be used, independently or jointly, for determining VRS coordinates (essentially gravity potential differences) over regional, continental or even global networks. Other important data sources, such as tide gauge records, absolute gravity measurements, altimetric data and sea surface topography models, can contribute additional information that should be consistently integrated within a VRS realization, particularly for applications related to sea level monitoring and vertical crustal deformation.

The aforementioned concept of a vertical reference system has been extensively studied in the geodetic literature for its theoretical and practical aspects (e.g. Heck 2004, Hipkin 2003, Rummel and Teunissen 1988, Ihde et al. 2007). Moreover, the synergetic geometrical/physical approach for VRS definition and realization has been endorsed by several scientific organizations (IAG, FIG, IHO) and their special study groups (Ihde 2007a), and it provides a standard framework for modernization efforts in regional and continental vertical datums (e.g. EVRS, SIRGAS), as well as for the future development of a unified global VRS and a related world height system in support of the International Terrestrial Reference System (ITRS) and its various realizations (ITRFs); see Ihde and Sanchez (2005), Ihde (2007b).

Regardless of its spatial coverage, a geodetic VRS relies on the specification of two essential datum parameters, namely the *origin* and the *scale* for the corresponding vertical (1D) coordinates. The stipulation of these VRS-related datum parameters (and their temporal evolution in case that time-variable phenomena need to be taken into account) is a key issue for the establishment of modern cm/mm-level height systems in support of various geodetic applications and other interdisciplinary studies in Earth sciences.

Given that the primary vertical coordinate at any terrestrial point P, with respect to a conventional geodetic VRS, corresponds to its geopotential number:

$$c(P) = W_0 - W(P), \tag{1}$$

the notion of the VRS origin is identical with a reference equipotential surface $W(\cdot) = W_0$ of Earth's gravity field, commonly called the «geoid». Since all types of physical heights used in geodesy are linearly scaled versions of $c(P)$, the VRS origin serves as a uniform zero-height level regardless of the specific type of physical heights that will be used within the vertical reference system. Indeed, the geopotential number of any point located on the VRS origin is, by definition, zero and thus the magnitude of all physical heights will vanish there; although the actual 3D geometrical interpretation for some height types is rather cumbersome through the involvement of auxiliary reference surfaces and vertical mapping schemes (see Molodensky's normal heights). The realization of the VRS origin and its temporal evolution (e.g. conventional choice or empirical determination of $W_0/\dot{W}_0$), as well as the connection of vertical datums with different origins, are important topics that have been thoroughly investigated by geodesists over the last decades (e.g. Colombo 1980, Rummel and Teunissen 1988, Xu 1992, Ardalan et al. 2002, Burša et al. 2002) and almost all related issues have been well recognized and, to a large extent, resolved.

On the other hand, the notion of the VRS scale has not been sufficiently treated in the geodetic literature, despite some recent efforts to introduce a consistent set of VRS conventions and «heighting» procedures in a harmonized way with the IERS guidelines underlying the ITRS/ITRF framework (Ihde 2007a, b). The inclusion of a scale component in the definition and, more importantly, in the realization of a vertical reference framework is a critical issue in view of the heterogeneous observables and their combination strategies that need to be employed in modern geodetic practice of height-related positioning applications. Each data type that can contribute to VRS realization inherently defines its own spatial «metric» (at a certain accuracy level) which is dictated by its underlying observation technique and other related modeling assumptions. As a result, the physical heights (either absolute or relative) obtained from different methodologies and/or data sets are likely affected by spatial scale inconsistencies that should be properly taken into account for their joint analysis within a VRS framework, in the same spirit that alternative ITRS realizations from different space geodetic techniques and/or network adjustment procedures are typically validated and connected through a 3D similarity transformation model.

The aim of this paper is to identify the importance of the spatial scale component in vertical positioning and physical height determination, and to promote its role as a conventional datum-transformation parameter that should be used for the formal evaluation and connection of different VRFs. The paper tackles a number of theoretical and practical aspects related to the notion of VRS scale, and it concludes with a list of relevant open questions that need to be answered and agreed upon by the geodetic community.

2. – VRS SCALE

The scale of a geodetic VRS can be regarded as a metric property of the Earth's space along a privileged spatial direction that is used for height (and height velocity) determination. From a practical viewpoint, one may approach this concept more pragmatically by adopting a factor of spatial (vertical) scale change as one of the conventional parameters for height transformation between different realizations of the same or different vertical reference systems.

Presumably, the geopotential numbers and the equivalent physical heights obtained from different geodetic techniques and/or data sets exhibit a discrepancy in their realized spatial scale due to hidden systematic effects in their respective measurement procedures and data modeling assumptions. The appraisal of such scale inconsistencies not only contributes to the identification of problematic issues in heighting methodologies, but it could also support the evaluation of the uniformity between different VRFs on the basis of their spatial (vertical) scale variation.

Let us try to elucidate some of these problems by considering the usual constraint among orthometric (H), ellipsoidal (h) and geoid (N) heights

$$h - H - N = 0 \tag{2}$$

or, in relative form

$$\Delta h - \Delta H - \Delta N = 0 \tag{3}$$

Both of these constraints are theoretically based not only on the «origin consistency» for the various data sets (i.e. H and N should refer to the same equipotential surface of Earth's gravity field, while h and N must use a common geodetic terrestrial reference system (TRS) with a conventional Earth reference ellipsoid), but they further require that all three height types refer to the same metric scale.

However, in practice, the gravimetric and the geometric geoid heights do not conform to a unified spatial scale realization. Equivalently, the orthometric heights obtained through spirit leveling and certain theoretical hypotheses about the Earth's local crustal density and terrain roughness (e.g. Prey's reduction) do not rigorously match the spatial scale of orthometric heights derived from GPS measurements and a global geopotential model. The previous relationships will thus hold at an accuracy level that is dictated not only by the random errors in the available data, but also from their inherent scale inconsistencies. In fact, the latter represent nothing more than the contribution of various systematic errors and other modeling problems in the original data, whose combined effect is describable by a required re-scaling for (some of) the height types that appear in eqs. (2) and (3). To the author's knowledge, such scaling corrections have never been practically implemented for the combined analysis and least-squares adjustment of heterogeneous height data, and the inference of their reciprocal vertical scale variations.

Two basic aspects can be identified for the scale analysis of height-related quantities within a geodetic VRS, namely:

– the minimization of vertical scale differences among the individual height components (e.g. h, H, N or Δh, ΔH, ΔN) forming a single integrated VRF over a network of fiducial stations;

– the assessment of vertical scale differences between VRFs obtained from different techniques, data sets, estimation methodologies, modeling assumptions, time epochs, etc.

The first of the above aspects is related to the internal consistency of the spatial scale that is realized from the individual (geometric and physical) components of a modern vertical reference framework. Although no standard methodology currently exists for this task, a reasonable strategy should include a «fine-tuning» of the geopotential model that accompanies a geodetic VRS, in order to minimize any height scaling errors in eqs. (2) and (3) at a number of fundamental stations. On the other hand, the second aspect is concerned with the scale consistency between different heighting techniques that may be independently used for absolute and/or relative vertical positioning over a particular area, as well as for the regional densification of an existing vertical reference frame.

The treatment of the previous issues requires the adoption of a height transformation model which incorporates, among other parameters, a scale change factor between different or alternative VRS realizations. Such a transformation model (either in static or time-dependent form) is a key tool for the optimal combination of heterogeneous height data and the consistent connection of traditional vertical datums with modern GPS/geoid leveling networks.

3. – VRS ORIGIN VS. VRS SCALE

Often, in the relevant geodetic literature, the notion of the VRS scale is intrinsically linked with the W_0 value that is adopted for defining absolute vertical coordinates (i.e. geopotential numbers) and their equivalent physical heights at points on the Earth's surface. For example, in the drafted guidelines for the development of a Conventional International Vertical Reference System given in Ihde (2007a), the VRS scale is explicitly associated with «the reference level W_0 = const realized by the combination of a conventional mean sea surface model with a conventional global gravity model following the geoid definition formulated by Gauss-Listing».

The above perspective corresponds to a geodetically meaningful approach for the standardization of Earth's global «radial» scale in terms of gravity field information. Indeed, the ratio of W_0 with a mean terrestrial gravity value g, or alternatively with the geocentric gravitational constant GM, yields the average size of the reference surface used for physical height determination (Burša et al. 2002). Specifically, the following expressions:

$$R = \frac{W_0}{g} \quad \text{or} \quad R = \frac{GM}{W_0} \tag{4}$$

give, in a very good approximation, the mean radius of the geoid, which essentially defines a reference spatial scale for the geocentric position of terrestrial points with zero heights!

Obviously, any change of the VRS origin (W_0) creates a direct offset on the values of physical heights, which can be successively perceived as an apparent scaling effect due to the varying size of the reference equipotential surface with respect to a conventional representation $W(\cdot)$ of Earth's gravity field. Note that, theoretically, such an offset is not equivalent to a constant height bias since its magnitude depends on the actual gravity value on the geoid (in the case of orthometric heights), or on the normal gravity value on the reference ellipsoid (in the case of normal heights).

The previous viewpoint gives a rough geometrical interpretation for the physical height variation due to a change of the fundamental W_0 datum parameter. However, it should be emphasized that the radial scale of the zero-height level is conceptually different from the notion of VRS scale that was outlined in Sect. 2. In fact, a change of W_0 is related to a straightforward transformation from a conventional VRS origin to another one, whereas the scope of a VRS scale change is to account for the discrepancy of the spatial metric realized by different heighting techniques and/or datasets with respect to an arbitrarily fixed VRS origin.

In general, both types of vertical datum perturbation (*origin* and *scale*) are feasible and they can co-exist in the joint analysis of different VRS realizations. The estimation of the underlying datum perturbation parameters signifies an important task that can provide the basis for a geodetically meaningful comparison between different VRFs, in analogy with similar procedures that are currently applied in the evaluation of 3D spatial reference frames.

Lastly, let us mention that the apparent relationship between the origin and the scale of a *purely geometric* VRF (i.e. based solely on space geodetic measurements) has been discussed many years ago by Soler and van Gelder (1987). In their study, a general transformation procedure for geometric (ellipsoidal) heights with respect to a geodetic TRS and an associated Earth reference ellipsoid was presented, which explicitly considered the indirect effect of TRS scale change on the size of the accompanying reference ellipsoid! Note, however, that the current IERS conventions (McCarthy and Petit 2004) imply that the geometric/physical parameters of the Earth reference ellipsoid that supports any 3D Cartesian coordinate frame (ITRF, ETRF) should remain unaffected under a scale change of the underlying spatial coordinates. A similar conventional postulation or recommendation does not currently exist for the VRS origin parameter (W_0) in the context of physical height determination from global geopotential models and space geodetic measurements with respect to different ITRS realizations.

4. – DATUM TRANSFORMATION MODEL FOR PHYSICAL HEIGHTS

A conventional transformation model between different realizations of a vertical reference system is a necessary tool for geodetic studies on precise vertical positioning and terrestrial height determination. From a practical viewpoint, such a model is required to play a similar role to that of the linearized similarity transformation which is commonly employed in geometric positioning studies and the evaluation of 3D spatial reference frames.

A conventional transformation model for physical heights must incorporate the basic perturbation parameters of a geodetic vertical datum. More specifically, it has to account for the variation in the VRS *origin* and *scale* that are respectively realized by different heighting techniques and/or datasets over a number of common terrestrial stations. For the purpose of the present paper, let us restrict ourselves to the case of static (time-independent) orthometric heights, whose variations from a VRS realization (H) to another one (H') can be conventionally described by the following model:

$$H_i' = (1 + \delta s)\, H_i + \frac{W_0' - W_0}{g_i^0} \tag{5}$$

where g_i^0 denotes the geoidal gravity value at the projected location of the terrestrial benchmark P_i. The gravity potential terms W_0 and W_0' correspond to the equipotential reference surfaces that are respectively associated with the orthometric heights H_i and H_i', while the unitless factor δs refers to a differential change of the metric scale along the physical plumbline of Earth's gravity field.

Note that the second term in Eq. (5) corresponds to a linear approximation of the vertical separation between two equipotential surfaces of Earth's gravity field. For practical purposes, instead of the geoidal gravity value, we may alternatively use the normal gravity value determined through Somigliana's formula for the geodetic latitude of the terrestrial benchmark (P_i) with respect to an Earth reference ellipsoid. Such a simplification causes an error in Eq. (5) that is typically below the mm-level, and thus negligible for most geodetic purposes.

The previous height transformation model employs two conventional parameters, a geopotential offset $\delta W_0 = W_0' - W_0$ and a vertical scale-change coefficient δs, both of which can provide a formal assessment for the spatial consistency between different VRFs. Their estimation requires the inversion of eq. (5) via a least-squares adjustment of orthometric heights that have been determined through different techniques, modeling assumptions and/or data sets over a common group of terrestrial control stations. As with any datum transformation model, the estimated values of these parameters will inevitably absorb a part of hidden systematic errors within the height data that may not be strictly identified as «datum-related» disturbances (yet their presence inflicts an apparent variation in the realized VRS origin and/or scale). On the other hand, the remaining residuals after the least-squares adjustment of eq. (5) provide a valuable source for identifying possible systematic differences between the tested VRFs, which are not describable by the conventional VRS perturbation terms. The spatial analysis of such systematic differences is essential for a comprehensive comparison of the underlying VRFs, as well as for the development of augmented height transformation models of higher precision than the principal model of eq. (5).

A major concern regarding the inversion of eq. (5) is the likely strong correlation between the two height transformation parameters. Such an effect may obscure the significance of their estimated values and it could diminish their overall usefulness in VRF evaluation studies. Evidently, this is a general problem in most geodetic studies dealing with comparisons of spatial reference frames over terrestrial control networks with limited geographical coverage and/or poor spatial configuration. In the case of VRFs, the crucial aspect that controls the separability between the vertical scale change (δs) and the origin offset (δW_0) is the *height variability* in the corresponding vertical network. Due to the one-dimensional character of the problem at hand, the horizontal spatial distribution of the leveling benchmarks over the Earth's surface does not really affect the statistical correlation between the two height transformation parameters. An optimal VRF network geometry in this case is not related to a homogeneous global coverage, but rather to a large dispersion of the vertical coordinates (i.e. orthometric heights) over its control stations. Therefore, the least-squares inversion of eq. (5) can be effectively applied even in local vertical networks, provided that their height variability is high enough to distinguish the effects of the height-dependent scaling term and the (nearly constant) term of the VRS origin offset.

It needs to be emphasized that the scale-change factor in eq. (5) has a distinctively different meaning from the corresponding scale-change factor which is employed in the 3D similarity transformation between geometric TRFs. The latter describes a uniform scale variation for the 3D terrestrial position vector with respect to a geocentric, or almost geocentric, Cartesian reference frame, whereas the differential factor δs in eq. (5) reflects a vertical scale variation for physical heights with respect to an equipotential surface of Earth's gravity field. Since the establishment of vertical reference frames is presently less accurate by 1-2 orders of magnitude than the 3D spatial reference frames obtained by space geodetic techniques (Ihde and Sanchez 2005), it should be expected that the scale factor between different VRFs is significantly larger than the scale variation commonly encountered among the modern geometric TRFs.

5. – CONCLUSIONS

Several aspects concerning the role of spatial scale within a vertical reference framework have been discussed in this paper, in an attempt to 'streamline' the evaluation procedure for 1D VRFs with the conventional strategy that is typically followed for the comparison of 3D geometric TRFs. Our aim was to present some of the key issues involved in the above subject, rather than providing fixed answers for their treatment. Moreover, a number of additional scale-dependent issues related to VRS/VRF theory and practice have not been discussed here, yet they are included in the following list of open questions for further study:

– Should a spatial (vertical) scale-change factor be included as a conventional transformation parameter for the evaluation and connection of different VRS realizations?

– For the establishment of an integrated time-dependent VRF from heterogeneous multi-epoch geodetic data, what should be the conventional constraints for the temporal evolution of the VRS origin and scale?

– How should spatially inhomogeneous vertical scale changes due to localized geodynamic phenomena (e.g. Fennoscandian postglacial rebound) be handled in the context of a unified VRS?

– Are there scale-like systematic discrepancies in height velocities obtained from heterogeneous data sources (e.g. repeated leveling measurements, continuous or repeated GPS positioning, repeated absolute gravity measurements, geophysically derived models)? Could such vertical velocity differences be taken into account through a time-dependent version of the height transformation model given in eq. (5)?

– How do the inherent differences in the treatment of permanent tidal effects in various physical/geometrical VRS components affect its internal scale consistency?

The study of the above issues, among others, seems to be a pre-requisite before a unified global VRS and a world height system can be implemented for precise vertical positioning and Earth monitoring, in conjunction with the existing ITRS realizations and modern global geopotential models.

REFERENCES

A. Ardalan, E. Grafarend, J. Kakkuri (2002), *National Height Datum, the Gauss-Listing Geoid Level Value w_0 and its Time Variation w_0 (Baltic Sea Level Project: Epochs 1990.8, 1993.8, 1997.4)*. Journal of Geodesy, no. 76, pp. 1-28.

M. Burša, E. Groten, S. Kenyon, J. Kouba, K. Raděj, V. Vatrt, M. Vojtiškova (2002), *Earth dimension specified by geoidal geopotential*. Studia Geophysica et Geodaetica, no. 46, pp. 1-8.

O. Colombo (1980), *A World Vertical Datum*. OSU Report no. 196, Dept. of Geodetic Science, Ohio State University, Columbus, OH.

B. Heck (2004), *Problems in the definition of vertical reference frames*. IAG Symposia, vol. 127, Springer, Berlin, pp. 164-173.

R.G. Hipkin (2003), *Defining the geoid by : Theory and practice of a modern height system*. Proceedings of the 3rd meeting of the International Gravity and Geoid Commission, pp. 367-377, Thessaloniki, Greece, August 26-30, 2002.

J. Ihde, L. Sanchez (2005), *A unified global height reference system as a basis for IGGOS*. Journal of Geodynamics, no. 40, pp. 400-413.

J. Ihde, J. Makinen, M. Sacher (2007), *Conventions of the European Vertical Reference System*. Presented at the EUREF Annual Symposium, June 6-8, London, UK.

J. Ihde (2007a), *Conventions for the Definition and Realization of a Conventional Vertical Reference System*. Report of the IAG Inter-Commission Project (ICP 1.2) on Vertical Reference Frames.

J. Ihde (2007b), *Consideration of a Global Vertical Reference System (GVRS) in the IERS Conventions*. Presented at the IERS Workshop on Convention. September 20-21, Sevres, France.

D.D. McCarthy, G. Petit (2004), *IERS Conventions 2003*. International Earth Rotation and Reference Systems Service (IERS) Technical Note, no. 32, Verlag des Bundesamts für Kartographie und Geodäsie, Frankfurt am Main.

R. Rummel, P. Teunissen (1988), *Height datum definition, height datum connection and the role of the geodetic boundary value problem*. Bulletin Geodesique, no. 62, pp. 477-498.

T. Soler, B.H.W. van Gelder (1987), *On differential scale changes and the satellite Doppler system z-shift*. Geophysical Journal of the Royal Astronomical Society, 91 (3), pp. 639-656.

P. Xu (1992), *A quality investigation of global vertical datum connection*. Geophysical Journal International, no. 110, pp. 361-370.