

Let  $A = \{\mathbf{a}_i \mid 1 \leq i \leq m\}$  be a subset of  $\mathbb{Z}^n$  such that the semigroup generated by  $A$ ,  $\mathbb{N}A = \{\sum_{i=1}^m n_i \mathbf{a}_i \mid \mathbf{a}_i \in A, n_i \in \mathbb{N}\}$ , is an affine semigroup. An *affine semigroup*  $S$  is a finitely generated subsemigroup of  $\mathbb{Z}^n$  with no invertible elements, that means  $S \cap (-S) = \{0\}$ . To the set  $A$  we associate the *toric ideal*  $I_A$  which is the kernel of the  $K$ -algebra homomorphism

$$\phi : K[x_1, \dots, x_m] \rightarrow K[t_1, \dots, t_n, t_1^{-1}, \dots, t_n^{-1}]$$

given by

$$\phi(x_i) = \mathbf{t}^{\mathbf{a}_i} = t_1^{a_{i,1}} \dots t_n^{a_{i,n}} \text{ for all } i = 1, \dots, m,$$

where  $\mathbf{a}_i = (a_{i,1}, \dots, a_{i,n})$ . The set of zeroes of  $I_A$ ,  $V(I_A) \subset K^m$  is an *affine toric variety*. To the toric variety  $V(I_A)$  we associate a strongly convex rational polyhedral cone

$$\sigma = \text{pos}_{\mathbb{Q}}(A) := \{l_1 \mathbf{a}_1 + \dots + l_m \mathbf{a}_m \mid l_i \in \mathbb{Q}^+\} := \mathbb{Q}^+ A,$$

where  $\mathbb{Q}^+$  denotes the set of nonnegative rationals.

We will discuss the interaction between algebraic properties of the toric ideal  $I_A$  and geometric properties of the cone  $\sigma = \text{pos}_{\mathbb{Q}}(A)$ .