Lie and Super Lie Hopf algebras

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Notions discussed:

- LIE ALGEBRAS: antisymmetry, Jacobi identity, Lie subalgebras, Lie ideals, Lie homomorphisms
- LIE FUNCTOR $\mathbb{L}$: k-ALGEBRAS$\rightarrow$ k-LIE ALGEBRAS, $gl(V) := \mathbb{L}(End_k(V))$.
- $U(g)$: THE UNIVERSAL ENVELOPING ALGEBRA OF g and the Poincaré -Birkhoff-Witt theorem for the monomial basis of $U(g)$.
- THE FUNCTOR $\mathbb{U}$: k-LIE ALGEBRAS$\rightarrow$ k-ALGEBRAS, and it's adjointness with the functor $\mathbb{L}$.
- REPRESENTATIONS OF THE LIE ALGEBRA g.
  - Each representation $T : g \rightarrow gl(V)$ gives a g-module structure to V (Due to the universality property of $U(g)$, V becomes automatically a $U(g)$-module also).
  - If T_1 and T_2 are two representations of g with carrier space V, such that any two linear transformations $(T_1)(l)$, $(T_2)(m)$ mutually commute, then T_1+T_2 is also a representation (If l is an element of the Lie algebra g, the linear transformation $(T_1+T_2)(l)$ is defined to be the superposition of the (commuting) linear transformations $(T_1)(l)$, $(T_2)(l)$ i.e: $(T_1 + T_2)(l) = (T_1)(l) + (T_2)(l)$).
  - There is no analogue of the latter fact in the associative algebra case.
  - Any two representations $T_V : g \rightarrow gl(V)$ and $T_W : g \rightarrow gl(W)$ give rise to a new representation $g \rightarrow gl(V\otimes W)$ where $l \mapsto T_V(l)\otimes 1_w + 1_V \otimes T_W(l)$
- THE HOPF ALGEBRA STRUCTURE OF $U(l)$ (comultiplication $\Delta(l) = l \otimes 1 + 1 \otimes l$ and antipode $S(l) = -l$.)
- Γ-GRADED STRUCTURES where $\Gamma = \mathbb{Z}$ or $\mathbb{Z}_2$.

References

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