Symmetric and asymmetric librations in planetary and satellite systems at the 2/1 resonance

George Voyatzis and John D. Hadjidemetriou
Department of Physics, University of Thessaloniki
54124 Thessaloniki, Greece,
e–mail: voyatzis@auth.gr, hadjidem@auth.gr

Abstract. Families of asymmetric periodic orbits, at the 2/1 resonance are computed for different mass ratios. The existence of the asymmetric families depends on the ratio of the planetary (or satellite) masses. As models we used the Io-Europa system of the satellites of Jupiter for the case $m_1 > m_2$, the system HD82943 for the new masses, for the case $m_1 = m_2$ and the same system HD82943 for the values of the masses $m_1 < m_2$ given in previous work. In the case $m_1 \geq m_2$ there is a family of asymmetric orbits that bifurcates from a family of symmetric periodic orbits, but there exist also an asymmetric family that is independent of the symmetric families. In the case $m_1 < m_2$ all the asymmetric families are independent from the symmetric families. In many cases the asymmetry, as measured by $\varpi_2 - \varpi_1$ and by the mean anomaly M of the outer planet when the inner planet is at perihelion, is very large. The stability of these asymmetric families has been studied and it is found that there exist large regions in phase space where we have stable asymmetric librations. It is also shown that the asymmetry is a stabilizing factor. A shift from asymmetry to symmetry, other elements being the same, may destabilize the system.

1. Introduction

The study of extrasolar planetary systems is a new field of research in dynamical astronomy, following the discovery of planetary systems around distant stars. A complete catalogue of extrasolar planetary systems can be found in the web site *http://www.obspm.fr/encycl/catalog.html* maintained by Jean Schneider. There are 136 confirmed extrasolar planetary systems at the time of writing this paper, with 14 of them having two or more planets. In some planetary systems the two planets do not come close to each other. In such a case the ratio of the planetary periods is not important for the stability of the system. There are however planetary systems, many of them with large eccentricities, where the two planetary orbits are close to each other, and may even intersect, but evidently they are stable. Many of these systems are close to a *mean motion resonance* and, as we shall see in the following, this provides a phase protection mechanism, which results to the avoidance of close encounters and thus the system is stable. We remark also that *periodic motion* of the planetary system, in the sense that the *relative* configuration is repeated in space, implies exact mean motion resonance.

There are several studies on the dynamics of extrasolar planetary systems. One field of research is to study the stability of a particular extrasolar system, considering all possible configurations at a certain resonance (Ford *et al.*, 2001; Gozdziewski *et al.*, 2002; Kinoshita and Nakai, 2001a,b; Kiseleva *et al.*, 2002; Laughlin and Chambers, 2001; Lissauer and Rivera, 2001; Malhotra, 2002a,b; Murray *et al.*, 2001; Peale and Lee, 2002; Rivera and Lissauer, 2001; Ferraz-Mello *et al.* 2005). A different field of research refers to the question of how the observed extrasolar planetary systems evolved to their present form. It is generally believed that the observed planetary systems, many of them having large eccentricities, were not formed in their present form, but were trapped in the present form following a migration process. Lee (2004) studied the migration of a planetary system, starting with circular planetary orbits, and considering planet-disk interaction, forcing the outer planet to migrate at a rate of the form $\dot{a}_2/a_2 \propto T_2^{-1}$, where a_2 is the semimajor axis and T_2 the period of the outer planet. A similar study is made by Beaugé *et al.* (2003). The migration process was also studied by Ferraz-Mello *et al.* (2003) who considered the action of anti-dissipative tidal forces and integrated the exact equations of motion. In all these cases it was shown that the system is trapped to a resonant configuration, which, as we shall show in this paper, is close to a symmetric or asymmetric *periodic* orbit of the planetary system, in a rotating frame. This shows the importance of the periodic orbits in the study of the dynamics of a planetary system.

A third field of research is to study the dynamics of a resonant planetary system, as it is at its present configuration. The planetary system is considered as a dynamical system and the topology of its phase space is studied, with the aim to find what are the regions where stable motion can be found, or what are the regions where the motion is chaotic and consequently no planetary system could exist. As it will be made clear in the following, it is the exact periodic orbits that determine the topology of the phase space. The study of periodic orbits has been made by Beaugé *et al.* (2003) for the 2/1 and 3/1 resonance, by finding the fixed points of an averaged Hamiltonian. These fixed points correspond to symmetric and asymmetric periodic orbits in a rotating frame. A similar study was made by Hadjidemetriou (2002), Hadjidemetriou and Psychoyos (2003), Psychoyos and Hadjidemetriou (2005a,b). They found families of symmetric resonant periodic orbits for several resonant cases.

In many cases, the motion of a planetary system in resonance is *symmetric* with respect to the line of apsides : The lines of apsides of the two planetary orbits coincide and the perihelia are either aligned, $\Delta\varpi = 0°$, or antialigned, $\Delta\varpi = 180°$, where $\Delta\varpi = \varpi_2 - \varpi_1$. Moreover, there exists a moment that when one planet is at perihelion or aphelion, the other planet is, at that moment, also in perihelion or aphelion.

Apart from the symmetric resonant periodic orbits, there exist resonant configurations where the planetary orbits are *asymmetric*, which means that the angle $\Delta\varpi$ of the lines of apsides of the two planets is not equal to 0° or 180°. Together with this *geometric asymmetry*, we have in this case a *dynamic asymmetry*, in the sense that when one planet is at perihelion (or aphelion) the second planet, at that moment, is in a position different from perihelion or aphelion.

In the present study we propose a new approach to detect stable planetary motion. We do not focus our attention to a particular planetary system (although we shall apply the results to specific extrasolar systems), but we present the main properties of the phase space of a planetary system close to resonance, for three different mass ratios: $m_1/m_2 < 1$, $m_1/m_2 = 1$ and $m_1/m_2 > 1$. The 2/1 resonance will be analyzed here, but this method of work can be extended to all other main resonances. A systematic method is presented, based on *periodic orbits*, to find regions of the phase space where stable motion exists and consequently a real planetary system could be found in nature. A study on the dynamics, based on families of periodic orbits, was made in Hadjidemetriou (2002), Hadjidemetriou and Psychoyos (2003) and Psychoyos and Hadjidemetriou (2005a), for the 2/1, 3/2 and 5/2 resonant systems. In these papers, all the periodic orbits are symmetric with respect to the common line of apsides of the planetary orbits. In the present paper we extend the study of the dynamics at the 2/1 resonance, by including asymmetric resonant (periodic) motion. This means that the lines of apsides of the two planetary orbits are no longer aligned or antialigned.

Although the planetary masses are in most cases small, compared to the mass of the star, the gravitational interaction between the two planets cannot be neglected, and in some cases it even dominates. For this reason, we used the model of the *general three-body problem.* The exact equations of motion were used in the computations.

It is known that the evolution of any dynamical system in general, and of a planetary system in particular, depends on the topology of its phase space. We remark at this point that the topology of the phase space is shaped by the position and the stability character of the periodic orbits (or, equivalently, the fixed points of the Poincaré map on a surface of section). Since around a stable periodic orbit there exists an island where we have stable librations, the knowledge of the position of the stable periodic orbits provides important information

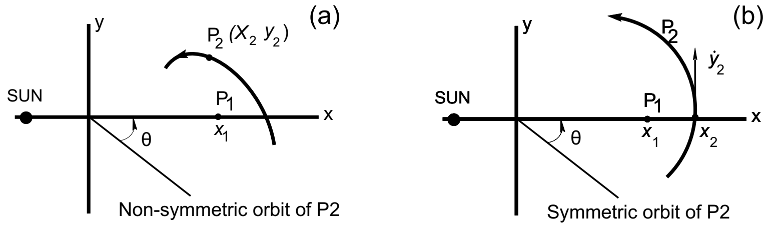

Figure 1. The rotating frame xOy. (a) asymmetric orbit and (b) Symmetric orbit.

on where an extrasolar planetary system could be found. In fact, in this way, we obtain a chart of the whole phase space which shows all the possible positions where a 2/1 resonant planetary system could exist. These are the configurations to which a planetary system could be trapped, if in the past it had followed a migration process (Beaugé *et al.* 2003; Ferraz-Mello *et al.*, 2003; Lee and Peale 2003; Lee, 2004). This study is also useful, because there are uncertainties in the orbital elements of the observed planetary systems, and we can at least know if these elements could, in principle, correspond to a stable system.

The above remarks make clear the importance of the periodic orbits in the study of the dynamics of a planetary system. Although periodic orbits form a subset of measure zero in the complete set of all possible motions, they are the "framework" of the phase space.

In what follows we shall call the main attracting body the *sun* and the two small bodies the *planets*, $\mathcal{P}_1$, $\mathcal{P}_2$, with the index 1 referring to the inner planet and the index 2 to the outer planet. In the present study only planar motion will be considered, so we shall refer to the angle between the lines of apsides of $\mathcal{P}_1$ and $\mathcal{P}_2$ as $\Delta\varpi = \varpi_2 - \varpi_1$.

2. Periodic orbits of the planetary problem

2.1. General remarks

As we mentioned above, the dynamical model that we use in the study of a planetary system is the general three-body problem. The center of mass of the planetary system is considered as fixed in an inertial frame, and the study is made in a *non-uniformly rotating* frame of reference xOy, whose x-axis is the line *sun* - $\mathcal{P}_1$, the origin O is the center of mass of these two bodies and the y-axis is perpendicular to the x-axis (figure 1a). In this rotating frame $\mathcal{P}_1$ moves on the x-axis and $\mathcal{P}_2$ in the xOy plane. The total mass and the gravitational constant are normalized to unit.

The coordinates are the position x_1 of $\mathcal{P}_1$, the position x_2, y_2 of $\mathcal{P}_2$ and the angle θ between the x-axis and a fixed direction in the inertial frame. The coordinates x_1, x_2, y_2, define the position of the system in the rotating frame and the angle θ defines the orientation of the rotating frame. This is a system of four degrees of freedom, but it turns out that the angle θ is ignorable, and consequently the angular momentum integral exists, $L = \partial\mathcal{L}/\partial\dot{\theta}$=constant, where $\mathcal{L}$ is the Lagrangian of the system. So the study is reduced to a system of three degrees of freedom, in the rotating frame only, and the angular momentum L is a fixed parameter (Hadjidemetriou, 1975).

Monoparametric families of periodic orbits exist in this rotating frame (Hadjidemetriou, 1976). The periodic orbits are either *symmetric*, with respect to the rotating x-axis, or *asymmetric*. In a symmetric periodic orbit of period T the periodicity conditions are

$$\dot{x}_1(0) = \dot{x}_1(T/2) = 0, \quad \dot{x}_2(0) = \dot{x}_2(T/2) = 0, \quad y_2(0) = y_2(T/2) = 0.$$

The symmetry implies that $\Delta\varpi$ is equal to 0 or π and there exists a moment that when one planet is at perihelion (or aphelion) the other planet is also at perihelion or aphelion.

For an asymmetric periodic orbit the periodicity conditions are

$$x_1(T) = x_1(0), \;\; \dot{x}_1(T) = \dot{x}_1(0) \neq 0,$$

$$x_2(T) = x_2(0), \;\; \dot{x}_2(T) = \dot{x}_2(0), \;\; \dot{y}_2(0) = \dot{y}_2(T),$$

provided that $y_2(0) = y_2(T) = 0$ and T is the period. For these initial conditions the two planets are in conjunction. The above initial conditions imply that the planet $\mathcal{P}_2$ starts from the x-axis (non-perpendicularly) and the planet $\mathcal{P}_1$ is not at rest on the x-axis, and after a time $t = T$, when $\mathcal{P}_2$ crosses again the x-axis, the planets $\mathcal{P}_1$,$\mathcal{P}_2$ have the same initial position and velocity as at $t = 0$.

An equivalent set of initial conditions for an asymmetric periodic orbit is

$$x_1(T) = x_1(0), \;\; x_2(T) = x_2(0), \;\; y_2(T) = y_2(0) \neq 0,$$

$$\dot{x}_2(T) = \dot{x}_2(0), \;\; \dot{y}_2(0) = \dot{y}_2(T),$$

provided that $\dot{x}_1(T) = \dot{x}_1(0) = 0$. This implies that we start, at $t = 0$, at the moment when $\mathcal{P}_1$ has zero velocity on the x-axis, which means that $\mathcal{P}_1$ is either at perihelion or at aphelion, ($\mathcal{P}_2$ is not on the x-axis), and after a time $t = T$, when $\mathcal{P}_1$ has again zero velocity on the x-axis, the planets $\mathcal{P}_1$,$\mathcal{P}_2$ have the same initial position and velocity as at $t = 0$.

The first or the second set of periodicity conditions is solved using differential approximations. In practice, this is realized by a Newton-Raphson shooting algorithm. We performed the integration of the differential equations of motion of the planetary system in the inertial frame (where the center of mass is fixed) and the reduction to three degrees of freedom, in the rotating frame, was made by a coordinate transformation. The method of integration was based on Taylor series expansion and/or on a control step Bulirsch-Stoer algorithm with accuracy 10^{-14}. In many cases the convergence of the algorithm to a periodic solution with a prescribed accuracy depends on the particular choice of the set of initial conditions.

Note that periodicity in the rotating frame means that the *relative* configuration of the three bodies is repeated after one period. The system *is not*, in general, periodic in the inertial frame. The orbits of the two planets (nearly Keplerian ellipses) in the inertial frame precess with a small angular velocity, which depends on the particular periodic orbit.

2.2. Circular or elliptic, resonant, planetary orbits

In the planetary problem, with small planetary masses, the orbits of the two planets are nearly Keplerian, due to their weak gravitational interaction, although in some resonant cases the deviations from Keplerian motion are important.

There are two types of periodic orbits, *circular* and *resonant elliptic*. The unperturbed motion with zero masses and *zero* eccentricities of both planets is periodic in the rotating frame for *any* value of the radii. If we switch on the masses, this periodic orbit is continued as a periodic orbit in the rotating frame, defined in figure 1. These are the *circular* orbits mentioned above. The continuation is possible for all values of the ratio of the semimajor axes (radii), except for those values corresponding to the resonances 2/1, 3/2, 4/3,..., where gaps appear (e.g. see figure 2b)

The unperturbed motion with zero masses and *nonzero* eccentricities of the two planets is periodic, for all possible orientations of the apsidal lines of the two planetary Keplerian orbits, provided that they are in mean motion resonance. If we switch on the masses, then out of the infinite set of resonant periodic orbits, only a finite number survives according to the Poincaré-Birkhoff theorem. The orbits that survive (usually one stable and one unstable, see Hadjidemetriou (2005)) may be symmetric with respect to the rotating x-axis (this is the most common case), but they can also be asymmetric. The symmetric 2/1, 3/2,.. resonant families "bifurcate" from the circular family, at the point of

Table I. All possible symmetric phases at $t = 0$ and $t = T/2$ for the 2/1 resonance

Type 1:	$\mathcal{P}_2$(ap)	-	sun	-	$\mathcal{P}_1$(per)	$\rightarrow$	sun	-	$\mathcal{P}_1$(per)	-	$\mathcal{P}_2$(per)
Type 2:	$\mathcal{P}_2$(per)	-	sun	-	$\mathcal{P}_1$(ap)	$\rightarrow$	sun	-	$\mathcal{P}_1$(ap)	-	$\mathcal{P}_2$(ap)
Type 3:	$\mathcal{P}_2$(per)	-	sun	-	$\mathcal{P}_1$(per)	$\rightarrow$	sun	-	$\mathcal{P}_1$(per)	-	$\mathcal{P}_2$(ap)
Type 4:	$\mathcal{P}_2$(ap)	-	sun	-	$\mathcal{P}_1$(ap)	$\rightarrow$	sun	-	$\mathcal{P}_1$(ap)	-	$\mathcal{P}_2$(per)

the gap mentioned before (see figure 2b). We do have however families of symmetric periodic orbits that do not bifurcate from the circular family. The existence of these families depends on the ratio m_1/m_2 of the planetary masses, and the family may disappear if this ratio is changed. The bifurcations of asymmetric families will be discussed in details through the following sections.

We remark that the system of differential equations that describe the motion of the three bodies in the rotating frame are invariant under the transformation

$$x_1 \rightarrow x_1,\ \ x_2 \rightarrow x_2,\ \ y_2 \rightarrow -y_2,\ \ t \rightarrow -t.$$

This means that to each asymmetric periodic orbit there exists also its mirror image. Consequently, the families of asymmetric periodic orbits are always in equivalent pairs. Evidently, the semimajor axis and the eccentricity of the corresponding periodic orbits along these two equivalent families are identical while the mean (or true) anomalies and the arguments of pericenter are of opposite sign.

3. Families of periodic orbits for different mass ratios

3.1. The different symmetric configurations

For any resonant symmetric periodic orbit there exist eight different configurations, for all possible combinations: perihelia of the two planets in the same direction or in opposite directions and position of each planet at perihelion or at aphelion. These eight configurations are not independent from each other, but are equivalent in pairs, depending on the particular resonance. By "equivalence" we mean that if we start from a certain configuration at $t = 0$, we come to the equivalent configuration at $t = T/2$. It is simple geometry to verify that for the 2/1 resonance we have the equivalent pairs shown in Table 1. *Type 1* and *type 2* correspond to alignment of perihelia ($\varpi_2 = \varpi_1$), and *type 3* and *type 4* to antialignment of perihelia, ($\varpi_2 = \varpi_1 + \pi$).

3.2. Families of periodic orbits at the 2/1 resonance: Basic properties

We present here some basic properties of the families of periodic orbits at the 2/1 resonance. Such families have been computed by Psychoyos and Hadjidemetriou (2005a), for the masses of HD 82943, Gliese 876, HD 160691 and also for the inverse masses of HD 82943. Beaugé et al. (2003) determined 2/1 resonant periodic orbits as equilibrium points of an averaged Hamiltonian and for several mass ratios. There is a good agreement between his results and the results of the present paper, obtained by integrating the exact equations of motion. There exist two resonant families of symmetric periodic orbits at the 2/1 resonance, *family 1*, which starts as *type 3* and ends as *type 1*, and *family 2*, corresponding to *type 4*. These families bifurcate from the circular non-resonant family, at the points where a gap develops at this resonance. When $m_1 \leq m_2$ there is also a third family, *family 3*, which starts as *type 2* and ends as *type 3*. The type of periodic orbits changes when the eccentricity of $\mathcal{P}_1$ crosses zero, while the eccentricity of $\mathcal{P}_2$ stays always at high values. The *family 3* is independent of the circular families. For $m_1/m_2 > 1$ this third family disappears. The mass ratio m_1/m_2 plays an important role for the stability. For $m_1/m_2 < 0.97$ the whole *family 1* is stable, but when $m_1/m_2 > 0.97$, an unstable region appears on this family. The unstable region increases as the ratio m_1/m_2 increases.

In the case $m_1/m_2 > 0.97$, where an unstable region appears on *family 1*, we have a bifurcation of a new family of periodic orbits, from the critical points at both ends of this unstable region. As we shall see in the following, these two families are families of *asymmetric* periodic orbits, at the 2/1 resonance, and in fact coincide to a single family: This asymmetric family starts from one end of the unstable region and ends to the other end. In addition to this asymmetric family, there exist also families of asymmetric periodic orbits, that are independent of the families of symmetric periodic orbits.

In the case $m_1/m_2 < 0.97$ there is not any unstable region on *family 1*, so the asymmetric family that we have for $m_1/m_2 > 0.97$ no longer exists (In Beauge et al. (2003) is also indicated that the unstable region survives for m_2 slightly larger than m_1). There are however families of asymmetric periodic orbits in this case, which are independent of the families of symmetric periodic orbits.

In sections 5-7 we present families of *asymmetric periodic orbits* for three different cases: $m_1/m_2 > 1$, $m_1/m_2 = 1$ and $m_1/m_2 < 1$. For the first case we studied the system Io-Europa, and for the second and third case the HD 82942 extrasolar planetary system. These are typical cases, and the results are the same for all other systems, with different values of m_1 and m_2. What is important is the ratio of the masses and not their absolute values. Lee (2004) has extended the study of resonant motion to values of m_1/m_2 from 0.1 up to 10, much larger than the values used in the present study.

4. Families of symmetric periodic orbits for $\mathbf{m_1} > \mathbf{m_2}$. Jupiter-Io-Europa mass ratio

We start with the system *Io-Europa* and we present families of resonant 2/1 periodic orbits, in the rotating frame, for the masses

$$m_1 = 4.684 \times 10^{-5} m_0 \ , \quad m_2 = 2.523 \times 10^{-5} m_0.$$

These are the masses corresponding to the satellites *Io* and *Europa* of Jupiter, where m_0 is the mass of Jupiter. The eccentricities are very small, $e_1 = 0.004$, $e_2 = 0.009$. A study of planetary or satellite systems with these masses was made by Ferraz-Mello et.al (2003). The results of this section are applicable also to all planetary systems where $m_1 > m_2$.

In the present section we start with the symmetric families and in section 5 we continue with the asymmetric families.

4.1. Io-Europa system: symmetric periodic orbits

In our computations that follow, we used the value of angular momentum equal to $L = 0.000066700$, for all the orbits of all the families. All the periodic orbits that we present in this section are symmetric with respect to the x-axis of the rotating frame. This means that a family can be presented as a continuous curve in the space of initial conditions $x_1(0)$, $x_2(0)$ and $\dot{y}_2(0)$. In order to make more evident the role played by the elements of the orbit, we present the families of periodic orbits in the eccentricity space $e_1\, e_2$ also.

We found two families of 2/1 resonant periodic orbits, *family 1* and *family 2*. These families are shown in figure 2a (projection on the $x_1\, x_2$ plane). To show better the gap on the circular family at the 2/1 resonance, we present in figure 2b a detail of figure 2a close to the gap. The 2/1 gap is clearly seen. *Family 1* starts as a family of *type 3* (antialigned perihelia, $\Delta\varpi = \pi$, see table I). This holds up to $e_1 \approx 0.097$ when $e_2 = 0$. For greater values of e_1 *family 1* corresponds to the phase of *type 1*, which means that the lines of apsides are aligned, $\Delta\varpi = 0$. This can be easily explained geometrically: The eccentricity e_2 of P_2 at first increases, while the planet is at aphelion, and then decreases, passing from zero value and then increases again. At the transition of e_2 from the zero value we have a shift from aphelion to

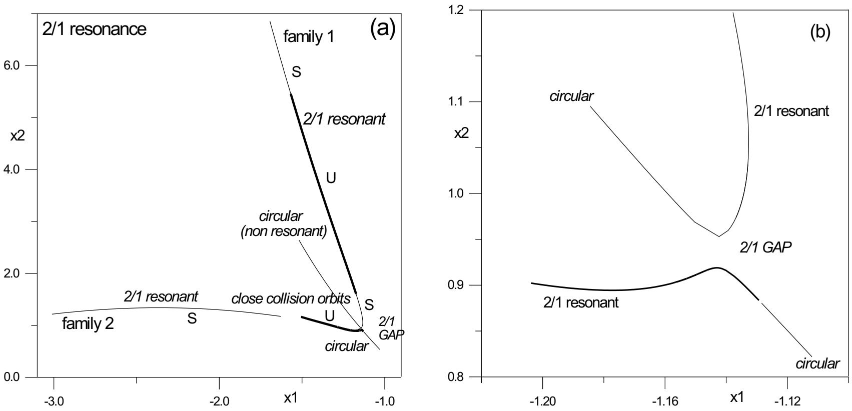

Figure 2. (a) 2/1 resonant families. Projection on the x_1x_2 plane of initial conditions. The thick line denotes the unstable part of the family. (b) Detail of the gap at the 2/1 resonance.

perihelion. *Family 2* corresponds to the phase of *type 4*, which means that the lines of apsides are antialigned, $\Delta\varpi = \pi$.

In figure 3a, we present the families 1 and 2 in the space of the eccentricities e_1 and e_2 of the two planets. We used the convention $e_i > 0$ for position of the corresponding planet at aphelion and $e_i < 0$ for position at perihelion. The linear stability along the two families is indicated. A thin curve indicates stability and a thick curve indicates instability. An unstable region appears on the *family 1*, because it is $m_1/m_2 > 0.97$, as we mentioned in section 3.2. The *family 2* starts as unstable, but after the collision area, the family is stable. Note that in this stable segment of the *family 2* the planetary eccentricities are larger than the eccentricities in the unstable part of the family.

Along a 2/1 resonant family, the ratio n_1/n_2, where n_i denotes the the mean motion of the planet $\mathcal{P}_i$ $(i = 1, 2)$, is almost equal to 2, but the eccentricities of the planets increase, starting from zero values. Along the *family 2* a collision orbit appears, at the eccentricities $e_1 = 0.12$ and $e_2 = 0.27$. At this region there is a gap along the *family 2*, as shown in figures 2a, 3a. We note that along the *family 1* the eccentricity e_1 of $\mathcal{P}_1$ is larger than the eccentricity e_2 of $\mathcal{P}_2$. The opposite is true along the *family 2*.

In Figure 3b we present a small section of the *family 1*, close to the gap, which corresponds to the *type 3* antialignment configuration (lower left section, $e_1 < 0$, $e_2 < 0$) and also the position of the Io-Europa system, for all possible configurations. The real Io-Europa system is at the *type 3* antialignment configuration and we note that its position is not exactly on the *family 1* of periodic orbits. This is due to the fact

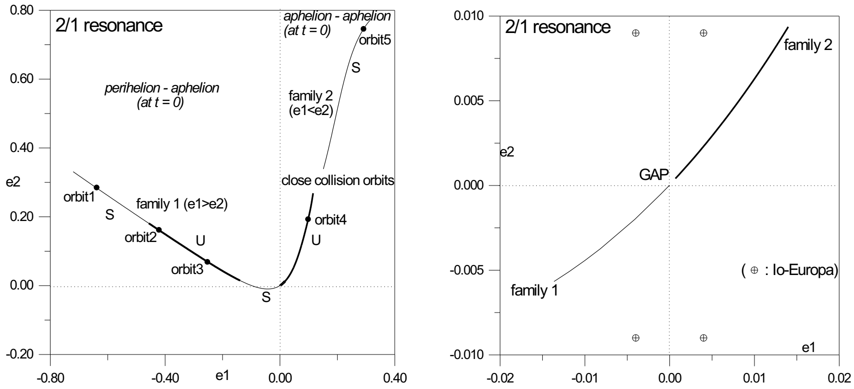

Figure 3. (a) The 2/1 resonant families of periodic orbits of the figure 2, in the eccentricity space. The orbits 1-5 of the figures 4 and 5 are indicated on the families. (b) Detail close to the gap. The position of Io-Europa, ($e_1 = 0.004$, $e_2 = 0.009$), for the four different symmetric configurations, is also indicated.

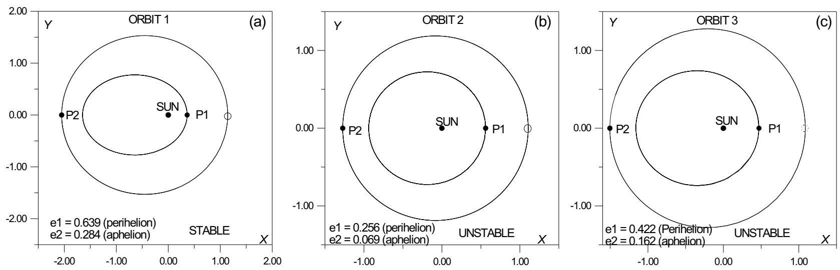

Figure 4. The orbits 1,2 and 3 along the *family 1*, in the inertial frame. The position of the two planets at $t = 0$ (full circles) is indicated. The position of planet 2 at $t = T/2$ is indicated as empty circle (the position of planet 1 is the same as at $t = 0$).

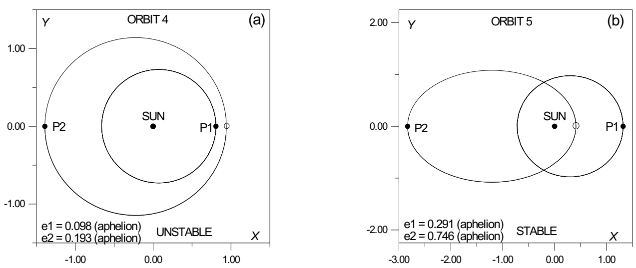

Figure 5. The orbits 4 and 5 of the *family 2* in the inertial frame as in figure 4. The orbit 4, panel (a), is close to a collision orbit.

that the Io-Europa system is a subsystem to the Io-Europa-Ganymede system, which is in the Laplace resonance 1:2:4. To take into account the effect of Ganymede on the Io-Europa system, one must consider periodic orbits or the general four-body problem, for the masses of the Sun-Io-Europa-Ganymede system, at the Laplace resonance. This has been done by Hadjidemetriou and Michalodimitrakis (1981), who computed four families of periodic orbits at the exact Laplace resonance 1:2:4. All these families are resonant families along which the eccentricities increase, starting with very small values. One of these families contains a periodic orbit which is very close to the real Io-Europa-Ganymede system, i.e. it has the correct configuration and the correct eccentricities (Figure 2 of the above paper). It is worth noting that this family is the only stable family among the above mentioned four families.

To have a better understanding of the families of resonant periodic orbits mentioned above, we show in figures 4 and 5 five typical periodic orbits. Their position on the two families is shown in figure 3a. The orbits are presented in the inertial frame, for a short time interval, of the order of the period. (For a longer time, the orbits would precess). The position of the planets at $t = 0$ and $t = T/2$ is shown on their corresponding orbits in the figures 4, 5. Note that in the periodic orbits of the *family 2*, after the collision orbit, the planetary orbits intersect, but due to a phase protection mechanism at this phase, because of the resonance, the planets do not come close to each other (figure 5b) and the system is stable. A 2/1 resonant stable orbit close to the system HD82943 where the two planetary orbits intersect, is given by Ji *et.al* (2004) and also by Hadjidemetriou and Psychoyos (2003). We remark that this refers to the old fit given by Israelinian et al. (2001).

4.2. Stability of the symmetric orbits

The stability of the periodic orbits along the families was investigated by computing the linear stability. The linear stability analysis showed that a large part of the *family 1*, where $e_1 > e_2$, is unstable, and the stable orbits correspond either to very small or to relatively large planetary eccentricities. On the *family 2* a collision orbit appears. The *family 2* is unstable from the beginning (zero eccentricities) up to the collision orbit. After the collision, where the eccentricities become quite large, the orbits are stable (figure 2a and 3). In this latter case, the planetary orbits intersect, but the motion is stable, because of the phase protection mechanism mentioned above. The nonlinear stability of the symmetric periodic orbits has been studied by Hadjidemetriou and Psychoyos (2003), and Psychoyos and Hadjidemetriou (2005a) and we shall not repeat it here.

5. Families of asymmetric periodic orbits, $\mathbf{m_1 > m_2}$. Jupiter-Io-Europa mass ratio

5.1. The asymmetric family A_1, which bifurcates from the symmetric *family 1*

We present in this section families of asymmetric periodic orbits for the masses of the Io-Europa system, given in section 4. It is known that if in a family of periodic orbits there exists a critical point, as far as the stability is concerned, (a point where we have a transition from stability to instability, or vice versa, in this family), then a new family of periodic orbits bifurcates from this point. In section 4.1 we found that along the *family 1* of symmetric periodic orbits there exists an unstable region (figure 3a), and consequently, from each of the two critical points at the two ends of this unstable region we have a bifurcation of a family of periodic orbits. These families are also resonant, at the 2/1 resonance. It turned out that the two families that bifurcate from the above two critical points are families of *asymmetric* periodic orbits. These two families meet and form one single family. This single family of asymmetric periodic orbits starts from one critical point, B_{11} and ends to the other critical point, B_{12}, as it is shown in figure 6, in the space $e_1\, e_2$. We call this asymmetric family, family A_1. In this figure the eccentricities are considered in all cases positive. The two symmetric families, *family 1* and *family 2*, are also shown (with dashed lines).

The family A_1 is linearly stable and the corresponding orbits of the planets $\mathcal{P}_1$ and $\mathcal{P}_2$ in the inertial frame do not intersect. The family A_1 has been found also by Beaugé *et al.* (2003), as a family of fixed points, and by Ferraz-Mello *et al.* (2003) and Lee (2004), as a capture domain of a migration process.

The family A_1 bifurcates from the *family 1* of symmetric periodic orbits, along which the lines of apsides are aligned, $\Delta\varpi = \varpi_2 - \varpi_1 = 0$ (for $e_1 > 0.097$). As a consequence, the angle $\Delta\varpi$ starts with zero value on the family A_1 and ends with zero value also, at the other end, but along the family $\Delta\varpi$ increases up to $104°$, as shown in figure 7a. This angle is a measure of the *geometric asymmetry* of a periodic orbit. The *dynamic asymmetry* along the family A_1 is measured by the value of the mean anomaly M of the second planet $\mathcal{P}_2$ when the first planet $\mathcal{P}_1$ is at perihelion (i.e. $M = M_2$ when $M_1 = 0$), and is given in figure 7c. We remark that since the orbits are 2/1 resonant, i.e. during one

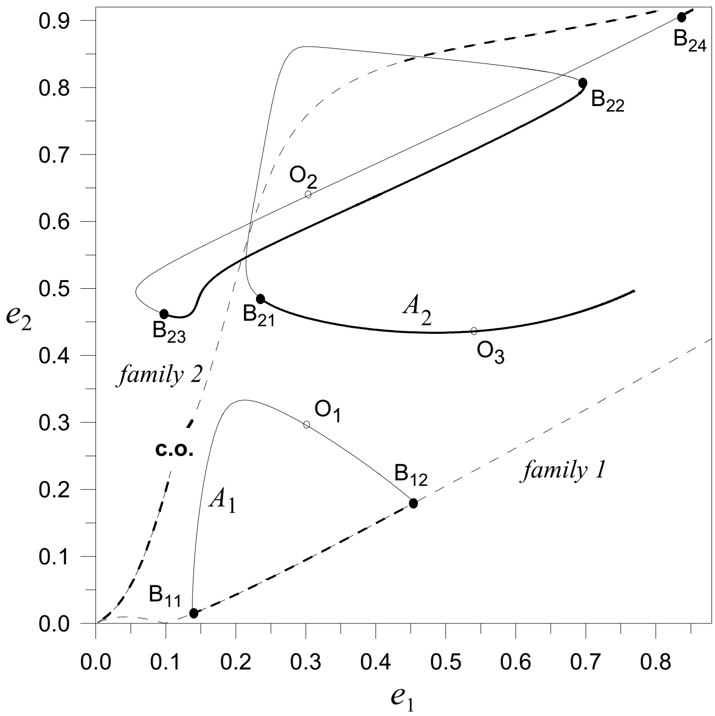

Figure 6. Io-Europa system. The 2/1 resonant families of periodic orbits in the eccentricity space. The symmetric families 1,2 of the figure 3a are shown by dashed lines and the asymmetric ones by solid lines. All eccentricities are now considered positive. Stability is indicated by thin line and instability by thick line. Two asymmetric families, family A_1 and family A_2, are shown. The position of the three orbits O_1 on A_1 and O_2, O_3 on A_2, which are discussed in the text, is indicated. The notation "c.o." denotes a collision orbit.

period the planet $\mathcal{P}_1$ passes twice from its pericenter, there exist two values of M, which differ by about $180°$. In the corresponding figures of the paper only one value of M is presented.

A typical asymmetric periodic orbit on this family, particularly the orbit O_1 indicated in figures 6 and 7a,c, is shown in figure 8 (first column). It is given both in the rotating frame, where it is exactly periodic, and in the inertial frame. In the latter case, the orbit is given for a time interval of one period, because the system is not periodic in the inertial frame and the planetary orbits precess slowly. Note that the lines of apsides of the two planetary orbits are not aligned.

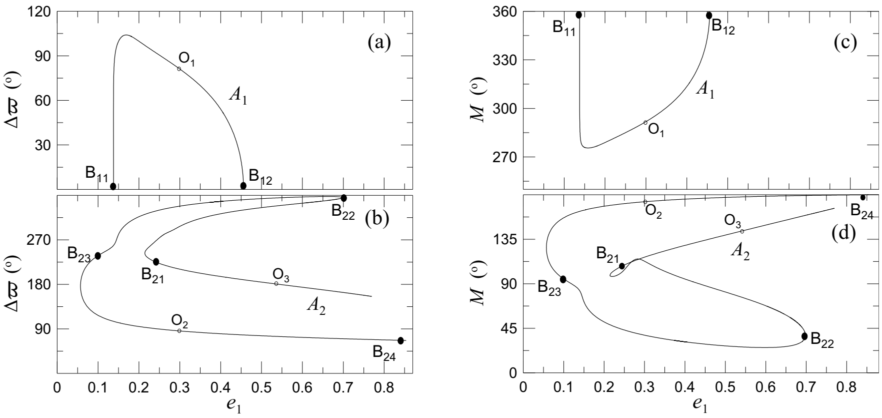

Figure 7. Io-Europa system. Left column : The variation of the angle $\Delta\varpi = \varpi_2 - \varpi_1$ along the asymmetric families A_1 (panel (a)) and A_2 (panel (b)). Right column: The variation of the mean anomaly M of planet $\mathcal{P}_2$ when planet $\mathcal{P}_1$ is at perihelion, along the asymmetric families A_1 (panel (c)) and A_2 (panel (d)). The points B_{ij} are the critical points where the stability type changes.

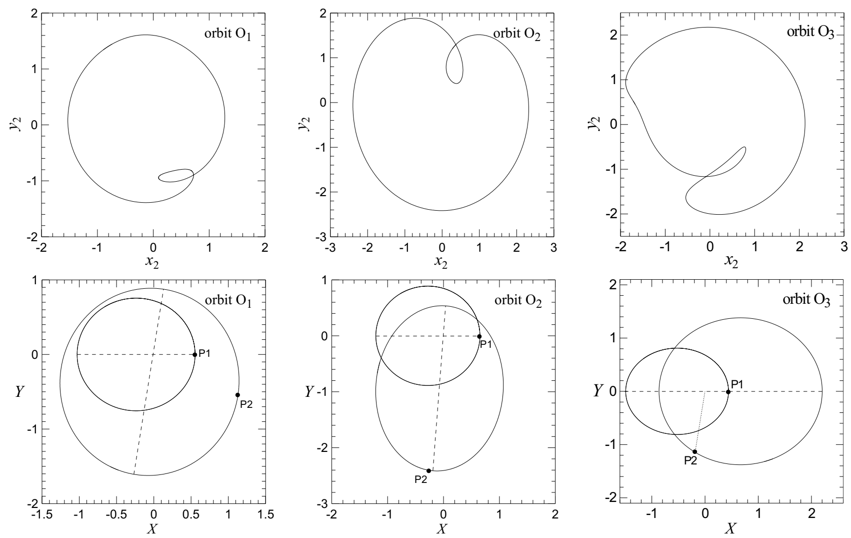

Figure 8. Typical asymmetric periodic orbits along the families A_1 and A_2. The position of these orbits on the corresponding families is indicated in figures 6 and 7. In the top panels the orbits are in the rotating frame and in the bottom panels they are in the inertial frame, for one period. P_1 and P_2 denote the initial position of the planets and the line of apsides is indicated by a dashed line. The orbit O_3 is an example of an asymmetric periodic orbit where we have geometric symmetry, but dynamic asymmetry.

5.2. The asymmetric family A_2

In addition to the asymmetric family A_1, there exists also one more family of asymmetric periodic orbits, family A_2, which is independent of the symmetric families mentioned in section 4.

The graph of the family A_2 in the space $e_1\,e_2$, as shown in figure 6, is quite complicated. Neither of the generalized variables of the system nor any of the orbital elements of the orbits vary monotonically along the family. There are two stable regions on this family, and the critical points, where we have a change of the stability, are denoted by the points B_{2i} $(i = 1, .., 4)$. Along the family A_2 the corresponding orbits of the planets $\mathcal{P}_1$ and $\mathcal{P}_2$ intersect in the inertial frame except for the family segment $0.5 < e_1 < 0.7$ located between the critical points B_{22} and B_{23}.

The variation of the angle $\Delta\varpi$, which is a measure of the geometric asymmetry, is also quite complicated, as shown in figure 7b. $\Delta\varpi$ does not vary monotonically along the family and takes values in a wide range ($66^\circ < \Delta\varpi < 360^\circ$). The dynamic asymmetry along the family A_2 is measured by the value of the mean anomaly M, defined in section 5.1, and is given in figure 7d. In figure 8 (second and third column) we present two typical asymmetric periodic orbits, orbit O_2 and orbit O_3, both in the rotating and the inertial frame. Note that in orbit O_3 it is $\Delta\varpi = 180^\circ$, but M is not equal to 0° or 180°. This means that we have geometric symmetry, but dynamic asymmetry. Note also that along the family A_2 the value of M tends to 180° as $e_1 \rightarrow 1$.

The approximation, followed by Lee (2004) indicates the existence of family $A2$. It has been shown that the stable segment from B_{23} to B_{24} can be reached by differential migration for $m_1/m_2 \gtrsim 2.75$. For more details on the migration process see Lee (2004) and Ferraz-Mello *et al.* (2003).

We remark that, as mentioned in section 2, to each of the two asymmetric families presented above, there correspond two equivalent asymmetric families, whose orbits are the mirror image of the former families. We remind that this is due to the system's fundamental symmetry mentioned in section 2.1. The semimajor axes and the eccentricities of the corresponding periodic orbits are identical, but the mean (or true) anomalies and the arguments of pericenter are opposite in sign.

6. Families of asymmetric periodic orbits, $m_1 = m_2$. HD82943 (new values) mass ratio

In this section we present families of asymmetric periodic orbits, for the masses of the extrasolar planetary system HD 82943, using the new elements. New values for the system HD 82943 were given recently by Mayor et al. (2004). The orbital elements and the values of the masses are quite different from those published before. The new values are: $m_0 = 1.05\ M_{SUN}$, $m_1 \sin i = 1.85$ MJ, $m_2 \sin i = 1.84$ MJ, $a_1 = 0.75$ AU, $a_2 = 1.18$ AU, $T_1 = 219.4 \pm 0.2$ d, $T_2 = 435.1 \pm 1.4$ d, $e_1 = 0.38 \pm 0.01$, $e_2 = 0.18 \pm 0.04$, $\varpi_1 = 124 \pm 3$, $\varpi_2 = 237 \pm 13$. We remark that Ferraz-Mello *et al.* (2005) showed that this particular system with the above elements is unstable.

The families of symmetric periodic orbits for the above masses are given in figure 9 (dashed lines). Note that *family 1* and *family 2* are similar to the corresponding families of the Io-Europa system, given in figure 6, but now the unstable region on the symmetric *family 1* is smaller, because $m_1 = m_2$. For $m_1/m_2 < 0.97$ this unstable region disappears. A third family of symmetric periodic orbits, *family 3*, also appears in this case. This family is unstable. The symmetric families *1* and *2* have been studied in Psychoyos and Hadjidemetriou, (2005a), and in the following we shall focus our attention on the asymmetric families.

Due to the smallness of the unstable region on *family 1*, the asymmetric family A_1 that bifurcates from this unstable region, is much smaller. However, the variation of $\Delta\varpi$ and M is rather significant (figures 10a,c). Their extremum is found at $e_1 = 0.32$ where $\Delta\varpi = 40°$ and $M = 34°$ (or $M = 326°$).

Similarly to the Io-Europa system, a second family, *family* A_2, exists. This family is independent of the symmetric families. All the symmetric and asymmetric families are shown in figure 9, in the $e_1\, e_2$ space. The linearly stable regions are indicated by a thin line and the unstable regions by a thick line. The points where the stability type changes are indicated by the points B_{2i} ($i = 1, ..4$). In the middle unstable part of the family, which is located between the points B_{22} and B_{23}, there are two intervals, $0.33 < e_1 < 0.47$ and $0.49 < 0.68$, where the periodic orbits correspond to planetary orbits that do not intersect during one period. In figures 10b and 10d we present the angles $\Delta\varpi$ and M, respectively, along the family A_2 of the asymmetric periodic orbits. We remind that M is the mean anomaly of the second planet $\mathcal{P}_2$ when the first one, $\mathcal{P}_1$, is at perihelion.

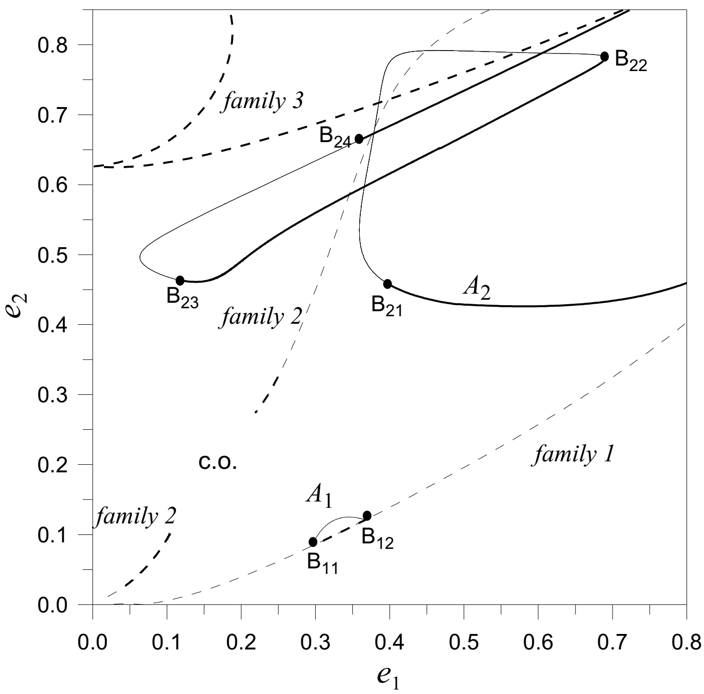

Figure 9. The symmetric and asymmetric families of periodic orbits at the 2/1 resonance, for the *new* masses $m_1 = m_2$ of HD 82943. The notation is the same with that of figure 6.

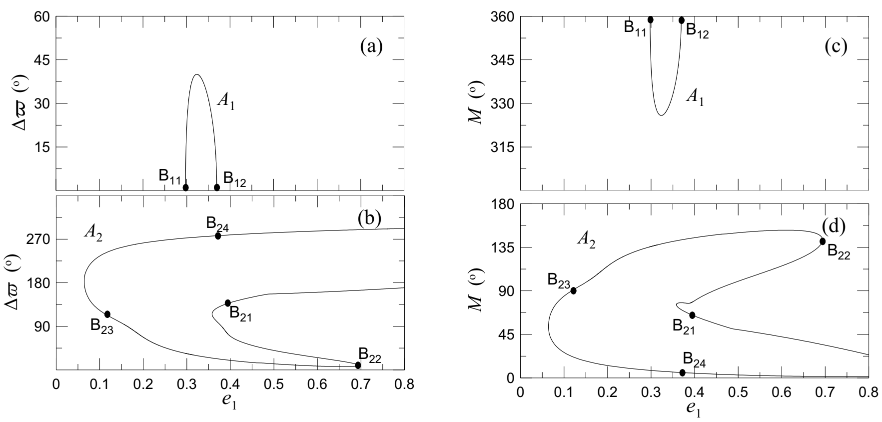

Figure 10. System HD82943 for $m_1 = m_2$. The variation of the angles $\Delta\varpi$ and M (as in figure 7) along the asymmetric families A_1 and A_2.

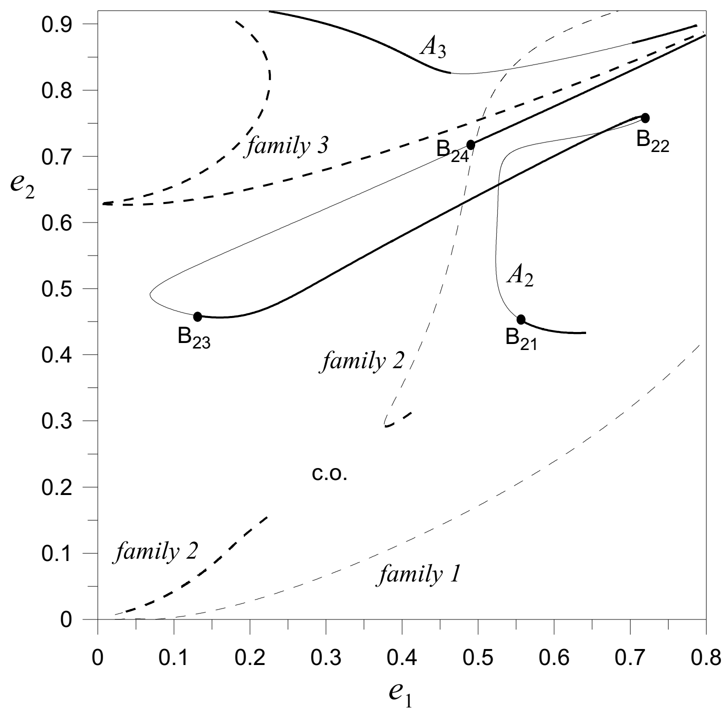

Figure 11. The symmetric and asymmetric families of periodic orbits at the 2/1 resonance, for the *old* masses of HD 82943. The notation is the same with that of figure 6. Note that no unstable region appears on the *family 1* of the symmetric orbits.

7. Families of asymmetric periodic orbits, $\mathbf{m_1 < m_2}$. HD82943 (old values) mass ratio

As a typical example of asymmetric periodic orbits of a planetary system with the mass of the inner planet smaller than the mass of the outer planet, $m_1 < m_2$, we study the planetary system HD82943, with the old masses. The elements of this system are (Israelinian et al., 2001): $m_1 \sin i = 0.88J$, $m_2 \sin i = 1.63J$, $a_1 = 0.73$AU, $a_2 = 1.16$AU, $T_1 = 221.6$d, $T_2 = 444.6$d, $e_1 = 0.54$, $e_2 = 0.41$ and the mass of the sun is $m_{sun} = 1.05$ solar masses. The normalized masses that we used in our computations are $m_0 = 0.9978$, $m_1 = 0.0008$, $m_2 = 0.0014$.

Symmetric periodic orbits for this system have been computed by Psychoyos and Hadjidemetriou (2005a). In this section we compute families of asymmetric periodic orbits. The three symmetric families, *family 1*, *family 2* and *family 3*, also exist in this case, as in the case $m_1 = m_2$, but now there is no an instability region on *family 1* and

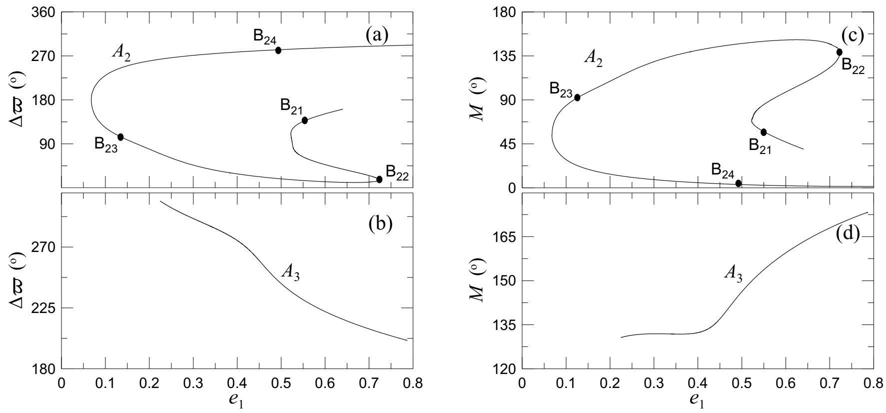

Figure 12. System HD82943 for $m_1 < m_2$. The variation of the angles $\Delta\varpi = \varpi_2 - \varpi_1$ and M (as in figures 7,10) for the indicated families A_2 and A_3.

consequently, we *do not have* a bifurcation of an asymmetric family A_1 (see figures 6 and 9). There exist however in the present case, $m_1 < m_2$, two families of asymmetric periodic orbits, family A_2 and family A_3, that are independent of the symmetric families.

In figure 11 we present the above two families of asymmetric periodic orbits, in the $e_1\, e_2$ space. The graph of the family A_2 is quite complicated and is similar to the previous cases studied (figures 6,9). The planetary orbits, generally, intersect except for the segments that correspond to the intervals $0.25 < e_1 < 0.39$ and $0.68 < e_1 < 0.71$ located between the critical orbits B_{22} and B_{23}. The new family A_3 is described by very high values of the eccentricity e_2 and a significant portion of it is stable. For all periodic orbits of family A_3 the planetary orbits intersect.

In figure 12 we present the variation of the angle of apsides $\Delta\varpi$ and the mean anomaly difference M along these two families. The family A_2 shows similar characteristics to those of the previous cases. Along the family A_3 the value of $\Delta\varpi$ and M tends to 180° as $e_1 \to 1$.

8. Stability analysis of the phase space regions near the asymmetric families of the Io-Europa system.

The linear stability of the asymmetric families A_1 and A_2 of periodic orbits for the mass ratio of Io-Europa is given in figure 6: Family A_1 is linearly stable, while family A_2 has linearly stable and linearly unstable sections. In order to study the nonlinear stability of the asymmetric periodic orbits we change the asymmetry of the two planetary orbits,

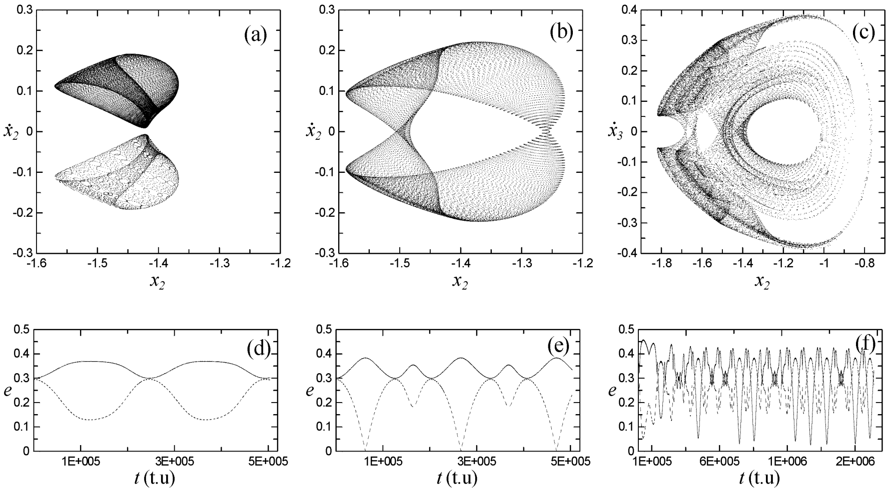

Figure 13. The Poincaré map (projection on the $x_2\,\dot{x}_2$ plane) of the orbit O_1 of the family A_1 in figure 6, for a fixed value $\Delta\varpi = 81°$, corresponding to the exact periodic orbit, and a shift of M from its value $M = 292°$ at the exact periodic orbit to: (a) $M = 302°$, (b) $M = 307°$ and (c) $M = 0°$. The panels under the Poincaré maps give the variation of the eccentricities of $\mathcal{P}_1$ (solid line) and $\mathcal{P}_2$ (dashed line) in time (t.u. denotes the time units, which result for the normalized system described in Section 2.1).

keeping the same values for the semimajor axes and the eccentricities. We use two types of perturbation: (a) We keep the angle $\Delta\varpi$ fixed, equal to the value at exact periodicity and shift the position of $\mathcal{P}_2$ on its orbit, from the position of exact periodicity at the moment when $\mathcal{P}_1$ is at perihelion. This means that we change the mean anomaly value M, starting from the exact periodicity. (b) We keep the value of M fixed, equal to the value M_2 at the exact periodicity when $M_1 = 0$, and change the angle $\Delta\varpi$, starting from the exact periodicity. We remark that this is not a complete exploration of the four-dimensional phase space in the neighbourhood of a periodic orbit, but it is an indication that stable librations do exist close to a linearly stable periodic orbit.

As we will see in the following, we have stable asymmetric librations even in the case where the two planetary orbits intersect, for a region of the phase space.

8.1. The family A_1

As a typical example of an asymmetric periodic orbit on the family A_1 of figure 6, we consider the orbit O_1 which is linearly stable (figure 8,

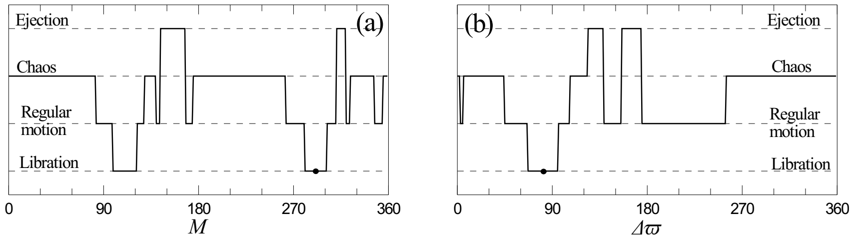

Figure 14. (a) The regions of the value of M of the orbit O_1, corresponding to different behaviour. The exact periodic motion is at $M = 292^\circ$ and is indicated by a dot. (b) The regions of the value of $\Delta\varpi$, corresponding to different behaviour. The exact periodic motion is at $\Delta\varpi = 81^\circ$ and is indicated by a dot.

left column). The elements of this 2/1 resonant orbit are $e_1 = 0.30$, $e_2 = 0.29$, $\Delta\varpi = 81^\circ$ and $M = -68^\circ$.

Let us start first by keeping the angles of apsides fixed, equal to the one corresponding to the exact periodic motion and vary the value of M, starting from the exact periodic motion. We found that there are three typical behaviours, plus the case of ejection (in a relatively short time interval) of one planet from the system:

– For a small deviation of M, we have a libration with small amplitude close to the exact periodic motion. At this point we remark that to each asymmetric periodic orbit there corresponds its mirror image, and we apply the same perturbation to the mirror image periodic orbit (presented with fewer points). We have a new, distinct, libration, which is the mirror image of the former one. These two mirror image librations are shown in figure 13a. The motion is clearly on a torus. In figure 13d we present the corresponding variation of the eccentricities. We can observe that when e_1 increases e_2 decreases and vice versa. This is due to the conservation of the angular momentum, obtained from the averaged system (Michtchenko and Ferraz-Mello; 2001, Beaugé *et al.*, 2003).

– For a larger deviation of M from the exact periodic motion, we still have a regular, bounded, motion on a torus. The difference from the previous case is that now the amplitude of the variation is much larger, because the two distinct tori that we had in the case of figure 13a merge into one large torus. This is shown in figure 13b, and the variation of the eccentricities is shown in figure 13e.

– If the deviation of M from the exact periodic motion is still larger, we have chaotic motion, which however is bounded for very long time intervals, at least up to 10^7 time units. This is shown in figure 13c and the corresponding variation of the eccentricities is shown in figure 13f.

- We found that there exists a range of values of M for which the motion is strongly chaotic and one of the planets escapes from the system in a relatively short time interval.

In all cases, the variation of the semimajor axes is very small (except in cases of ejection). All the above mentioned regions of ordered or chaotic motion are summarized in figure 14a.

A similar behaviour exists if we keep M fixed, equal to the exact periodic motion and vary $\Delta\varpi$. The four typical evolutions, which mentioned above, also appear in this case. The results are summarized in figure 14b.

In both the above two perturbation cases, there is a rather complicated change from one type of motion to another one. It is important to note however, that close to the exact periodic motion there exists a region in phase space where we have ordered motion with small variation of the orbital elements, appearing on the Poincaré map as motion on a torus. Note that due to the 2/1 resonance there is a symmetry of the behaviour close to the exact periodic motion, if M is changed by $180°$.

8.2. The family A_2

As a typical example of an asymmetric periodic orbit on the family A_2 of figure 6, we consider the orbit O_2 which is linearly stable (figure 8, middle column). The elements of this 2/1 resonant periodic orbit are $e_1 = 0.30$, $e_2 = 0.64$, $\Delta\varpi = 86°$ and $M = 173°$. Note that *the two planetary orbits intersect* in this case. We made the same analysis as that for the orbit O_1. The three typical behaviours are shown in figure 15. The regions in the range of values of M (for $\Delta\varpi$ fixed) and the regions in the range of values of $\Delta\varpi$ (for M fixed) are shown in figures 16a and 16b, respectively. Comparing with figure 14 we note that we have in this case also ordered motion with small amplitude of the orbital elements, close to the exact periodic motion, but for a larger deviation we go directly to strongly chaotic motion resulting to ejection of one planet.

We remark that the total stable region in phase space, where we have bounded motion with a small variation of the orbital elements, is considerable though the two planetary orbits intersect.

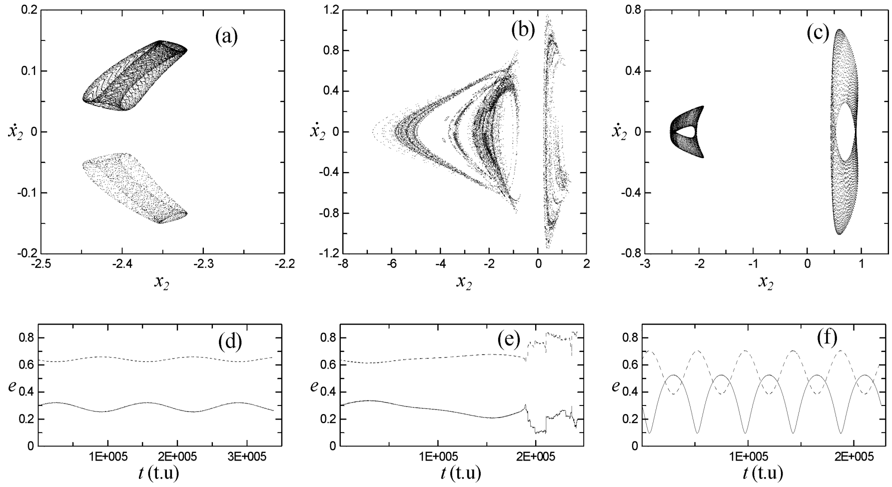

Figure 15. Poincaré sections and the eccentricity evolution, as in figure 12, for the orbit O_2 of the family A_2 in the Io-Europa system.

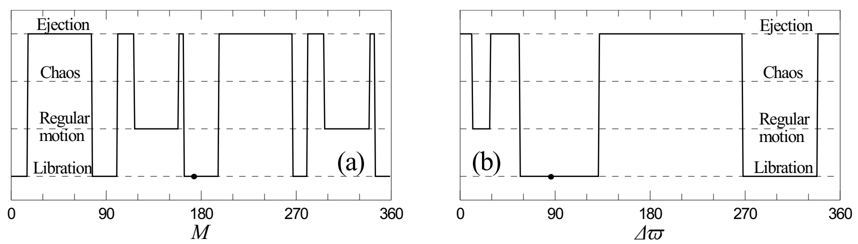

Figure 16. (a) The regions of the value of M of the orbit O_2, corresponding to different behaviour. The exact periodic motion is at $M = 173°$ and is indicated by a dot. (b) The regions of the value of $\Delta\varpi$, corresponding to different behaviour. The exact periodic motion is at $\Delta\varpi = 86°$

9. "Almost periodic" orbits and secondary resonances

The computation of periodic orbits is based on the satisfaction of the periodicity conditions given in Section 2.1. In numerical manner the periodicity conditions (a system of four equations) are satisfied up to a prescribed accuracy ε. In our study we set $\varepsilon = 10^{-13}$ (except for some cases of highly eccentric or/and strongly unstable motion where we set $\varepsilon = 10^{-12}$). We observed that in many cases the convergence of the associated numerical Newton-Raphson algorithm computationally terminates before achieving the prescribed accuracy, because the computed determinant of the system of equations is almost critical ($\sim 10^{-7}$). Finally, we obtain an orbit which satisfies the periodicity

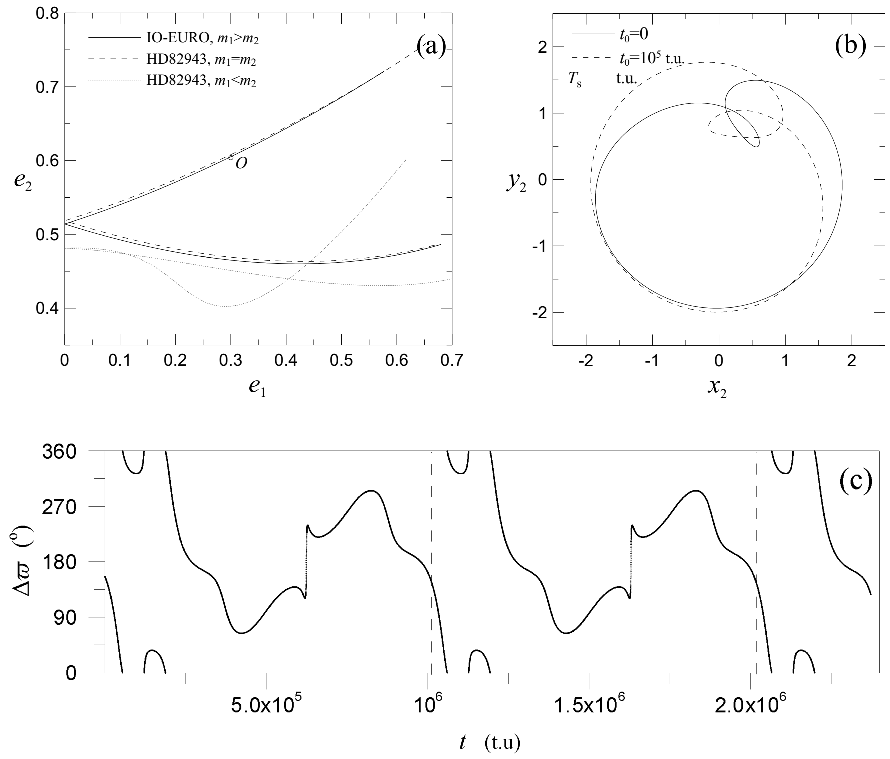

Figure 17. (a) Families of "almost periodic orbits" for the three systems studied (projection in the $e_1\,e_2$ plane) (b) The projection of the orbit O, shown in panel (a), in the plane of the rotating variables x_2 and y_2. The orbit is displayed for the time intervals $t_0 \le t \le t_0 + T_s$ where $t_0 = 0$ (solid line) and $t_0 = 10^5$ time units (dashed line). T_s is the "short period" of the orbit O (see the text) (c) The evolution of $\Delta\varpi$ along the orbit O. The evolution has a long period T_L, which is about 10^6 time units and is indicated by the vertical dashed lines.

conditions with accuracy ε' of few orders greater than ε. Such orbits are "almost periodic" orbits in the sense that their starting and ending points in phase space are in distance $\varepsilon' \ll 1$. Furthermore we can construct families of "almost periodic" orbits by continuation. It is not in the scope of this paper to study in details the above mentioned orbits. We restrict our discussion in describing their main characteristics and claiming their importance for the phase space topology.

Indicative families of "almost periodic" orbits, one for each system studied, are shown in figure 17a. In all these families the eccentricity e_1 of the inner planet passes through the value zero as we move along the family. The projection of the orbit O on the plane of the rotating variables x_2 and y_2 is shown in figure 17b. The orbit is presented for

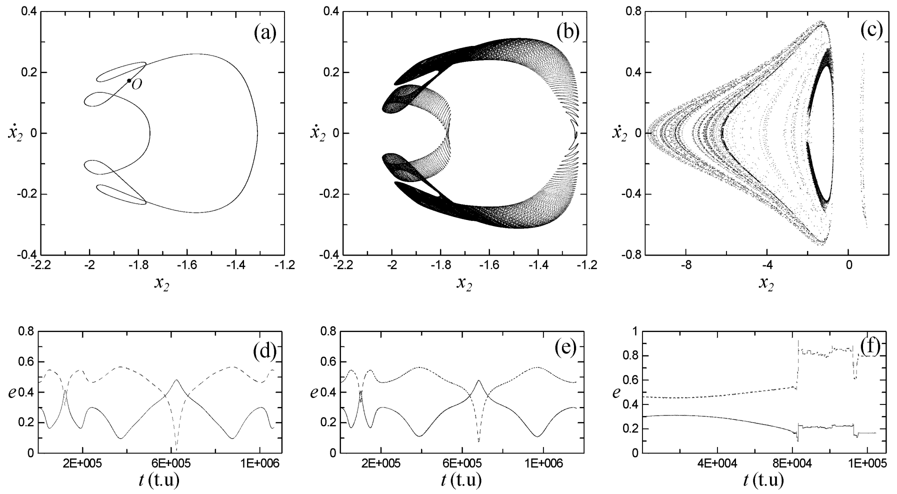

Figure 18. The Poincaré map (projection on the $x_2\,\dot{x}_2$ plane) of the orbit O indicated in figure 17a, for a fixed value $\Delta\varpi = 157°$, corresponding to the exact initial conditions of O, and a shift of M from its initial value $M = 144°$ to: (a) $M = 144°$, (b) $M = 138°$ and (c) $M = 130°$. The panels under the Poincaré maps show the variation of the eccentricities, correspondingly.

two equal but distinct time spans, i.e. they correspond to different initial time t_0 and are of size $\Delta t = T_s$, where T_s is the time span between two successive intersections of the orbit with the Poincare section ($y_2 = 0$, $\dot{y}_2 > 0$) and is called "short period". In each time span of "short period" the orbit looks like an asymmetric periodic orbit. Integrating the orbit for a longer time interval we obtain a deformation of the orbit with respect to its initial form, which takes place slowly in time. The computations indicate that this slow deformation of the orbit is also periodic and the orbit takes its original form after a "long period" T_L. This characteristic is shown in figure 17c, where the variation of the angle $\Delta\varpi$ is presented.

The long periodicity of the above mentioned orbits is verified by constructing the Poincaré sections. If we start with the exact initial conditions of O, we obtain the smooth closed invariant curve shown in Figure 18a, which takes long time to close and is symmetric with respect to the axis $\dot{x}_2 = 0$. We remind that two successive points of the Poincaré map are very close to each other. The formed curve denotes a two dimensional torus in the four dimensional phase space of the Poincaré map. In order to study the stability of this low dimensional torus we perform similar computations as that in the previous section. Initially, the orbit O correspond to $\Delta\varpi = 157°$ and $M = 144°$. A slight shift of the position of $\mathcal{P}_2$ on its orbit to $M = 138°$ results to motion on a torus, around the invariant curve of panel (a) of figure 18. This is shown in panel (b). A larger shift of $\mathcal{P}_2$ to $M = 130°$ results to chaotic motion and ejection of planet $\mathcal{P}_2$ (figure 18c,f).

We may claim that the "almost periodic" orbits described above are of a different kind of resonant orbits, called in the literature *secondary resonances*. The phase space of the Poincaré map is four-dimensional and in the regions of the phase space where we have ordered motion, the motion takes place on a 2-torus (with actions J_1, J_2 and angles θ_1, θ_2).This is the case close to the "almost periodic" orbits mentioned above. These orbits are represented as *closed curves* on a 2-torus, which means that we have a resonance between the two angles θ_1 and θ_2, $\dot{\theta}_1/\dot{\theta}_2 = p/q$, where p, q are integers. For this reason we called these orbits *secondary resonances*, although we did not relate the angles θ_1 and θ_2 with the usual libration and circulation frequencies.

The "almost periodic" orbits are in fact *periodic orbits of the averaged Hamiltonian*, and they have been proved to play an important role on the stability and the long term evolution of an asteroid in the asteroid belt (e.g. see Henrard *et al.*, 1995; Hadjidemetriou and Voyatzis, 2000). These resonances may play also an important role in the stability of extrasolar planetary systems.

10. Discussion

All the results obtained in this paper refer to resonant motion at the 2/1 resonance, for planetary orbits in the same plane. Our computations are based on the exact differential equations of the general planar three body problem.

It is clear that the periodic orbits play a crucial role in detecting the stable regions of the phase space. The numerical results indicate that there is a large region around a linearly stable periodic orbit, where we have stable motion, even in the case where the planetary orbits intersect. A phase protection mechanism operates, due to the resonance, so that the planets do not come close to each other, even in this latter case.

The stability of the *symmetric* periodic orbits of HD82943 has been studied extensively in previous papers of Hadjedemetriou and Psychoyos cited in the bibliography. In the present work the results about the symmetric orbits of the Io-Europa system are given. There exist, for this system, regions in phase space where stable motion exists, where the elements of the planetary orbits undergo librations with small amplitude. The symmetry in this case plays a stabilizing role, and a deviation from symmetry destabilizes the system.

In the present study we focus our attention mainly to stable *asymmetric* librations. The “backbone” of the regions of the phase space where we should expect asymmetric librations is provided by the families of asymmetric periodic orbits that were computed both for $m_1 \leq m_2$ and for $m_1 > m_2$. Parts on these families are linearly stable and the non linear analysis showed that close to the exact periodic motion there exists a region of the phase space where bounded, asymmetric, motion with small amplitude of the orbital elements exists. This means that a real *asymmetric* planetary system can exist for a rather large set of orbital elements.

We remark that in some regions of the phase space, i.e. for a set of values of the orbital elements of the two planetary orbits, it is the *asymmetry* that plays a stabilizing role, and the deviation from asymmetry destabilizes the system. As a consequence, we should expect real planetary systems with asymmetric librations.

The families of periodic orbits can be obtained by considering the averaging method. This has been done by Beaugé et al. (2003). The fixed points of the averaged Hamiltonian that they found correspond to the asymmetric family A_1 that we mention in sections 5 and 6 (figures 6 and 9). In the present paper we found also new asymmetric families of periodic orbits.

The knowledge of the location of the resonant, periodic, librations is important in the study of the migration of a planetary system. It is widely accepted that the observed planetary systems were not formed in their present configuration, but started with different elements and migrated to their present situation by the action of dissipative forces. Studies at the 2/1 resonance by Ferraz-Mello et al. (2003) and Lee (2004), show that a planetary system under the action of dissipative forces is trapped to an asymmetric (or symmetric) resonant periodic motion which coincides with the family A_1 that we found in this study. Their study included several mass ratios. Additionally, the work of Lee (2004) indicates the existence of periodic orbits as those of the asymmetric family A_2. For further information on how to reach stable, resonant, configurations, the reader is referred to the work of Lee (2004) and Ferraz-Mello *et al.* (2003).

Finally, we have shown the existence of a different kind of resonant orbits, called “almost periodic” orbits. These orbits form families by continuation and, at the regions where they are stable, they reveal regions of regular motion in phase space. We claim that they are associated with secondary resonances.

Acknowledgement: We thank Dr. Beaugé and the unknown referee for critically reading the manuscript and many useful comments.

This work was supported by the research programme *Pythagoras*, Nr.21878 of the Greek Ministry of Education and the E.U.

References

Beaugé, C., Ferraz-Mello, S. and Michtchenko, T.: 2003, 'Extrasolar Planets in Mean-Motion Resonance: Apses Alignment and Asymmetric Stationary Solutions', *ApJ*, **593**, 1124.

Callegari, N., Michtchenko, T. and Ferraz-Mello, S. : 2002, 'Dynamics of two planets in the 2:1 and 3:2 mean motion resonances', in 34th DPS Meeting, *BAAS*, **34**, 30.01.

Ferraz-Mello, S.: 2002, 'Tidal Acceleration, Rotation and Apses Alignment in Resonant Extra-Solar Planetary Systems', in 34th DPS Meeting, *BAAS*, **34**, 30.08

Ferraz-Mello, S., Michtchenko, T. and Beaugé, C.: 2005, 'The orbits of the extrasolar planets HD 82943c and b', ApJ, **621**, 473–481.

Ferraz-Mello, S., Beaugé, C. and Michtchenko T. : 2003, 'Evolution of migrating planet pairs in resonance', *Cel.Mech.Dyn.Astr.*, **87**, 99–112.

Ford, E.B., Havlikova, M. and Rasio,F.A. : 2001, *Icarus*, **150**, 303.

Gozdziewski, K., Bois, E. and Maciejewski, A.J. : 2002, 'Global dynamics of the Gliese 876 planetary system', *Monthly Not. R. Astron. Soc.*, **332**, 839–855.

Hadjidemetriou, J.D. : 1975, 'The continuation of periodic orbits from the restricted to the general three-body problem', *Cel.Mech.Dyn.Astr.*, **12**, 155–174.

Hadjidemetriou, J.D. : 1976, 'Families of Periodic Planetary Type Orbits in the Three-Body Problem and their Stability', *Astrophys.Sp.Science*, **40**, 201–224.

Hadjidemetriou, J.D. : 2002, 'Resonant periodic motion and the stability of extrasolar planetary systems', *Cel.Mech.Dyn.Astr.*, **83**, 141–154.

Hadjidemetriou, J.D. : 2005, 'Periodic Orbits in Gravitational Systems', Proceedings of the A.S.I., Cortina.

Hadjidemetriou, J.D. and Michalodimitrakis, M. : 1981, 'Periodic Planetary-type Orbits of the General 4-Body Problem with an Application to the Satellites of Jupiter', *Astron. Astrophys.*, **93**, 204-211.

Hadjidemetriou, J.D. and Voyatzis, G. :2000, 'The 2/1 and 3/2 resonanct asteroid motion: A symplectic mapping approach', *Cel.Mech.Dyn.Astron.*, **78**, 137–150.

Hadjidemetriou, J.D. and Psychoyos, D : 2003, 'Dynamics of Extrasolar Planetary Systems', *LNP*, **626**, 412.

Henrard, J., Watanabe, N. and Moons, M.: 1995, 'A bridge between secondary and secular resonances inside the hecuba gap', Icarus, **115**, 336-346.

Israelinian, G., Santos, N., Mayor, M. and Rebolo, R. : 2001, 'Evidence for planet engulfment by the star HD82943', *Nature*, **411**, 163.

Ji, J., Kinoshita, H., Liu, L., Li, G. and Nakai, H. : 2003, 'The apsidal antialignment of the HD 82943 system', *Cel.Mech.Dyn.Astr.*, **87**, 121–127.

Kinoshita, H. and Nakai, H. : 2001a, 'Stability of the GJ876 Planetary System', *PASJ*, **53**, L25–L26.

Kinoshita, H. and Nakai, H. : 2001b, 'Stability Mechanism of Planetary System of υ' Andromedae', in *Proc. of the IAU Symposium*, 202, Manchester 2000.

Kiseleva-Eggleton L., Bois E., Rambaux, N. and Dvorak R. : 2002, 'Global dynamics and stability limits for planetary systems around HD 12661, HD 38529, HD 37124 and HD 160691', *ApJ Letters*, **578**, L145.

Laughlin, G. and Chambers, J. : 2001, 'Short-term dynamical interactions among extrasolar planets', *ApJ*, **551**, L109–113.

Lee, M. H. and Peale, S.J. : 2003, 'Extrasolar planets and mean motion resonances', in *Scientific Frontiers in research of extrasolar planets*, D.Deming and S.Seager (eds), ASP, **197**.

Lee, M. H. : 2004, 'Diversity and Origin of 2:1 Orbital Resonance in Extrasolar Planetary Systems', *ApJ*, **611**, 517.

Lissauer, J.J. and Rivera, E.J. : 2001, 'Stability analysis of the Planetary System orbiting Andromedae', *ApJ*, **554**, 1141–1150.

Malhotra, R. : 2002a, 'A dynamical mechanism for establishing apsidal resonance', *ApJ Letters*, **575**, L33–36.

Malhotra, R. : 2002b, 'Eccentricity excitation and apsidal alignment in exo-planetary systems', in 34th DPS Meeting, *BAAS*, **34**, 42.05

Mayor, M., Udry, S., Naef, D., Pepe, F., Queloz, D., Santos, N.C. and Burnet, M.: 2004, 'The CORALIE survey for southern extra-solar planets. XII. Orbital solutions for 16 extra-solar planets discovered with CORALIE', *A&A*, **415**, 291.

Michtchenko, T.A. and Ferraz-Mello, S. :2001, 'Modeling the 5:2 mean motion resonance in Jupiter-Saturn planetary system', *Icarus*, **149**, 357–374.

Murray, N., Paskowitz, M. and Holman, M. : 2001, 'Eccentricity evolution of resonant migrating planets', *ApJ*, **565**, 608.

Peale, S. and Lee, M. : 2002, *Extrasolar Planets and the 2:1 Orbital Resonances*, in DDA 33rd Meeting, *BAAS*, **34**, 933.

Psychoyos, D. and Hadjidemetriou, J.D. : 2005a, 'Dynamics of 2/1 resonant extrasolar systems. Application to HD82943 and Gliese876', *Cel.Mech.Dyn.Astr.*, to appear.

Psychoyos, D. and Hadjidemetriou, J.D. : 2005b, 'Dynamics of Extrasolar systems at the 5/2 resonance: Application to 47 UMa', in *Proceedings of IAU Colloquium*, No197, 55–62.

Rivera, E.J. and Lissauer, J.J. : 2001, 'Dynamical models of the resonant pair of planets orbiting the star GJ 876', *ApJ*, **558**, 392–402.

Schneider, J. : 2002, *http://www.obspm.fr/encycl/catalog.html*.