Performance Analysis of M-ary PPM TH-UWB Systems in the Presence of MUI and Timing Jitter

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Abstract—The symbol error probability (SEP) performance of time-hopping (TH) ultra-wideband (UWB) systems in the presence of multiuser interference (MUI) and timing jitter is considered without making the usual Gaussian approximation (GA). Contrary to previous studies restricted to binary modulation formats, we analyze and evaluate the performance of such multiuser TH-UWB systems employing M-ary pulse position modulation (PPM) and a correlation receiver. Using the Gaussian quadrature rules (GQR) method and by considering the correlator outputs to be statistically independent, a semi-analytical method for the SEP evaluation is proposed. Furthermore, by modeling the timing jitter as a Gaussian process, we study the degradation it causes on the overall system performance (SEP and capacity). The accuracy of the proposed approach has been verified by means of computer simulations. Furthermore, a comparison with equivalent performance results obtained using the GA has provided precise conditions where this approximation should not be used.

Index Terms—Multiuser interference (MUI), pulse position modulation (PPM), symbol error probability (SEP), time hopping (TH), timing jitter, ultra-wideband (UWB).

I. INTRODUCTION

LTRA-WIDEBAND (UWB) technology is the primary candidate for the physical layer of the upcoming standards for wireless personal area networks [1], since it provides reliable high-speed data transmission at short ranges over severe multipath conditions. It also exhibits robust multiple-access (MA) performance with little interference to other communication systems sharing the same bandwidth due to its very low-power spectral density (PSD). The fundamental characteristic of UWB is the extremely large bandwidth, which is required since very narrow pulses of appropriate shape and subnanosecond duration, referred to as monocycles, are being used by the transmitted signal.

One of the most widely studied schemes for UWB communications employs pulse position modulation (PPM) combined with time hopping (TH) as its MA technique [2]. The UWB

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pulses are time hopped within a fixed time window (frame) and each transmitted symbol is spread over several pulses in order to facilitate multiple users. The MA performance of this scheme is commonly evaluated by modeling the multiuser interference as a Gaussian random process, an approach known as Gaussian approximation (GA) (e.g., see [4]–[8]). Although the GA does not always provide accurate results [11], this approximation makes closed-form analysis possible.

Other studies on TH-UWB systems that are not based on the GA have shown that accurate evaluation of the symbol error probability (SEP) is possible for 2-PPM (i.e., binary) systems. One method for computing the SEP based on the Gaussian quadrature rule (GQR) has been discussed in [9]. Another SEP performance evaluation approach using the characteristic function has been presented in [10]. The most important advantage of these two methods is that they do not require knowledge of the multiuser interference (MUI) probability density function (PDF), which is extremely difficult to obtain in closed form [12]. However, for multilevel (i.e., M-ary with M > 2) PPM systems, the SEP performance has been only obtained by means of computer simulation (e.g., [8]).

Another important problem when considering the performance of such systems is that the impulsive nature of the UWB signal imposes the requirement for extremely precise synchronization. Thus, it is important to study the effect of timing jitter on their performance [16]-[21]. Past work related to the subject of our paper includes [19] which investigates the UWB system performance by evaluating the degradation of the signal-to-noise ratio due to timing jitter. Furthermore, in [20], the bit-error rate (BER) performance of bipolar pulse waveform and position modulation UWB systems is evaluated. However, in these two studies and in [21], the MUI was not taken into account. The analysis presented in [17], although it takes MUI into account, is limited to 2- and 4-PPM schemes. It should be noted that the model for timing jitter suggested in [17] is to treat the timing inaccuracy of the UWB pulses as a Gaussian random process.

Motivated by the above, this paper proposes a semi-analytical GQR-based method for analyzing and evaluating the SEP performance of M-ary PPM TH-UWB systems operating in the presence of MUI and timing jitter. The remainder of this paper is organized as follows. In Section II, the model of the UWB signal is introduced and its characteristics are analyzed. Section III covers the analysis of MUI and timing jitter and the evaluation of SEP performance with the GQR. In Section IV, numerical results for the GQR method are presented and discussed. These results are also compared with equivalent simulations, as well as

with results obtained using the GA approach. Finally, the concluding remarks are provided in Section V.

II. SYSTEM MODEL

A multiuser TH-UWB system employing M-ary PPM as its modulation scheme is considered. Assuming that N_u users transmit asynchronously, the signal of the ν th user can be modeled as follows:

$$s^{(\nu)}(t) = \sum_{j} \sqrt{E_{p}^{(\nu)}} \times p \left(t - \tau^{(\nu)} - jT_{f} - c_{j}^{(\nu)}T_{c} - d_{|j/N_{p}|}^{(\nu)} \delta - \epsilon_{j} \right)$$
(1)

where p(t) is the UWB pulse of duration T_p and $E_p^{(\nu)}$ is the received energy per pulse from user ν ($1 \le \nu \le N_u$). Furthermore, T_f is the frame duration, $c_j^{(\nu)}$ is the TH sequence of user ν , T_c is the TH slot duration, and $d_{\lfloor j/N_p \rfloor}^{(\nu)}$ is the transmitted symbol $\left(0 \le d_{\lfloor j/N_p \rfloor}^{(\nu)} \le M - 1\right)$, where $\lfloor x \rfloor$ is the largest integer y with $y \le x$. Similar to [17], the timing jitter, ϵ_j , is assumed to be a zero-mean Gaussian random variable. An evenly spaced M-PPM scheme is considered, where δ is the time shift difference between two subsequent PPM signals. We further assume that $\delta \ge T_p$ resulting in an orthogonal signal set. N_p pulses/symbol are transmitted providing a symbol rate of $R_s = 1/(N_p T_f)$ and energy per symbol $E_s = N_p E_p$. The asynchronous transmission delay $\tau^{(\nu)}$ is typically several times the frame duration T_f and the spreading ratio is $\beta = T_f/T_p$. Similar to others (e.g., [2] and [3]), we consider for p(t) a Gaussian monocycle given by

$$p(t) = \left[1 - \left(\frac{t}{z}\right)^2\right] \exp\left(-\frac{t^2}{2z^2}\right) \tag{2}$$

where z is a pulse width parameter. The effective time duration of the pulse is $T_p=2\pi z$, which practically contains all the energy in the monocycle. Its autocorrelation function h(t) is given by

$$h(t) = \left[1 - \frac{t^2}{z^2} + \frac{t^4}{12z^4}\right] \exp\left(-\frac{t^2}{4z^2}\right). \tag{3}$$

Considering an additive white Gaussian noise (AWGN) channel, the optimum demodulator is a pulse correlation receiver implemented by M filters matched to the template functions $u_i(t) = p(t-i\delta)$ $(0 \le i \le M-1)$ followed by a detector which selects the correlator with the maximum signal output at the end of the symbol duration. In the special case of 2-PPM, the receiver can be implemented by a single correlator with the template function $u(t) = u_0(t) - u_1(t)$.

The received signal can be expressed as

$$r(t) = \sum_{\nu=1}^{N_u} s^{(\nu)}(t) + n(t) \tag{4}$$

where n(t) is the AWGN waveform with single sided PSD N_0 . Without loss of generality, it is assumed that the desired user is 1 and the reception of the first transmitted symbol is examined.

The receiver is considered perfectly synchronized to the desired user, i.e., $\tau^{(1)}$ and $c_j^{(1)}$ are known. The sampled signal at the output of each of the correlators is

$$y_{i} = \sum_{j=0}^{N_{p}-1} \int_{jT_{f}}^{(j+1)T_{f}} r(t+\tau^{(1)}) u_{i} \left(t - jT_{f} - c_{j}^{(1)}T_{c}\right) dt$$

$$= \begin{cases} S + N_{I} + N_{G}, & i = d_{[0/N_{p}]}^{(1)} \\ N_{I} + N_{G}, & i \neq d_{[0/N_{p}]}^{(1)} \end{cases}$$
(5)

where S is the contribution of the signal of user 1, N_I is the MUI component, and N_G is the AWGN component which remains Gaussian with zero-mean and variance $\sigma_G^2 = N_p N_0$. S is given by

$$S = \sqrt{E_p^{(1)}} \sum_{j=0}^{N_p - 1} h(\epsilon_j).$$
 (6)

III. PERFORMANCE ANALYSIS

A restriction imposed by the nature of TH-PPM systems is that the MUI affecting each of the correlator outputs y_i are not statistically independent. This is the main difficulty in analyzing M-ary (M > 2) PPM UWB systems, as the output of each correlator cannot be examined independently. For 2-PPM, this problem is bypassed as only one correlator is used (e.g., see [2] and [9]-[11]). However, this approach cannot be applied to M-ary systems where the selection among the M correlator outputs is necessary. On the other hand, the GA requires a sufficiently large number of users to invoke the central limit theorem and approximate y_i as a Gaussian random variable. This has also the effect of making the various y_i statistically independent as an arbitrarily large number of independent interfering UWB signals is integrated in each correlator, thus facilitating the analysis. In this paper, we adopt the assumption of statistical independence, but without making use of the assumption that y_i is Gaussian. Thus, by appropriately modifying the well-known equation for the SEP of an M-ary orthogonal signal in an AWGN channel [13, eq. 4-2-53], the following SEP expression for a multiuser system with timing jitter can be obtained:

$$P_{E} = 1 - \int_{-\infty}^{+\infty} p_{\text{MUI}+N}(x) \int_{-\infty}^{+\infty} p_{S}(\psi)$$

$$\times \left[\int_{-\infty}^{+\infty} \frac{1}{2} \operatorname{erfc} \left(-\frac{x + \psi - q}{\sqrt{2}\sigma_{G}} \right) p_{\text{MUI}}(q) dq \right]^{M-1}$$

$$\times d\psi dx. \tag{7}$$

In the above equation p_{MUI} is the PDF of MUI in a single PPM slot, p_S is the PDF of S, and $p_{\mathrm{MUI}+N}$ is the joint PDF of MUI and AWGN in a PPM slot given by

$$p_{\text{MUI}+N}(x) = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma_G} \exp\left[-\frac{(x-q)^2}{2\sigma_G^2}\right] p_{\text{MUI}}(q) dq.$$
(8)

Since no closed-form expression is available for $p_{\rm MUI}$, the GQR can provide a convenient solution as it requires only the moments which can be obtained analytically.

In the following, first the effects of timing jitter are analyzed. Then, the evaluation of the MUI moments is carried out and finally the application of GQR is presented.

A. Timing Jitter

Since S is a function of N_p independent random variables $\epsilon_0, \epsilon_1, \dots, \epsilon_{N_p-1}$, the evaluation of p_S would normally require the evaluation of an N_p -dimensional integral. Thus, even for small values of N_p the complexity of (7) becomes prohibitive, and it is clear that another method of analysis is required. Our approach is, similarly to [21], to invoke the central limit theorem and consider S to be Gaussian. Its mean μ_S and variance σ_S^2 are given by

$$\mu_S = N_p \sqrt{E_p^{(1)}} \int_{-\infty}^{+\infty} h(\tau) N_{0,\sigma_{\epsilon}}(\tau) d\tau$$
 (9a)
$$\sigma_S^2 = N_p E_p^{(1)} \int_{-\infty}^{+\infty} h^2(\tau) N_{0,\sigma_{\epsilon}}(\tau) d\tau - \frac{1}{N_-} \mu_S^2$$
 (9b)

where $N_{0,\sigma_{\epsilon}}$ is the Gaussian PDF with zero-mean and variance $\sigma_{\epsilon}^2 = E\left\langle \epsilon_j^2 \right\rangle$, where $E\langle \rangle$ is the expectation operator. After evaluating the integrals of (9) using (3) the following expressions are obtained:

$$\mu_S = \frac{N_p \sqrt{E_p^{(1)}}}{(1 + 0.5Q^2)^{5/2}}$$
(10a)
$$\sigma_S^2 = N_p E_p^{(1)} \frac{(0.729Q^8 + 2.5Q^6 + 3.5Q^4 + 2Q^2 + 1)}{(1 + 0.5Q^2)^{5/2}} - \frac{\mu_S^2}{(1 + 0.5Q^2)^{5/2}}$$

$$\sigma_S^2 = N_p E_p^{(1)} \frac{(0.729Q^8 + 2.5Q^6 + 3.5Q^4 + 2Q^2 + 1)}{(1 + Q^2)^{9/2}} - \frac{\mu_S^2}{N_p}$$
(10b)

where $Q = \sigma_{\epsilon}/z$. It should be noted that in [21], μ_S and σ_S^2 were evaluated only by numerical integration and no analytical expressions have been obtained.

It is convenient for our analysis to express S as $S=\mu_S+S_\epsilon$ where S_ϵ is zero-mean Gaussian with variance σ_S . With the appropriate transformations on (7), the following SEP expression is obtained

$$P_E = 1 - \int_{-\infty}^{+\infty} p_{\text{MUI}+N+S}(x)$$

$$\times \left[\int_{-\infty}^{+\infty} \frac{1}{2} \text{erfc} \left(-\frac{x + \mu_S - q}{\sqrt{2}\sigma_G} \right) p_{\text{MUI}}(q) dq \right]^{M-1} dx \quad (11)$$

where $p_{\text{MUI}+N+S}$ is the joint PDF of MUI, AWGN, and S_{ϵ} .

B. Moments

Since for TH-UWB systems the pulse collision is the primary source of interference, our analysis will focus on the pulse time-of-arrival statistics. Furthermore, in this subsection, the timing jitter of the interfering pulses will not be considered since it is insignificant as compared with the timing inaccuracy due to the lack of synchronization among users. Thus, the MUI component comprising of the correlation of the signals from all interfering users at the receiver is

$$N_{I} = \sum_{\nu=2}^{N_{u}} \sum_{j=0}^{N_{p}-1} \int_{jT_{f}}^{(j+1)T_{f}} s^{(\nu)}(t+\tau^{(1)}) \times u_{i} \left(t-jT_{f}-c_{j}^{(1)}T\right) dt. \quad (12)$$

Based on the TH structure of the signal, only one pulse from user ν can interfere with any one pulse of user 1. Thus, N_I can be expressed as

$$N_{I} = \sum_{v=2}^{N_{u}} \sum_{j=0}^{N_{p}-1} \sqrt{E_{p}^{(v)}} h(\Delta)$$
 (13)

where Δ is the time-of-arrival difference between the jth pulse of user 1 and a possibly interfering pulse of user ν . The time delay of user ν relative to user 1 spans typically over several frame intervals, so that $\tau^{(\nu)} - \tau^{(1)} = kT_f + \tau_f, 0 \leq \tau_f < T_f, k \in \mathbb{N}$. Furthermore, the PPM time shift δ is not taken into account since typically $M\delta \ll T_f$, and therefore has only a minor effect on Δ . Thus, Δ is given by

$$\Delta = \begin{cases} \left(c_j^{(1)} - c_{j-k}^{(\nu)}\right) T_c - \tau_f, & c_j^{(1)} T_c \ge \tau_f \\ \left(c_j^{(1)} - c_{j-k-1}^{(\nu)}\right) T_c - \tau_f + T_f, & c_j^{(1)} T_c < \tau_f \end{cases} . (14)$$

 P_{MUI} depends on the statistical characteristics of Δ and the shape of h(t). To derive the distribution of Δ , it is assumed that the TH shift $c_j^{(\nu)}T_c$ is a random variable independent and uniformly distributed over T_f . Under this assumption, Δ is equivalent to the difference of two independent uniformly distributed random variables, and thus has triangular PDF for any value of τ_f in the range $[0,T_f]$.

For our purpose, the case of pulse collision, i.e., when $|\Delta| < T_p$, is of special interest. Since for typical values of $\beta, T_f \gg T_p$, we can approximate the PDF of Δ as constant over $[-T_p, T_p]$. In fact, we are only interested for the PDF of Δ with $|\Delta| < T_p$, since otherwise $h(\Delta) = 0$. Thus, Δ can be approximated as uniformly distributed over the range $-(T_f/2), (T_f/2)$.

Since the transmitted PPM symbols are orthogonal with each other, the kth moment of the MUI detected at the output of each correlator of the receiver $\mu_k^{\text{MUI}} = E \left\langle N_I^k \right\rangle$ is given by

$$\mu_k^{\text{MUI}} = E \left\langle \left(\sum_{\nu=2}^{N_u} \sum_{j=0}^{N_p - 1} \sqrt{E_p^{(\nu)}} h(\Delta) \right)^k \right\rangle$$

$$= \sum_{i_1, i_2, \dots, i_{N_u}} \frac{k!}{i_1! i_2! \dots i_{N_u}!} \mu_{i_1}^{(1)} \mu_{i_2}^{(2)} \dots \mu_{i_{N_u}}^{(N_u)}. \quad (15)$$

In the above expression, $i_1 + i_2 + \cdots + i_{N_u} = k$ and $\mu_k^{(\nu)}$ is the kth moment of the interference contributed by user ν and can be expressed as

$$\mu_k^{(\nu)} = \left(E_p^{(\nu)}\right)^{k/2} E\left\langle \left(\sum_{j=0}^{N_p-1} h(\Delta)\right)^k \right\rangle$$

$$= \left(E_p^{(\nu)}\right)^{k/2} \sum_{j_1, j_2, \dots, j_{N_p}} \frac{k!}{j_1! j_2! \dots j_{N_p}!} \mu_{j_1}^h \mu_{j_2}^h \dots \mu_{j_{N_p}}^h$$
(16)

where $j_1+j_2+\cdots+j_{N_p}=k$ and $\mu_k^h=E\langle h^k(\Delta)\rangle$ can be obtained from

$$\mu_k^h = \frac{1}{T_f} \int_{-T_f/2}^{T_f/2} h^k(\tau) d\tau.$$
 (17)

Since h(t) is nonzero only for a small interval fully contained within $[-T_f/2, T_f/2]$, (17) can be rewritten as

$$\mu_k^h = \begin{cases} \frac{1}{T_f} \int_{-\infty}^{+\infty} h^k(\tau) d\tau, & k > 0\\ 1, & k = 0 \end{cases} \quad k \in \mathbb{N}$$
 (18)

and using (3), μ_k^h can be obtained from the above equation as

$$\mu_k^h = \frac{1}{2\sqrt{\pi}\beta} \sum_{l_1, l_2, l_3} \left[\frac{k!}{l_1! l_2! l_3!} (-1)^{l_2} \left(\frac{1}{12} \right)^{l_3} \times \frac{2^{l_2 + 2l_3 + 1}}{\sqrt{k^{2l_2 + 4l_3 + 1}}} \prod_{i=1}^{l_2 + 2l_3} (2i - 1) \right], \quad k > 0 \quad (19)$$

where $l_1 + l_2 + l_3 = k$.

The joint moments of MUI, AWGN, and $S_\epsilon, \mu_k^{\text{MUI}+N+S}$, can be similarly derived as

$$\mu_k^{\text{MUI}+N+S} = E \left\langle (N_I + N_G + S_\epsilon)^k \right\rangle$$

$$= \sum_{i=0}^k \frac{k!}{(k-i)!i!} \mu_{k-i}^{\text{MUI}} \mu_i^{N+S}$$
(20)

where μ_i^{N+S} is the *i*th joint moment of N_G and S_{ϵ} , given by

$$\mu_i^{N+S} = \begin{cases} 0, & i = 2n+1\\ \left(\sigma_G^2 + \sigma_S^2\right)^{i/2} \prod_{l=1}^{\lfloor i/2 \rfloor} (2l-1), & i = 2n\\ n \in \mathbb{N}. \end{cases}$$
(21)

By evaluating the first and second moments ($\mu_1^{\text{MUI}+N+S}$, $\mu_2^{\text{MUI}+N+S}$) using (20), the mean of the sum of MUI, AWGN, and S_ϵ is zero and its variance is given by the following expression:

$$\sigma^2 \simeq 0.291 \frac{N_p}{\beta} \sum_{\nu=2}^{N_u} E_p^{(\nu)} + \sigma_G^2 + \sigma_S^2.$$
 (22)

C. Application of GQR

The GQR can be efficiently applied for the evaluation of the integrals in (11) because the SEP computation involves averaging a function f(x) over a random variable X whose PDF is unknown. The calculation of GQR provides a set of weights w_j and abscissas x_j such that the approximation

$$\int_{a}^{b} f(x)W(x)dx \simeq \sum_{j=1}^{K} w_{j}f(x_{j})$$
 (23)

is exact if f(x) is a polynomial of degree up to 2K-1 [15]. The values of w_j and x_j depend on the weight function W(x) and the integration interval, and can be computed by finding a set of orthogonal polynomials over W(x) on [a,b]. Using the algorithm proposed in [14], if W(x) is the PDF of the random variable X, the K-point GQR can be computed by the first 2K-1 moments of X.

The application of GQR produces the following expressions for the two integrals of (11):

$$P_{E} = 1 - \sum_{j=1}^{K} w_{j}^{\text{MUI}+N+S} I\left(x_{j}^{\text{MUI}+N+S}\right)$$
 (24a)

$$I(x) = \left[\sum_{j=1}^{K} w_j^{\text{MUI}} \frac{1}{2} \text{erfc} \left(-\frac{x + \mu_S - x_j^{\text{MUI}}}{\sqrt{2}\sigma_G} \right) \right]^{M-1}$$
(24b)

where w_j^{MUI} and x_j^{MUI} are the weights and abscissas corresponding to p_{MUI} , and $w_j^{\mathrm{MUI}+N+S}$ and $x_j^{\mathrm{MUI}+N+S}$ are the weights and abscissas corresponding to $p_{\mathrm{MUI}+N+S}$.

The proposed approach, although similar to the method presented in [9], has important differences. Apart from the fact that (11) is more complex than the equivalent equation for 2-PPM as it requires the GQR method to be applied twice, the main difference lies in the actual computation of the required MUI moments. This is due to the fact that in [9] the receiver has a single correlator with the template function u(t), as is the case for 2-PPM systems, and thus its output signal has a very different distribution than the equivalent M-ary PPM signals examined in this paper.

IV. PERFORMANCE EVALUATION RESULTS AND DISCUSSION

The SEP performance, P_E , of the previously described TH-UWB system has been evaluated using (24) and representative results are presented in Figs. 1–7. In order to verify the validity of the proposed GQR method, complementary performance results have also been obtained by means of computer simulation. Furthermore, equivalent SEP performance evaluation results using the GA are presented. Unless otherwise noted, the presented results were obtained for 4-PPM and with the following parameters: $\beta=30, \delta=T_p, N_u=8, N_p=4, \sigma_\epsilon=0,$ and $E_s/N_0=15$ dB.

The number of points K chosen for the GQR method is important for obtaining reliable results as its accuracy can be improved by increasing K. In practice however, such an increase is not always possible since the matrix constructed from the coefficients of the three-term recurrence relation is sensitive to roundoff errors [22]. In fact, such errors can cause the resulting matrix to differ from its normal symmetric real form and produce complex eigenvalues. In all cases studied in this paper, the GQR method produced accurate results for $K \geq 7$. The maximum number of points before experiencing numerical problems was K=17, and this value was used for obtaining all performance results presented in this paper.

Fig. 1 presents P_E versus E_s/N_0 for the GQR and GA methods, as well as computer simulations for different values of T_c . It should be noted that, in our analysis, T_c was considered arbitrarily small since the TH shift $c_j^{(\nu)}T_c$ was assumed to be uniformly distributed over T_f . The presented simulation results were obtained for $T_c = T_p/2$, T_p , and $2T_p$, and show that if $T_c \leq T_p/2$, the analysis provides a good estimate of the average SEP performance. For $T_c > T_p/2$, the simulated SEP increases due to the autocorrelation properties of the UWB pulse, but even in that case the GQR method is shown to be a more accurate approach as compared with the GA.

Fig. 2 illustrates P_E versus N_u for $N_p = 2, 4$, and 8. Clearly, the performance results obtained with the GQR method are in excellent agreement with equivalent simulation results, even for a small number of users, which is very commonly the case for

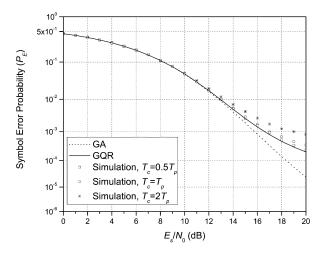


Fig. 1. SEP performance P_E versus E_s/N_0 for different values of T_c employing the two evaluation methods: GQR and GA.

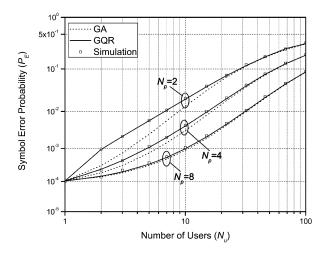


Fig. 2. SEP performance P_E versus the number of users N_u for $N_p=2,4$, and 8. The superiority of the GQR method as compared with the GA is observed.

practical UWB systems. On the contrary, the GA is very optimistic especially for small N_u . However, as N_u increases this difference becomes smaller making the GA an acceptable approximation. It is also of interest to note that the GA fits better the simulation data when the number of pulses per symbol is increased. This should not come as a surprise since more interfering pulses are integrated in each correlator, and thus due to the central limit theorem the MUI becomes more Gaussian. Our research has shown that as a rule of thumb, the GA appears to be acceptable for $N_u N_p > \beta$.

In Fig. 3, P_E versus N_u is shown, while in Fig. 4, it is plotted versus E_s/N_0 . Both figures compare the GQR method with simulations and with the GA for 2-, 4-, and 8-PPM. The obtained results show again that in all cases the GQR method provides a more accurate SEP performance evaluation as compared with the GA. The GA appears to be inadequate for high E_s/N_0 and small N_u . It should be noted that for high E_s/N_0 there is a small deviation of GQR results from the simulation results. This is due to the approximation made for the correlator outputs being statistically independent. Clearly, this has a greater impact when only a few interfering pulses are integrated in the correlators,

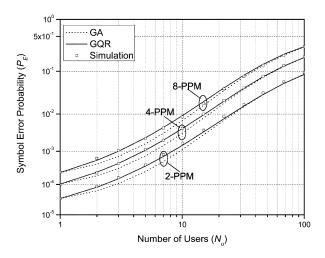


Fig. 3. SEP performance P_E versus the number of users N_u for 2-, 4-, and 8-PPM. It is observed that the GQR method can estimate P_E performance more accurately than the GA.

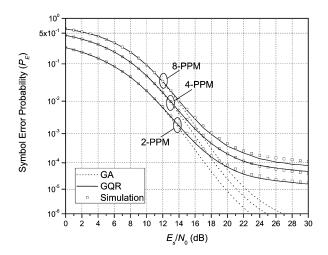


Fig. 4. SEP performance P_E versus E_s/N_0 for 2-, 4-, and 8-PPM. The slight inaccuracy of the GQR method observed at high E_s/N_0 is due to the assumption made that the correlator output signals are statistically independent.

i.e., a few users and small N_p . Furthermore, the deviation increases as M becomes larger, since there is a greater number of correlator outputs which are approximated as independent.

In Figs. 5 and 6, P_E is plotted versus σ_{ϵ}/T_p to illustrate the SEP performance degradation due to timing jitter. It is observed that for large σ_{ϵ}/T_p or small N_p the proposed GQR method is somewhat optimistic as compared with the simulation data. In the first case, the main factor for this is intersymbol interference caused when a pulse overlaps with adjacent PPM positions due to a large timing error. Note however, that such a large timing error is unrealistic in practical systems. In the latter case, this occurs because S deviates from the assumed Gaussian distribution when N_p is small. This is clearly illustrated in Fig. 5, where there are no interfering users, i.e., $N_u = 1$, and thus the primary source of performance degradation is timing jitter. It is noted that this deviation can only be observed for small N_u and high E_s/N_0 since otherwise the dominant factor affecting the SEP performance is MUI and AWGN. In general, however, the GQR method conforms to the simulation results, thus providing accurate performance evaluation. It is also observed that as σ_{ϵ}/T_{p}

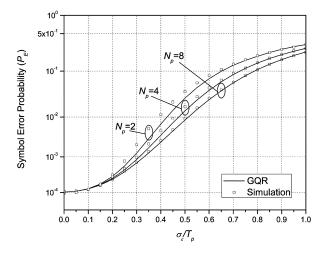


Fig. 5. SEP performance P_E versus σ_ϵ/T_p for $N_u=1$ and $N_p=2,4$, and 8. The inaccuracy observed in this case is due to the considered Gaussian approximation for the timing jitter.

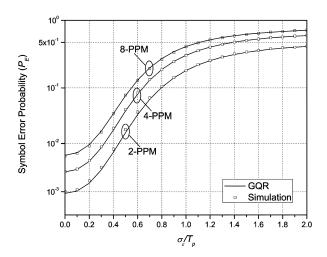


Fig. 6. SEP performance, P_E , versus σ_ϵ/T_p for 2-, 4- and 8-PPM. The rapid degradation of SEP performance as σ_ϵ/T_p increases is noted.

increases, the SEP performance degrades rapidly due to timing jitter. As a rule of thumb, for $\sigma_\epsilon/T_p < 0.2$, the degradation to SEP performance is considered acceptable.

Fig. 7 presents the capacity in terms of maximum number of users supported with $P_E \leq 10^{-3}$ versus E_s/N_0 for $N_p=8$, 16, 32 and 64, and $\sigma_\epsilon/T_p=0$, 0.1, and 0.5. The results were obtained by repetitive applications of the GQR method, which proved to be very computationally efficient. As observed from these results, although for $\sigma_\epsilon/T_p=0.1$ the capacity is not practically affected, it drops rapidly for greater values and is reduced by approximately 1/2 for $\sigma_\epsilon/T_p=0.5$.

V. CONCLUSION

A semi-analytical GQR-based method for evaluating the SEP performance of *M*-ary PPM TH-UWB systems in the presence of MUI and timing jitter has been presented. The analysis was conducted under the hypothesis of statistical independence of the correlator outputs and the approximation that the variation

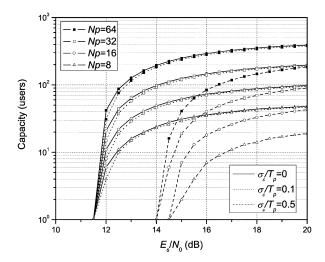


Fig. 7. Capacity, in terms of number of users, versus E_s/N_0 for $N_p=8$, 16, 32 and 64, and $\sigma_\epsilon/T_p=0$ 0.1, and 0.5.

of the correlator output due to timing jitter is Gaussian. The accuracy of the proposed methodology has been verified by means of computer simulation and several relevant SEP and capacity performance results have been provided, showing a very good agreement with the simulation data. A comparison with the GA has demonstrated the superiority of the proposed methodology and has provided conditions under which the GA should be considered an accurate approximation. The degradation on system performance caused by timing jitter has been also evaluated, and the range within which it is considered acceptable has been specified.

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