Two-Relay Distributed Switch and Stay Combining

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***Abstract*—We study a distributed version of switch-and-stay combining (DSSC) for systems that utilize two relays. In particular, four different scenarios are considered, depending on a) whether or not the source-destination channel is taken into account, and b) the type of relaying, i.e., decode and forward or amplify and forward. A performance analysis in terms of outage and bit error probability is presented, when operating over Rayleigh fading channels. Numerical results demonstrate that two-relay DSSC achieves the same diversity gain and outage performance as if the best relay is selected for each transmission slot, albeit simpler.**

***Index Terms*—Distributed switch and stay combining, relaying channel.**

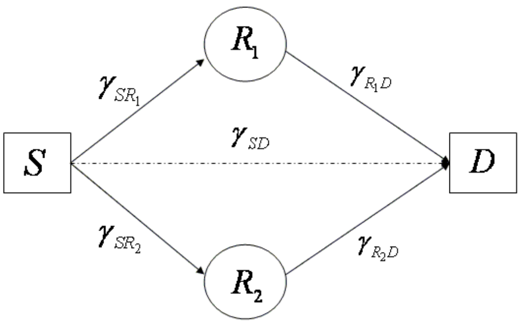

Fig. 1. The proposed setup.

I. Introduction

RELAYING transmissions have been recently proposed as a mean of attaining spatial diversity without using multiple antennas at either the transmitter or the receiver. Considering thus that relaying transmissions serve as a substitute of the common diversity techniques, which have been extensively analyzed in the literature, it naturally follows that they can be studied and thereby designed under that perspective.

This alternative diversity-achieving concept was initially studied in [1], where a set of relaying protocols for the single-relay scenario were proposed. The authors of [1] showed that utilizing channel knowledge can improve the performance, by activating the relay (and thus using only half of the degrees of freedom of the channel) only when necessary. In [2], the authors extended this single-relay diversity concept and proposed a distributed version of the well-known switch-and-stay combining (SSC) technique, where the same branch remains active as long as the signal-to-noise-ratio (SNR) of that branch is above a given threshold [3]- [4]. In cases where more than one relays are available, "opportunistic relaying" [5] was shown to attain diversity gain on the order of the number of relays, by activating only the best relay for each transmission slot, representing thus a distributed version of selection combining [6, ch. 9.8].

In this letter, we extend the distributed SSC (DSSC) concept proposed in [2], for the case where two relaying terminals are utilized; this may be the case in practical scenarios where deep shadowing renders it difficult to achieve diversity with single-relay usage. In particular, we study a two-relay cooperative scheme where only a single relay is activated in each transmission slot in a fashion similar to SSC, i.e., the same relay remains active as long as the corresponding equivalent SNR is sufficiently high. We consider four different scenarios, depending on a) whether the source-destination channel is taken into account (together with the relaying ones) or not, and b) the type of relaying, i.e., decode and forward (DF) or amplify and forward (AF). Closed-form expressions for the outage probability of the proposed schemes for each of the above cases are provided. Moreover, the bit error probability (BEP) for the case of uncoded BPSK modulation is studied, allowing for a broader view of the performance of the two-relay DSSC under different coding assumptions.

II. System Model

The system under consideration is depicted in Fig. 1. In particular, we consider a source node S which wants to communicate with a destination one, D. Two relays, namely R_1 and R_2, are willing to assist this communication, either by demodulating the received signal and then remodulating and forwarding it to D, or by acting as simple analog repeaters, i.e., amplifying and forwarding the signal to D without any further process. The former type of relaying is widely known as DF; the latter as AF. The relays are assumed to operate in the half-duplex mode; that is, they cannot receive and trannsmit simultaneously, but on different timeslots. Hence, in the first subslot of each transmission slot they listen to the source, whereas in the second subslot they send the processed data, along with a packet that contains source-relay channel state information (CSI), to the destination.

We denote by $\mathcal{R}_i$ the relaying channel associated with R_i, with corresponding equivalent SNR represented by γ_i, $i \in \{1, 2\}$. In the sequel, we use the subscript i to refer to both subscripts 1 and 2 (i.e., $i \in \{1, 2\}$), and these relaying channels are termed branches, since they actually represent the input branches of the virtual SSC. The active branch is denoted

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by $\mathcal{A}$, so that the event that the $\mathcal{R}_i$ branch is active can be concisely written as $\mathcal{A} = \mathcal{R}_i$. The instantaneous SNRs of the S-D, S-R_i and R_i-D channels are represented by γ_{SD}, γ_{SR_i} and γ_{R_iD}, respectively. Further, we assume that these channels experience independent, flat and slow Rayleigh fading, with average SNRs denoted by $\overline{\gamma}_{SD}$, $\overline{\gamma}_{SR_i}$ and $\overline{\gamma}_{R_iD}$, respectively. Also, the transmission slots are considered small enough so that constant fading conditions during two consecutive slots can be assumed.

A. *Mode of Operation*

Being a distributed version of the SSC techniques, the proposed system activates only one of the two relays during each transmission slot, in a switch-and-stay fashion. More specifically, in each transmission slot the destination compares the equivalent SNR of the active branch with a switching threshold, denoted by T. If the SNR is lower than T, then a branch-switching occurs. This is implemented by appropriate feedback sent to both relays, which in the next slot switch from the active to the idle mode and vice versa. In a word, the destination keeps receiving from (and keeps estimating the equivalent SNR of) a single branch, regardless of the channel conditions of the other, until the equivalent SNR of that branch falls below T.

Depending on the hardware complexity the destination can tolerate, it can be adjusted to receive only from the relays, or from both the source and the relays during the first and second subslot, respectively. In the latter case, the destination combines the received signals in a maximal ratio combiner (MRC). Apparently, employing a MRC at the destination enhances the performance; this, however, comes at the cost of complexity, and in cases where the S-D channel is deeply shadowed this MRC employment might be useless. We note that the reader should not confuse this "actual" diversity combiner with the virtual SSC that our system employs, as described above.

III. Two-Relay DSSC without MRC at the Destination

In cases where employing a diversity combiner at the destination is either unfeasible due to hardware constraints, or just superfluous due to deep shadowing in the S-D channel, the proposed system can be thought of as a virtual SSC scheme, where the two input branches are $\mathcal{R}_1$ and $\mathcal{R}_2$.

A. *Relays Operate in the DF mode*

If DF relays are used, the signal that reaches the destination through $\mathcal{R}_i$ undergoes two demodulations in cascade. Thus, γ_i is not trivially derived. In the sequel, we adopt the outage-based definition for γ_i; specifically, γ_i is defined such that its cumulative density function (CDF) evaluated at the outage threshold SNR, γ_{th} (where $\gamma_{th} = 2^{2r} - 1$, with r representing the target rate), coincides with the outage probability of the $\mathcal{R}_i$ branch (i.e., $F_{\gamma_i}(\gamma_{th}) = \Pr\{\mathcal{O}\,|\mathcal{A} = \mathcal{R}_i\}$, where $F_Z(\cdot)$ stands for the CDF of the random variable Z and $\mathcal{O}$ denotes the outage event). Considering that $\Pr\{\mathcal{O}\,|\mathcal{A} = \mathcal{R}_i\}$ is the probability of the union of the outage events corresponding to the S-R_i and R_i-D channels, γ_i is defined as

$$\gamma_i := \min\left(\gamma_{SR_i}, \gamma_{R_iD}\right). \tag{1}$$

We emphasize here, however, that γ_i is the quantity associated with the $\mathcal{R}_i$ branch that is compared with T, and does not generally represent the equivalent SNR of that branch with the strict sense (i.e., if BPSK modulation is used, $\Pr\{\mathcal{E}\,|\mathcal{A} = \mathcal{R}_i, \gamma_i\} \neq \frac{1}{2}\mathrm{erfc}\left(\sqrt{\gamma_i}\right)$, where $\mathcal{E}$ represents the bit-error event and $\mathrm{erfc}(\cdot)$ is the complementary error function).

1) Outage Performance: The outage probability of the proposed system is straightforwardly derived by utilizing the outage analysis of SSC systems [6, eq. (9.327)] as

$$P_{out}(\gamma_{th}) = \begin{cases} \frac{F_{\gamma_1}(T)F_{\gamma_2}(T)\left(F_{\gamma_1}(\gamma_{th})+F_{\gamma_2}(\gamma_{th})\right)}{F_{\gamma_1}(T)+F_{\gamma_2}(T)}, \gamma_{th} < T \\ \frac{F_{\gamma_1}(T)F_{\gamma_2}(T)\left(F_{\gamma_1}(\gamma_{th})+F_{\gamma_2}(\gamma_{th})-2\right)}{F_{\gamma_1}(T)+F_{\gamma_2}(T)} \\ +\frac{F_{\gamma_1}(\gamma_{th})F_{\gamma_2}(T)+F_{\gamma_2}(\gamma_{th})F_{\gamma_1}(T)}{F_{\gamma_1}(T)+F_{\gamma_2}(T)}, \gamma_{th} \geq T \end{cases}, \tag{2}$$

where $F_{\gamma_i}(x) = 1 - \exp\left(-x/\overline{\gamma}_{SR_i}\right)\exp\left(-x/\overline{\gamma}_{R_iD}\right)$. We note that the choice of T significantly affects the outage performance: For a given γ_{th}, the outage probability is minimized by setting $T = \gamma_{th}$ (see [6, ch. 9.9.1.1]), since in that case (2) yields

$$\begin{aligned} P_{out}(\gamma_{th}) &= F_{\gamma_1}(\gamma_{th})\,F_{\gamma_2}(\gamma_{th}) \\ &= \prod_{i=1}^{2}\left[1 - \exp\left(-\frac{\gamma_{th}}{\overline{\gamma}_{SR_i}}\right)\exp\left(-\frac{\gamma_{th}}{\overline{\gamma}_{R_iD}}\right)\right], \end{aligned} \tag{3}$$

i.e., *the optimal outage probability of DSSC equals that of a system that selects the best of the $\mathcal{R}_1$ and $\mathcal{R}_2$ channels for each transmission slot.*

2) BEP Analysis: Let us denote with $p_{\mathcal{R}_i}$ the steady-state selection probability of $\mathcal{R}_i$, (i.e. $p_{\mathcal{R}_i} = \Pr\{\mathcal{A} = \mathcal{R}_i\}$), which has been evaluated in [4] as

$$\begin{aligned} p_{\mathcal{R}_i} &= \frac{F_{\gamma_j}(T)}{\sum_{k=1}^{2} F_{\gamma_k}(T)} \\ &= \frac{1 - \exp\left(-\frac{T}{\overline{\gamma}_{SR_j}}\right)\exp\left(-\frac{T}{\overline{\gamma}_{R_jD}}\right)}{\sum_{k=1}^{2}\left[1 - \exp\left(-\frac{T}{\overline{\gamma}_{SR_k}}\right)\exp\left(-\frac{T}{\overline{\gamma}_{R_kD}}\right)\right]}, \end{aligned} \tag{4}$$

where j is the complement of i with respect to $\{1, 2\}$, i.e., $j = 2$ if $i = 1$ and vice versa. Then, considering the system's mode of operation described in Section II-A, the BEP can be expressed as

$$\begin{aligned} \Pr\{\mathcal{E}\} = &\sum_{i=1}^{2} p_{\mathcal{R}_i}\left[F_{\gamma_i}(T)\Pr\{\mathcal{E}\,|\mathcal{A} = \mathcal{R}_j\}\right. \\ &\left.+ \left(1 - F_{\gamma_i}(T)\right)\Pr\{\mathcal{E}\,|(\mathcal{A} = \mathcal{R}_i \text{ and } \gamma_i \geq T)\}\right]. \end{aligned} \tag{5}$$

Assuming uncoded BPSK modulation, the conditional BEP (conditioned on the SNR, γ) is defined as $\Pr\{\mathcal{E}\,|\gamma\} = 1/2\ \mathrm{erfc}\left(\sqrt{\gamma}\right)$. Moreover, the $\mathcal{R}_i$ branch leads to an error if an error on either the S-R_i or the R_i-D link (but not on both) occurs. Therefore, the conditional BEP, conditioned on the event $\mathcal{A} = \mathcal{R}_i$ and $\gamma_i \geq T$, is obtained by averaging over the exponential probability density functions (PDFs) of γ_{SR_i} and γ_{R_iD} as

$$\begin{aligned} &\Pr\{\mathcal{E}\,|(\mathcal{A}=\mathcal{R}_i,\, \gamma_i \geq T)\} \\ =\;& \frac{1}{2\overline{\gamma}_{SR_i}} I\left(\frac{1}{\overline{\gamma}_{SR_i}},1,T\right) + \frac{1}{2\overline{\gamma}_{R_iD}} I\left(\frac{1}{\overline{\gamma}_{R_iD}},1,T\right) \\ & - \frac{1}{2\overline{\gamma}_{SR_i}\overline{\gamma}_{R_iD}} I\left(\frac{1}{\overline{\gamma}_{SR_i}},1,T\right) I\left(\frac{1}{\overline{\gamma}_{R_iD}},1,T\right), \quad (6) \end{aligned}$$

where the auxiliary function $I(\alpha,\beta,\omega)$ is defined as (see [2, Appendix])

$$\begin{aligned} I(\alpha,\beta,\omega) &= \int_{\omega}^{\infty} e^{-\alpha x}\mathrm{erfc}\left(\sqrt{\beta x}\right)dx \quad (7) \\ &= \frac{1}{\alpha}e^{-\alpha\omega}\mathrm{erfc}\left(\sqrt{\beta\omega}\right) - \frac{\sqrt{\beta}}{\alpha\sqrt{\alpha+\beta}}\mathrm{erfc}\left(\sqrt{(\alpha+\beta)\,\omega}\right). \end{aligned}$$

Consequently, a closed-form expression for the BEP is derived by inserting (4) and (6) in (5). Note that the probabilities $\Pr\{\mathcal{E}\,|\mathcal{A}=\mathcal{R}_i\}$ can be also expressed as shown in (6), by setting $T=0$.

B. Relays Operate in the AF mode

Let us now consider that both relays operate in the AF mode. We further assume that the relays are capable of eliminating half of the propagated noise power by using the signal-rotation technique proposed in [7], resulting in an equivalent SNR $\gamma_i = \gamma_{SR_i}\gamma_{R_iD}/\left(\gamma_{SR_i}+\gamma_{R_iD}+1/2\right)$. However, in the sequel we focus on the following tight upper bound of γ_i

$$\gamma_i = \frac{\gamma_{SR_i}\gamma_{R_iD}}{\gamma_{SR_i}+\gamma_{R_iD}}, \quad (8)$$

which in fact corresponds to an ideal relay gain capable of inverting the attenuation in the S-R_i link ignoring the noise. The same study regarding the bound of γ_i was also conducted in [8]- [9], where the authors showed that (8) results in a tight bound of the corresponding performance metrics (which is even tighter when the noise-reduction technique [7] is being used), especially for medium and high SNRs. Note that, contrary to the DF case, γ_i represents the metric of the $\mathcal{R}_i$ branch that is compared with T, and is also the SNR associated with the performance corresponding to this branch.

1) Outage Performance: Similarly to the DF case, the outage probability for the AF relaying scenario is obtained directly from (2) (and from (3) as well, assuming that the optimal $T=\gamma_{th}$ has been set), by substituting $F_{\gamma_i}(\cdot)$ with (see [8])

$$F_{\gamma_i}(x) = 1 - \frac{2x}{\sqrt{\rho_i}}e^{-\frac{\sigma_i}{\rho_i}x}K_1\left(\frac{2x}{\sqrt{\rho_i}}\right), \quad (9)$$

where $\sigma_i = \overline{\gamma}_{SR_i}+\overline{\gamma}_{R_iD}$, $\rho_i = \overline{\gamma}_{SR_i}\overline{\gamma}_{R_iD}$ and $K_v(\cdot)$ stands for the modified Bessel function of the second kind and order v.

2) BEP Analysis: For uncoded BPSK modulation, an analytical expression for the BEP is obtained from (5) by averaging the conditional BEP expressions over the PDF of γ_i given in [8, eq. (12)]. Unfortunately, such expression is not easily tractable and cannot be further simplified. In the high-SNR regime, however, the BEP for BPSK modulation can be approximated in closed-form by utilizing (2) and the alternative definition of the conditional BEP, $\Pr\{\mathcal{E}\,|\gamma\} = Q\left(\sqrt{2\gamma}\right)$, where $Q(\cdot)$ is the Gaussian Q-function, as follows: Let X be a Gaussian distributed random variable with zero-mean and unitary variance (i.e., $X\sim\mathcal{N}(0,1)$). Denoting with $\gamma_{\mathcal{A}}$ the system SNR, i.e., $\gamma_{\mathcal{A}}=\gamma_i$ if $\mathcal{A}=\mathcal{R}_i$, and using the definition of the Gaussian Q-function, the BEP is expressed as

$$\begin{aligned} \Pr\{\mathcal{E}\} &= \Pr\left\{X > \sqrt{2\gamma_{\mathcal{A}}}\right\} = \Pr\left\{\gamma_{\mathcal{A}} < \frac{X^2}{2}\right\} \quad (10) \\ &= \int_0^{\infty} P_{out}\left(\frac{X^2}{2}\right)\frac{e^{-\frac{X^2}{2}}}{\sqrt{2\pi}}dX = \int_0^{\infty}\frac{P_{out}(y)\,e^{-y}}{2\sqrt{\pi}\sqrt{y}}dy. \end{aligned}$$

Using the approximation $K_1(z)\approx 1/z$ for $z<<1$ [10, eq. (9.6.9)], (9) yields

$$F_{\gamma_i}(x) \approx 1-e^{-\frac{\sigma_i}{\rho_i}x} = 1-\exp\left(-x/\overline{\gamma}_{SR_i}\right)\exp\left(-x/\overline{\gamma}_{R_iD}\right). \quad (11)$$

Notice that (11) gives a high-SNR approximation for $F_{\gamma_i}(\cdot)$ which is identical with the CDF of $\min\left(\gamma_{SR_i},\gamma_{R_iD}\right)$; that is, in the high-SNR region we may define γ_i as in (1), instead of (8), a fact which was also addressed in [11, Property 1].

By substituting (11) into (2) and then inserting (2) in (10), after some manipulations we infer

$$\begin{aligned} \Pr\{\mathcal{E}\} \approx\; & \tfrac{1}{2} + \frac{\mathrm{erf}\left(\sqrt{T}\right)\left(e^{\frac{\sigma_1}{\rho_1}T}+e^{\frac{\sigma_2}{\rho_2}T}-2\right)}{2\sum_{i=1}^{2}\left(e^{\frac{\sigma_i}{\rho_i}T}-e^{\left(\frac{\sigma_i}{\rho_i}+\frac{\sigma_j}{\rho_j}\right)T}\right)} \\ & + \frac{\sum_{i=1}^{2}\left[\frac{\left[1-e^{\frac{\sigma_i}{\rho_i}T}\left(1+\mathrm{erfc}\left(\sqrt{\frac{T(\rho_i+\sigma_i)}{\rho_i}}\right)\right)\right]\left(1-e^{\frac{\sigma_j}{\rho_j}T}\right)}{\sqrt{\frac{\rho_i+\sigma_i}{\rho_i}}}\right]}{2\sum_{i=1}^{2}\left(e^{\frac{\sigma_i}{\rho_i}T}-e^{\left(\frac{\sigma_i}{\rho_i}+\frac{\sigma_j}{\rho_j}\right)T}\right)} \quad (12) \end{aligned}$$

where we have used the fact that $\int x^{-1/2}e^{-zx}dx = \sqrt{\pi/z}\mathrm{erf}\left(\sqrt{zx}\right)$ [12, eq. (8.251.1)].

IV. Two-Relay DSSC with MRC at the Destination

Now, let us assume that the destination is equipped with a MRC, so that it can optimally combine the signals received from the S-D and one of the $\mathcal{R}_1,\mathcal{R}_2$ branches. Specifically, in the first subslot of each transmission slot the destination receives the signal incident from S, and inserts it into a time-diversity MRC. At the same time, the relays also receive the same signal but only the active relay (as this is determined by the switch-and-stay process) forwards it to the destination in the second subslot, in order to be inserted into the MRC. Then, the destination compares the SNR at the MRC output with the switching threshold, T, and if this SNR is lower than T, it sends appropriate feedback to the relays, indicating their next-slot transition from the active to the idle mode and vice versa.

A. Relays Operate in the DF mode

1) Outage Performance: Using the outage-based definition of γ_i (i.e., $\gamma_i : F_{\gamma_i}(\gamma_{th}) = \Pr\{\mathcal{O}\,|\mathcal{A}=\mathcal{R}_i\}$), together with the fact that an outage occurs if neither the direct nor the

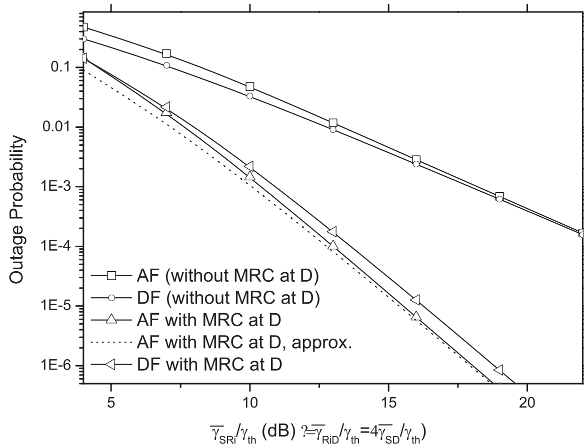

Fig. 2. The proposed scheme's optimal outage probability.

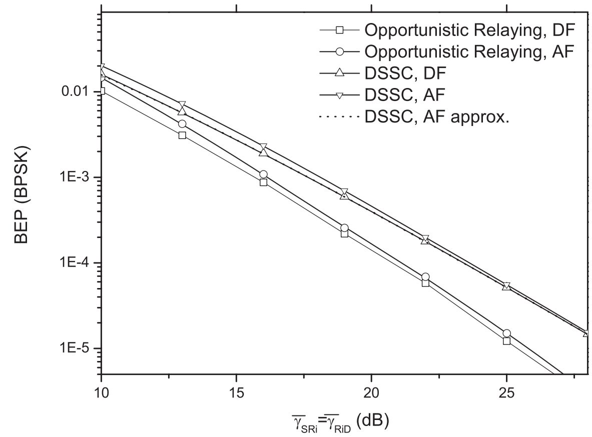

Fig. 3. BEP performance of the proposed and the OR scheme, assuming BPSK modulation and that no diversity combiner is employed at the destination.

relayed branch together with the direct one can support the target rate r, it holds[1]

$$F_{\gamma_i}(\gamma_{th}) = F_{\gamma_{SR_i}}(\gamma_{th}) F_{\gamma_{SD}}(\gamma_{th}) + \left(1 - F_{\gamma_{SR_i}}(\gamma_{th})\right) F_{g_i}(\gamma_{th}), \tag{13}$$

where $g_i = \gamma_i + \gamma_{SD}$ is the SNR at the combiner output when the $\mathcal{R}_i$ branch is active; $F_{g_i}(\cdot)$ is thus derived by convoluting $F_{\gamma_{R_iD}}(\cdot)$ with the exponential PDF of γ_{SD}. Hence, (13) yields

$$F_{\gamma_i}(\gamma_{th}) = \left(1 - e^{-\frac{\gamma_{th}}{\overline{\gamma}_{SR_i}}}\right)\left(1 - e^{-\frac{\gamma_{th}}{\overline{\gamma}_{SD}}}\right) \tag{14}$$
$$+ e^{-\frac{\gamma_{th}}{\overline{\gamma}_{SR_i}}} \frac{\overline{\gamma}_{R_iD}\left(1 - e^{-\frac{\gamma_{th}}{\overline{\gamma}_{R_iD}}}\right) - \overline{\gamma}_{SD}\left(1 - e^{-\frac{\gamma_{th}}{\overline{\gamma}_{SD}}}\right)}{\overline{\gamma}_{R_iD} - \overline{\gamma}_{SD}},$$

and therefore the outage probability is derived by substituting (14) in (2) (or in (3), in case of setting $T = \gamma_{th}$).

B. Relays Operate in the AF mode

In such case, γ_i is expressed as

$$\gamma_i = \frac{\gamma_{SR_i}\gamma_{R_iD}}{\gamma_{SR_i} + \gamma_{R_iD}} + \gamma_{SD}. \tag{15}$$

1) Outage Performance: An analytical expression for the CDF of γ_i is obtained by convoluting (9) with the exponential PDF of γ_{SD}, $f_{\gamma_{SD}}(\cdot)$; however, such expression is not easily tractable and cannot yield a closed-form expression for the $F_{\gamma_i}(\cdot)$. In the high SNR regime, using the approximation [10, eq. (9.6.9)], we infer

$$F_{\gamma_i}(x) \approx \frac{\rho_i\left(1 - e^{-\frac{\sigma_i}{\rho_i}x}\right) - \overline{\gamma}_{SD}\sigma_i\left(1 - e^{-\frac{x}{\overline{\gamma}_{SD}}}\right)}{\rho_i - \overline{\gamma}_{SD}\sigma_i}. \tag{16}$$

2) BEP Analysis: Working similarly as in Section III-B2, we may obtain a high-SNR approximation for the BEP, by utilizing (16), (2), (10) and the indefinite integral $\int x^{-1/2}e^{-zx}dx = \sqrt{\pi/z}\,\mathrm{erf}(\sqrt{zx})$; this BEP expression is given in eq. (17) shown at the top of the next page.

[1]We note that the protocol presented here differs from the DF protocol proposed in [1] in the sense that even if the source-relay link is in outage the destination still attempts to decode the message from the source-destination channel.

V. Numerical Examples and Discussion

In this section, we present a schematic illustration of the proposed schemes' performance, along with that of the opportunistic relaying (OR), where in each transmission slot the branch with the highest instantaneous SNR is selected (i.e., $\mathcal{A} = \mathcal{R}_{\arg\max_{i\in\{1,2\}}(\gamma_i)}$) [5]. In all figures, the S-R_i and R_i-D channels are assumed to experience independent and identically distributed Rayleigh fading, with average value equal to four times that of the (also Rayleigh distributed) S-D channel, i.e., $\overline{\gamma}_{SR_i} = \overline{\gamma}_{R_iD} = 4\overline{\gamma}_{SD}$.

Fig. 2 depicts the outage probability of the proposed scheme versus the normalized value of $\overline{\gamma}_{SR_i}$ with respect to γ_{th}. The switching threshold used is the optimal one, i.e., $T = \gamma_{th}$, and thus the curves shown in this figure portray also the outage probability of the corresponding OR schemes. In Fig. 2, both complexity-tolerance assumptions (i.e., with and without MRC at D) and both relaying modes (DF and AF) are considered. We note that for the AF case with MRC at the destination the solid line was derived through simulations; the dotted one using (16) and (3). Interestingly, one may notice that for the former assumption (i.e., no diversity combiner employed at the destination) the DF performance is silghtly better than the AF one, whereas for the latter AF outperforms DF. This is due to the fact that the combining weights employed by the MRC in the DF case do not take into account the source-relay channel, resulting in sub-optimum combining of the received signals; this fact was also addressed in [11].

The BEP performance of the DSSC and the OR systems versus $\overline{\gamma}_{SR_i}$ is depicted in Figs. 3 and 4, assuming uncoded BPSK modulation; in the former figure, the destination is assumed not to employ any diversity combiner, whereas in the latter, it employs a MRC. Each curve in these figures was generated by using the optimal switching threshold T, which is derived numerically by minimizing the corresponding BEP expressions with respect to T. For the case of DF relaying with MRC at the destination, the curves are derived via simulations, using the same switching threshold as that with the "no MRC" case. As expected, all OR schemes outperform the equivalent DSSC ones albeit achieving the same diversity order since, generally speaking, selection combining can be seen as an

$$
\begin{aligned}
\Pr\{\mathcal{E}\} \approx\; & \frac{\operatorname{erf}\left(\sqrt{T}\right) + \frac{\overline{\gamma}_{SD}^{3/2}\operatorname{erf}\left(\sqrt{\left(1+\frac{1}{\overline{\gamma}_{SD}}\right)T}\right)\sum_{i=1}^{2}\left(\rho_i\sigma_j-\overline{\gamma}_{SD}\sigma_i\sigma_j\right)-\sum_{i=1}^{2}\rho_i\sqrt{\frac{\rho_i\left(\overline{\gamma}_{SD}+1\right)}{\rho_i+\sigma_i}}\left(\rho_j-\overline{\gamma}_{SD}\sigma_j\right)\operatorname{erf}\left(\sqrt{\frac{T(\rho_i+\sigma_i)}{\rho_i}}\right)}{2\sqrt{\overline{\gamma}_{SD}+1}\prod_{i=1}^{2}\left(\rho_i-\overline{\gamma}_{SD}\sigma_i\right)}}{\sum\limits_{i=1}^{2}\left[\left(\rho_i\left(1-e^{-\frac{\sigma_i}{\rho_i}T}\right)-\overline{\gamma}_{SD}\sigma_i\left(1-e^{-\frac{T}{\overline{\gamma}_{SD}}}\right)\right)\left(\rho_j-\overline{\gamma}_{SD}\sigma_j\right)\right]\left(\prod\limits_{i=1}^{2}\left[\rho_i\left(1-e^{-\frac{\sigma_i}{\rho_i}T}\right)-\overline{\gamma}_{SD}\sigma_i\left(1-e^{-\frac{T}{\overline{\gamma}_{SD}}}\right)\right]\right)^{-1}} \\
& + \frac{\left[2\sigma_1\sigma_2\overline{\gamma}_{SD}^2\left(1-e^{-\frac{T}{\overline{\gamma}_{SD}}}\right)-\overline{\gamma}_{SD}\sum_{i=1}^{2}\sigma_i\rho_j\left(2-e^{-\frac{T}{\overline{\gamma}_{SD}}}-e^{-\frac{\sigma_j}{\rho_j}T}\right)+\rho_1\rho_2\left(2-\sum_{i=1}^{2}e^{-\frac{\sigma_i}{\rho_i}T}\right)\right]\operatorname{erfc}\left(\sqrt{T}\right)}{2\sum_{i=1}^{2}\left[\left(\rho_i\left(1-e^{-\frac{\sigma_i}{\rho_i}T}\right)-\overline{\gamma}_{SD}\sigma_i\left(1-e^{-\frac{T}{\overline{\gamma}_{SD}}}\right)\right)\left(\rho_j-\overline{\gamma}_{SD}\sigma_j\right)\right]} \\
& + \frac{\sum_{i=1}^{2}\left[\overline{\gamma}_{SD}\sigma_i\left(\rho_j\left(1-e^{-\frac{\sigma_j}{\rho_j}T}\right)-\overline{\gamma}_{SD}\sigma_j\left(1-e^{-\frac{T}{\overline{\gamma}_{SD}}}\right)\right)\left(1+\frac{\rho_i\left(1-e^{-\frac{\sigma_i}{\rho_i}T}\right)-\overline{\gamma}_{SD}\sigma_i\left(1-e^{-\frac{T}{\overline{\gamma}_{SD}}}\right)}{\rho_i-\overline{\gamma}_{SD}\sigma_i}\right)\right]\operatorname{erfc}\left(\sqrt{T+\frac{T}{\overline{\gamma}_{SD}}}\right)}{2\sqrt{1+\frac{1}{\overline{\gamma}_{SD}}}\sum_{i=1}^{2}\left[\left(\rho_i\left(1-e^{-\frac{\sigma_i}{\rho_i}T}\right)-\overline{\gamma}_{SD}\sigma_i\left(1-e^{-\frac{T}{\overline{\gamma}_{SD}}}\right)\right)\left(\rho_j-\overline{\gamma}_{SD}\sigma_j\right)\right]} \\
& - \frac{\sum_{i=1}^{2}\left[\sqrt{\frac{\rho_i^{3/2}}{\rho_i+\sigma_i}}\operatorname{erfc}\left(\sqrt{\frac{T(\rho_i+\sigma_i)}{\rho_i}}\right)\left(\rho_j\left(1-e^{-\frac{\sigma_j}{\rho_j}T}\right)-\overline{\gamma}_{SD}\sigma_j\left(1-e^{-\frac{T}{\overline{\gamma}_{SD}}}\right)\right)\left(1+\frac{\rho_i\left(1-e^{-\frac{\sigma_i}{\rho_i}T}\right)-\overline{\gamma}_{SD}\sigma_i\left(1-e^{-\frac{T}{\overline{\gamma}_{SD}}}\right)}{\rho_i-\overline{\gamma}_{SD}\sigma_i}\right)\right]}{2\sum_{i=1}^{2}\left[\left(\rho_i\left(1-e^{-\frac{\sigma_i}{\rho_i}T}\right)-\overline{\gamma}_{SD}\sigma_i\left(1-e^{-\frac{T}{\overline{\gamma}_{SD}}}\right)\right)\left(\rho_j-\overline{\gamma}_{SD}\sigma_j\right)\right]}
\end{aligned}
\tag{17}
$$

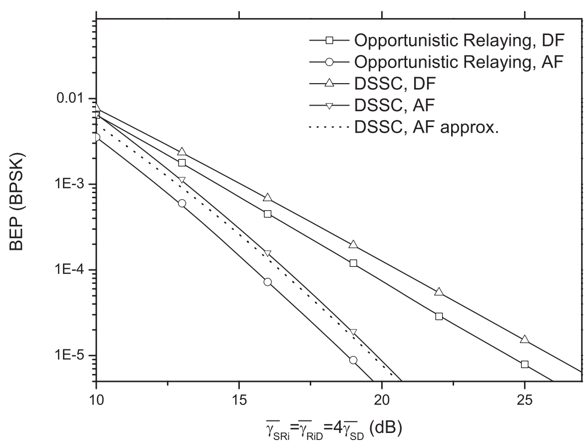

Fig. 4. BEP performance of the proposed and the OR scheme, assuming BPSK modulation and MRC at the destination.

optimal yet more complex implementation of SSC.

The simplicity of DSSC lies in the fact that only a single branch is estimated in each transmission slot, and that feedback to the relays is not sent continuously, but only after a branch-switching decision. That is, neither global CSI nor feedback in each transmission slot is needed. Finally, it is interesting to note the difference between the slope of the BEP curves for the DF case without MRC employment and the corresponding outage curves shown in Fig. 2, leading to superior AF performance compared to the DF one. This stems from the fact that, contrarily to Fig. 2, uncoded modulation is assumed in Figs. 3 and 4, resulting in a significant DF performance degradation due to error propagations.

VI. Conclusions

We presented the concept of two-relay distributed switch and stay combining, where only one out of two available relays is activated in a switch and stay fashion. This allows for only a single end-to-end branch to be estimated in each transmission period, offering thus a simpler alternative of relay selection while still attaining the same diversity gain, as well as identical outage probability.

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