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SEISMIC HAZARD ASSESSMENT OF EARTHQUAKES IN GREECE USING DIRECTED EARTHQUAKE NETWORKS

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ABSTRACT

The objective of this study is the seismic hazard evaluation in the area of Greece based on modeling of the transition probabilities of seismicity as a Markov and semi-Markov chain model. The data consist of strong earthquakes with magnitude $M_{thr} \geq 5.5$ that occurred during the period 1911-2015 are considered. The study area is divided into 5 subareas (seismic zones) that are homogenous from the seismotectonic point of view and the seismic catalog is divided into subsets for four magnitude ranges. Two Markov chains are defined with respect to predefined time window, one having as states the occurrence or not of strong earthquakes in any of the 5 subareas ($2^5 = 32$ states), and one having as states the occurrence or not of strong earthquakes anywhere in Greece at any of the four magnitude ranges ($2^4 = 16$ states). The states constitute the nodes of a network with weighted directed connections defined by the transition probabilities of the Markov chain. The null hypothesis that the Markov chain has no memory is rejected using test statistics for three memoryless models (uniform, Poissonian and fixed Markov chain). It is confirmed that the degree (strength) distribution of the network matches well the limiting state distribution of the Markov chain. In a different approach, two semi-Markov models are developed, one for subareas (5 states) and one for magnitudes (4 states), for the sequence of strong earthquakes using appropriate time step and core matrices. The semi-Markov model on the subareas and magnitudes is found to give satisfactory aftcast (estimation of the next transition considering the data until the time of forecast), which is regarded an estimate of seismic hazard. Finally, a new approach that combines the Markov and semi-Markov models is attempted in order to estimate the occurrence probability of the next strong earthquake assuming that the previous strong earthquake (for semi-Markov model) and the previous state (for Markov chain) are given.

Keywords: Complex networks, Seismic zones, Magnitude ranges, Markov chain, Semi-Markov chain.

1. INTRODUCTION

The seismic hazard assessment is one of the main targets of seismological research aiming to contribute in reducing the catastrophic consequences from strong earthquakes occurrence. By seismic hazard assessment we mean the probability of occurrence of strong earthquakes within a given space, time, and magnitude ranges. The most known model, which is referred for seismic hazard assessment, is the Poisson model for random series of events (earthquakes). The main conditional of this model is that the earthquake occurrences are independent in space and time. Therefore, the Poisson model is frequently applied for statistical analysis of seismicity (Lomnitz 1974, Bath 1978, Brillinger 1982, Lomnitz and Nava 1983). The Markov chain model was introduced as a suitable means for earthquake probability estimation, which contrary to the Poisson model, assumes that all events are dependent on one another in space and time (Tsapanos and Papadopoulou 1999, Console 2001, Console et al. 2002, Nava et al. 2005). The semi-Markov model employed in order to estimate the waiting time and magnitudes of strong earthquakes (Altınok 1991, Altınok and Kolçak 1999). According to the semi-Markov model, the next strong earthquake depends on the previous one and the time elapsed between them. A different emerging field for seismic hazard assessment is based on network theory. The complex network analysis was introduced by Abe and Suzuki (2004) in order to study seismicity as a spatiotemporal complex system. Considerable research work was accomplished on network theory and its applications in different disciplines ranging from communication and economics to biology and neuroscience (Wang and Chen 2003, Emmert-Streib and Dehmer 2010, Rubinov and Sporns 2010).

The main purpose of this study is to provide earthquake estimates using earthquake data for the Greek area. It is shown that the seismicity can be modeled as a Markov and semi-Markov chain. Earthquake network is formed on the basis of the transition probability matrix of the Markov chain model and the core matrix of the semi-Markov chain model. A new approach that combines the Markov and semi-Markov models is attempted in order to identify the space, time, and magnitude of the next strong earthquake.

2. METHODOLOGY

The section of methodology is divided in 7 subsections. In the first subsection the magnitude threshold of data and the states of systems are defined, in the second subsection the time interval of chains is determined, in the next three subsections the models (Markov, semi-Markov and new approach) are presented, in the sixth subsection the evaluation scheme for the results is presented and in the seventh subsection the network measures used in the study are briefly described.

2.1 The definitions of data and states

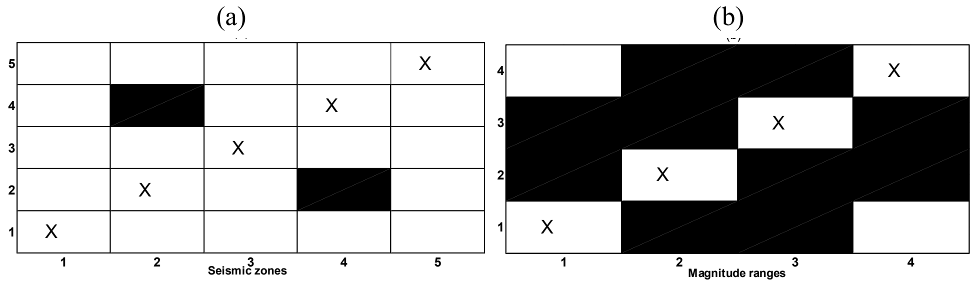
The data must satisfy the completeness requirements, namely, to contain all the earthquakes. Thus, the data include earthquakes from seismic catalog with magnitude $M \geq M_{thr}$, where M_{thr} the magnitude of completeness. Concerning the definition of states, we divide the study area into R seismic zones, which are homogenous from the seismotectonic point of view (faulting type, seismic moment rate) and taking into account previous results (Papaioannou and Papazachos 2000). In addition, to estimate the magnitude we create K magnitude ranges.

Regarding the Markov chain model, for each time interval $\Delta\tau$ (the way it is defined is shown below, see section 2.2), we create two chains one for the regions and one for the magnitudes. Each state of seismic zones (regions) or magnitude ranges takes the value 0 or 1, corresponding to the absence or presence of earthquakes in the respective seismic zones or magnitude ranges. The chain for the regions is created by the earthquakes which are occurred in the seismic zones regardless of the magnitude ranges. In the other case the chain for the magnitudes is created by the magnitude ranges regardless of the seismic zones in which are occurred the earthquakes. The total number of states of the system, for the seismic zones or the magnitude ranges, are 2^R (R denotes the number of seismic zones) or 2^K (K denotes the number of magnitude ranges). In the binary form, each state can be denoted $z_R \dots z_2 z_1$ ($z_1, z_2, \dots, z_R$ the seismic zones) or $m_K \dots m_2 m_1$ ($m_1, m_2, \dots, m_K$ the magnitude ranges) is simply the right to left concatenation of the binary seismic zones or magnitude ranges states. The disadvantage of this approach is that the succession of earthquakes in each $\Delta\tau$ is not to be taken into account. Two successive states formed at two subsequent time intervals $\Delta\tau$, define the transition between of the states.

If the seismic activity within the time window $\Delta\tau$ were to be independent with respect to the magnitude ranges or seismic zones, the construction of states presented above would not be required and the analysis could be done separately at each magnitude range or seismic zone. The correlation analysis showed that both magnitude ranges and seismic zones are indeed correlated, as shown in Fig. 1. Specifically, for a step $\Delta\tau$=0.5 year, we consider the five series of earthquake occurrence frequency of the five seismic zones as well as the four series of earthquake occurrence frequency of the four magnitude ranges, having 210 data points per series. For each pair of seismic zone series the Pearson correlation coefficient is computed and the parametric significance test using the t-statistic is performed at the significance level $a = 0.05$. The results are shown in matrix form in Fig. 1a and the same results are shown for the magnitude ranges in Fig.1b. The black color ($t < 0.05$) in the pairs of seismic zones and magnitude ranges reveals the correlation between them (Fig. 1).

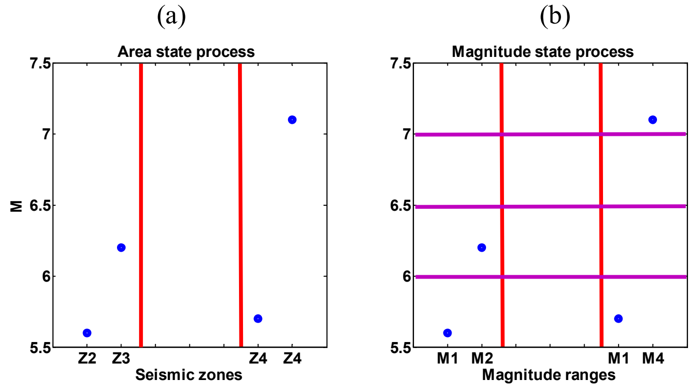
In case of a semi-Markov model we also create two chains where the state space S is simply the R seismic zones or K magnitude ranges, respectively, as the focus is on the region or magnitude of the next strong earthquake. Thus, the total number of states of the system, for the seismic zones and the magnitude ranges, are R and K, respectively. For the transitions we take into account the time units (holding time), integer multiple of $\Delta\tau$, where the process of semi-Markov chain may remain at state i before made the transition in state j. Thus, two successive earthquakes define the transition between the respective states for the semi-Markov chain taking into account the holding time.

Figure 1. *The value of the test statistic t among of seismic zones in (a) and magnitude ranges in (b). The black color in cells shows that the corresponding pairs are correlated ($t < 0.05$).*



For example, if $R = 4$ and $M = 4$ we define the two next transitions within the three predefined time intervals Δτ, separated by red lines in Figure 2, as described below. For the Markov chain about regions for each time interval Δτ, we have $z_4z_3z_2z_1 = 0110$ (state 7), $z_4z_3z_2z_1 = 0000$ (state 1) and $z_4z_3z_2z_1 = 1000$ (state 15), the occurrence of earthquake represented by 1, so the transitions are 7→1→15. For the semi-Markov chain the respective transitions are $2\xrightarrow{1}3\xrightarrow{2}4\xrightarrow{1}4$, where the numbers above the arrows represent the holding time. The transitions in case of magnitudes are defined similarly. Therefore, for a Markov chain we have $m_4m_3m_2m_1 = 0011$ (state 4), $m_4m_3m_2m_1 = 0000$ (state 1) and $m_4m_3m_2m_1 = 1001$ (state 10), so the transitions are 4→1→10. For the semi-Markov the respective transitions are $1\xrightarrow{1}2\xrightarrow{2}1\xrightarrow{1}4$ (Fig. 2).

Figure 2. *An example about the definition of states systems (Markov and semi-Markov) about the seismic zones in (a) and the magnitude ranges in (b). The red vertical lines divide the time intervals $\Delta\tau$ and the purple horizontal lines create the magnitude ranges.*



2.2 Determination of time interval of chains

For the threshold time chosen, $\Delta\tau$ should be large enough to allow interaction among seismic zones or magnitude ranges in order the transition probabilities to be robust. On the other hand, $\Delta\tau$ should be small enough such that the hazard assessment be useful. If $\Delta\tau$ is too small, the most frequently occurring transition is from state 1 to state 1 (no earthquake occurs), whereas if it is too large state 2^R or 2^K to state 2^R or 2^K (earthquakes occur in all possible zones or magnitudes ranges) transitions are dominant. We consider three criteria given by the following three functions in order to determine the time interval $\Delta\tau$ for the Markov chain model.

Function 1 is the difference between the number of transitions from state 1 to state 1 and the number of transitions from state 2^R or 2^K to state 2^R or 2^K, respectively.

Function 2 is the difference between the total number of transitions from state 1 and the total number of transitions from state 2^R or 2^K.

Function 3 is given by $F = -\sum_i \pi_i \sum_j p_{ij} log_2 p_{ij}$. This function is based on the maximum entropy principle as it is applied to finite Markov chains, where p_{ij} are elements of the transition probability matrix P and $\pi = \{\pi_i\}$ is the stationary distribution of the Markov chain.

To satisfy the functions 1-3, $\Delta\tau$ is chosen so that both Functions 1 and 2 show a value close to 0 and Function 3 is maximized.

2.3 Markov model

The Markov model is a probabilistic one useful in analysing stochastic phenomena. Suppose $S = \{1, 2, \ldots\ldots, 2^R\ or\ 2^K\}$ is the state space of a Markov chain. Let us define $\{X_t\}$, where t is a time index (at multiples of $\Delta\tau$), be a Markov chain formed by the time succession of states with values from the state space S. The Markov chain is defined in terms of a transition probability matrix: $P = P\{j|i\} = \{p_{ij}\}$, where p_{ij} is the probability that the state j follows state i with $i, j = 1, 2, \ldots, 2^R\ or\ 2^K$. From an observed Markov chain over n time units, the transition probability p_{ij} is estimated by the ratio of successions of the ordered pair (i,j),$\{l_{ij}\}$, over all observed successions starting at state i, $\{p_{ij}\} = \{l_{ij}\}/\sum_{j=1}^{N}\{l_{ij}\}$.

The transition probabilities p_{ij} satisfy $p_{ij} \geq 0$ and $\sum_{j=1}^{N} p_{ij} = 1$. Given p_{ij} and the system is in state i, we express the conditional probability of an earthquake occurring in seismic zone z_b, with $b = 1, \ldots, R$, or regional activity probability, p_{iz_b}as: $p_{iz_b} = \Pr\{z_b|i\} = \sum_{j \ni z_b} p_{ij}$, where $j \ni z_b$ means that state j includes seismicity in zone z_b (Nava et al. 2005). The regional activity probability implies that the probability of a Markov process which has entered state i will enter zone z_b on its next transition depends only upon the current state. The magnitude activity probability, p_{lm_b}, with $b = 1, \ldots, K$ is defined in a similar way.

2.4 Discrete semi-Markov model

The behaviour of a discrete semi-Markov model is similar with that of a pure Markov model. A discrete time semi-Markov process is defined completely by the transition probabilities, the holding time mass functions and the core matrices or discrete semi-Markov kernels (Pyke 1961, Barbu and Limnios 2009). The transition probability $\{p_{ij}\}$ is the probability that a semi-Markov chain that entered state i on its last transition will enter state j on its next transition, allowing for a holding time *m* for the transition to take place (Trevezas and Limnios 2011). The standard definition of a semi-Markov chain thus excludes the transition from one state to the same state. Applying directly the semi-Markov model to our setting, having the earthquakes at any seismic zone or magnitude range as the R or K states, respectively, would require to define the time steps by the running index in the sequence of successive earthquakes, which is not a natural time parameter. In our setting, we have defined the time steps in terms of the interval $\Delta\tau$. Since more than one earthquake at a different seismic zone or magnitude range can occur in the interval $\Delta\tau$ we modify the definition of the semi-Markov model and assume that transitions from state (earthquake) *i* to *j* are allowed within the interval $\Delta\tau$ and assign for such transitions the holding time t_{ij}=0. Accordingly, t_{ij}=1 regards the transition from a state (earthquake at a seismic zone or magnitude range) occurring at a time $t_k = k\Delta\tau$, where this is actually the whole interval ($(k-1)\Delta\tau, k\Delta\tau)$, to a state occurring in the next time $t_{k+1} = (k+1)\Delta\tau$. In this case, the state i refers to the last earthquake occurring at the time interval ($(k-1)\Delta\tau, k\Delta\tau)$. According to this definition of the semi-Markov model, transitions from one state to the same state are allowed. The probability mass function T_{ij} is called the holding time mass function for a transition from state i to j and is given as $Pr\{t_{ij} = m\} = T_{ij}(m)$, where m is the time unit (0 within $\Delta\tau$, and otherwise multiples of $\Delta\tau$). The final step is to define the core matrices (discrete semi-Markov kernels). The $c_{ij}(m)$ element of the core matrix $C(m)$ is the probability of the joint event that a system that entered state i makes its next transition to state j and this takes place after a holding time m (Altinok and Kolcak 1999). The core matrix is given by: $C(m) = c_{ij}(m) = p_{ij} \otimes T_{ij}(m)$, where $\otimes$ denotes multiplication of corresponding elements.

2.5 New approach combining the Markov and semi-Markov chain

Let us first concentrate on the seismic zones (the approach for the magnitude ranges is similar). The interest here is to determine the probability of having a strong earthquake at the next time interval $\Delta\tau$ in one of the seismic zones given the information of earthquake occurrences at the present time interval. The states of the Markov chain do not contain the information about the order of occurrences of earthquakes at a time interval, and particularly the last earthquake in this interval (referring to the states representing more than one earthquake occurrence in the time interval). On the other hand, as the semi-Markov model is defined, its state carries the information about the last earthquake occurrence regarding the present time interval. In the proposed approach, we attempt to combine the information of the state of the Markov model and the semi-Markov model for the same time interval, and further use it to predict the occurrence of an earthquake at a specific seismic zone within the next time interval. First, using the states of the Markov chain for the seismic zones, we calculate the regional activity probability matrix P_{reg} of size $2^R x\ R$ with components p_{iz_j}, where $i = 1, 2, \ldots, 2^R$, $j = 1, 2, \ldots, R$, and $z_1, z_2, \ldots, z_R$, are the R seismic zones. The p_{iz_j} denotes the probability that given the Markov model state i at time $t_k = k\Delta\tau$, a strong earthquake occurs at seismic zone z_j in the time interval $t_{k+1} = (t_k, t_k + \Delta\tau)$. We further involve the semi-Markov model defined in Section 2.4, and particularly the part of it that regards holding times t_{ij}=1, in order to use the information about the last earthquake in the current interval and predict that a strong earthquake occurs at seismic zone z_j in the time interval $(t_k, t_k + \Delta\tau)$. This information lies in the core matrix $c_{rj}(1)$ of size RxR with $r, j = 1, 2, \ldots, R$. The $c_{rj}(1) = c_{z_r z_j}(1)$, denoted for simplicity $c_{z_r z_j}$, is the probability that given the last strong earthquake is at seismic zone z_r in the current $\Delta\tau$ at time $t_k = k\Delta\tau$, a strong earthquake occurs at seismic zone z_j in the time interval $t_{k+1} = (t_k, t_k + \Delta\tau)$. The two transition probabilities p_{iz_j} and $c_{z_r z_j}$ from the Markov and the semi-Markov model, respectively, target in predicting the seismic zone an earthquake occurs in the next time interval $t_{k+1} = (t_k, t_k + \Delta\tau)$, the former on the basis of the state of seismic zones in $t_k = k\Delta\tau$, and the latter on the basis of the seismic zone of the last earthquake in t_k. We merge the two probabilities to the probability $q_{i,z_r,z_j} = p_{iz_r} c_{z_r z_j}$, which approximates the conditional probability $P(z_b|i, z_r)$, i.e. the occurrence of an earthquake in seismic zone z_j within the interval t_{k+1} when in the last interval t_k the state of seismic zones is i and the last earthquake in t_k is z_r. The probability q_{i,z_r,z_j} is an operationally suitable approximation rather than an exact expression of $P(z_b|i, z_r)$ using the probabilities obtained from the Markov and semi-Markov model. The probabilities q_{i,z_r,z_j} can be estimated by the corresponding frequencies of p_{iz_r} and $c_{z_r z_j}$. The probability $q_{im_r m_j} = p_{im_r} c_{m_r m_j}$ is defined similarly for the magnitude ranges.

2.6 Results Evaluation

For evaluating the performance of the Markov chain, we will compare the aftcast probability of the Markov model to a threshold formed according to the hypothesis that the system has no memory, and the seismicity cannot be modeled as a Markov chain. Three memoryless models are considered: uniform, Poisson and fixed Markov chain. The uniform model has transition probabilities $P_{ij}^{U} = P^{U} = S^{-1} = \{p_{ij}\}$, where the probabilities $\{p_{ij}\}$ correspond to purely random guessing. The Poisson model has transition probabilities $P_{ij}^{P} = P_{j}^{P} = \prod_{z_b \notin j}(1 - e^{-\lambda_{z_b}\Delta\tau})\prod_{z_b \in j}(e^{-\lambda_{z_b}\Delta\tau})$, where λ_{z_b} is the mean number of earthquakes per unit time in zone z_b and $z_b \in j$ means that state j includes seismicity occurrence in zone z_b. The third model is the fixed Markov chain model and is given by $p_{ij} = P_{j}^{O} = \frac{\xi_j}{\sum_i \xi_i}$, where $\xi_i = \sum_j t_{ij}$. For magnitude states the three models are defined accordingly. For the uniform model we consider a threshold probability $P^{H} = (1 + \mu)P^{U}$, where μ is an arbitrary non-negative constant. An aftcasted transition is defined as successful if $p_{ij} > P^{H}$. For the Poisson or fixed Markov chain model we have a successful aftcasted transition when $p_{ij} > P_{ij}^{P}$ or $p_{ij} > P_{j}^{O}$. Aftcasted, means that the probabilities p_{ij} are evaluated based on all available information considering the data until the time of forecast.

For evaluating the performance of the semi-Markov chain, it is tested whether the next predicted transition $c_{ij}(m)$ of system having the maximum probability among of the others pair of states is in agreement with the observed transition.

2.7 Network measures

Generally with the term «network» we mean the graph $G = (N, E)$ that is defined by the nodes and the connections between them, where N is the set of nodes and E the set of connections. In our analysis, the nodes are represented by the states of system and the directed weighted connections are defined by the elements of the transition probability matrix P (for Markov model) and core matrix C (for semi-Markov model), respectively. The network properties are quantified with a number of characteristics (network measures) computed on P and C.

The simplest and most known network measure is the degree (for binary connections) or strength (for weighted connections). This characteristic measures the number of connections or the sum of weights at each node *i* as $k_i^{P} = \sum_{j \in N} p_{ij}$. Essentially, the measure reveals whether a node is active in the network. Another well-known characteristic is the average clustering coefficient, which estimates the tendency of any node i to form connected triads given as $C^{\rightarrow} = \frac{1}{n}\sum_{i \in N}\frac{\sum_{j,h \in N} p_{ij} p_{ih} p_{jh}}{(k_i^{out} + k_i^{in})(k_i^{out} + k_i^{in} - 1) - 2\sum_{j \in N} p_{ij} p_{ji}}$, where k_i^{in} is the sum of connection weights directed to the node and k_i^{out} is the sum of connection weights leaving the node. A high value of the average clustering coefficient indicates higher likelihood existence of "clique" (clique is a group of fully connected nodes). The third characteristic is the betweenness centrality defined as the number of shortest paths between pairs of nodes that pass through a specific node given as $b_i^{\rightarrow} = \frac{1}{(N-1)(N-2)} \sum_{\substack{h,j \in \\ h \neq j, h \neq i, j \neq i}} \frac{\rho_{\vec{hj}}(i)}{\rho_{\vec{hj}}}$, where ρ_{hj} is the number of the shortest paths between h and j nodes and $\rho_{hj}(i)$ the number of the shortest paths between h and j nodes through the node i. Nodes with a high betweenness have a high probability to occur on a randomly chosen shortest path between two randomly chosen nodes. Such nodes are critical to the network since their removal would destroy many short paths in the network.

3. DATA AND APPLICATION

The data are obtained from the earthquake catalog compiled in the Geophysics Department of the Aristotle University of Thessaloniki (http://geophysics.geo.auth.gr/ss/). They comprise crustal earthquakes (focal depth less than 40 Km) that occurred in 1911-2015, and are divided in four subsets of magnitude $M_{thr} \geq 5.5$ (574 events), $M_{thr} \geq 5.5$ (444 events, after declustering), $M_{thr} \geq 6.0$ (188 events) and $M_{thr} \geq 6.0$ (154 events, after declustering). The data are complete for the study period and declustering was performed for testing subsets of the entire catalog or after removing dependent events with respect to foreshocks and aftershocks. Seismicity declustering is the identification and the separation of seismicity catalogs into main shocks (independent events), foreshocks and aftershocks (dependent events), so as to eliminate the interference of the already dense occurrence and strong dependence of the events which belong to an aftershock seismic excitation. The Reasenberg's algorithm (1985) is used, here, for the declustering procedure. The algorithm is used for identifying aftershock clusters based on a two-parameter earthquake interaction model producing a Poissonian declustered earthquake catalog which is deprived of correlated events.

The study area is divided in 7 subareas and we define 5 seismic zones to reduce complexity, which are homogenous from the seismotectonic point of view, and are shown in Figure 3 along with the epicentral distribution of the earthquakes used for the analysis. In addition, the seismic catalog is divided in 4 magnitude ranges for each of the 4 data subsets with the purpose of distinguishing the different levels of earthquake magnitudes (moderate, strong, major and great). In case of magnitude $M_{thr} \geq 5.5$ we define the following magnitude ranges: $5.5 \leq M_1 \leq 5.9$, $6.0 \leq M_2 \leq 6.4$, $6.5 \leq M_3 \leq 6.9$ and $M_4 \geq 7.0$. Then, with magnitude $M_{thr} \geq 6.0$ we define: $6.0 \leq M_1 \leq 6.2$, $6.3 \leq M_2 \leq 6.5$, $6.6 \leq M_3 \leq 6.9$ and $M_4 \geq 7.0$.

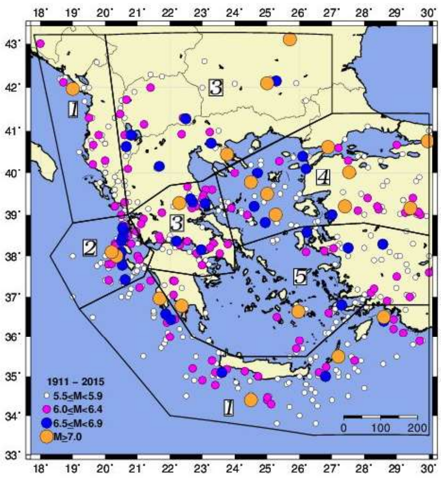
For the five seismic zones and for the four magnitude ranges there are $2^5 = 32$ and $2^4 = 16$ states, respectively. For example in the case of seismic zones, the state 1 (denoted 00000 in binary format) corresponds to no earthquake occurrence in all five seismic zones in the chosen time interval, $\Delta\tau$, unlike the state 32 (denoted 11111 in binary format) that represents earthquake occurrence in all five seismic. The combination of seismic zones or magnitude ranges gives the nodes of the network and the states of a Markov chain system.

In the case of semi-Markov chain, the nodes of the network and the states of the system correspond to the five seismic zones (5 states) and the four magnitude ranges (4 states), respectively. The directed weighted connections of a network are given by the transition probabilities p_{ij} of the Markov chain and the elements of core matrix $c_{ij}(m)$ of the semi-Markov chain.

For the selection of the time interval $\Delta\tau$ of chains, the optimal $\Delta\tau$ should give the largest entropy (Function 3) and the value of Functions 1 and 2 closes to zero (see Subsection 2.2). So, in our analysis the time window that best meet the three criteria is 6 months for $M_{thr} \geq 5.5$ and 12 months for $M_{thr} \geq 6.0$ as evidenced in Figure 4.

We created the earthquake network based on the seismic zones and the magnitude ranges for each model (Markov, semi-Markov) using the different data settings. The construction of network allows us to check which distribution of network measures (strength, clustering coefficient, betweenness centrality) matches well with the limiting state distribution of the Markov chain.

Figure 3. *Epicentral distribution of earthquake magnitudes $M_{thr} \geq 5.5$, that occurred in 1911-2015 in the broader area of Greece. The division of the area in seismic zones is also shown.*



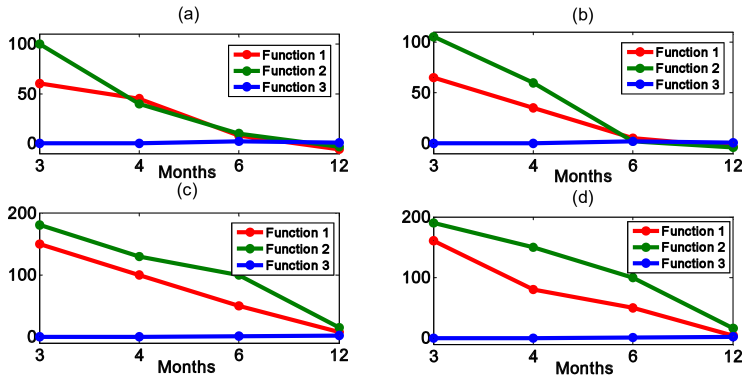
Having defined the data, the system states and nodes, the time interval of chains and the earthquake network, we now estimate the seismic hazard assessment for the next strong earthquake occurrence. Thus, we compute the transition probability and the regional or magnitude activity probability matrices of Markov chain and the core matrices of semi-Markov chain for all cases (data settings, models using as states the seismic zones and magnitude ranges).

The evaluation of the model performance is done by counting the number of successes of aftcasted regions or magnitude ranges occurrences, respectively. The aftcasts are used when the seismic catalog is not large and the number of transitions is not big in order to both obtain a robust transition probability matrix and have a statistically significant sample of forecasts.

4. RESULTS

With the chosen time interval $\Delta\tau$, which is 6 months for $M_{thr} \geq 5.5$ and 12 months for $M_{thr} \geq 6.0$, we compute the transition probability, the regional or magnitude activity and the core matrices for all cases. In particular, in the case of $M_{thr} \geq 5.5$ and seismic zones as states, it is evident from Figure 5a that the states of low occurrence of earthquakes in seismic zones have higher transition probabilities than the states of high occurrence. In addition, from the estimated regional activity probability matrix we notice that when the system is in state $i = 1, \dots, 2^R$, the probability of a strong earthquake occurring in seismic zone 1 in the next $\Delta\tau$ is very high for all cases (see Fig. 5b). Thus, we can assume that the seismic zone 1 has a highest risk for a strong earthquake than the other seismic zones. This is the reason that the states which include the seismicity of seismic region 1 have high transition probabilities in Figure 5a. Also, from the core matrix (see Fig. 5c) the key finding is that the main shocks have the tendency to recur in a short time in the same seismic zone that caused the earthquake. This fact is confirmed from the values at the main diagonal (transitions among the same seismic zones) which shows higher probabilities than the other pairs of states. This is in accordance with the clustering of strong earthquakes observed in many others cases in Greece and worldwide (Kagan and Jackson 1991, Papadimitriou 2002).

Figure 4. *Determination of time interval $\Delta\tau$ when $M_{thr} \geq 5.5$ in (a) the entire catalog and (b) after declustering, and respectively for $M_{thr} \geq 6.0$ in (c) and (d).*



Then, the limiting state distribution corresponding to the Markov chain model in relation with the distribution of network measures (strength, clustering coefficient, betweenness centrality) is shown in Figure 6. It can be observed that the limiting state distribution of the underlying directed network with weighted connections is in full agreement with the distribution of strength where the states (nodes) are represented by seismic zones and magnitude ranges in Figure 6a and 6d, respectively. This is due to the way that the weighted directed networks were constructed. Matching is also observed in the distributions between of the limiting state distribution and the betweenness centrality as shown in Figures 6c and 6f unlike the case of clustering coefficient (see Fig. 6b and 6e).

The last step in our analysis is the evaluation of the performance of each model with respect to the null model of no memory (see Fig. 7). The evaluation results for Markov and semi-Markov model are obtained by quantifying the aftcast success rate from the last 20 transitions corresponding to 20% of the total transitions in the case of $M_{thr} \geq 6.0$ and 10% of $M_{thr} \geq 5.5$. The transitions that are under testing come from the transition probability and core matrix for Markov and semi-Markov model, respectively (see Fig. 7a, 7b and 7c). The success rate is better for the magnitude states than the zoning states, the latter going beyond 90% for the Markov model and 80% for the semi-Markov model, respectively. On the other hand, the success rate for zoning states is about 50% for both cases (Markov, semi-Markov) if we exclude the 85% success rate when the null model is based on the uniform distribution constituting the most random scenario. The performance evaluation for the new approach is done on the basis of the latest 10 transitions for $M_{thr} \geq 6.0$ (without and after declustering) and the time step $\Delta\tau$ at 12 months, for data sets in 2005-2015.

Figure 5. *The transition probability matrix and the regional activity probability matrix, shown as color maps (black for zero and white for 1) in (a) and (b), respectively, which come from Markov chain and the core matrix for $m = 1$ in (c) which come from semi-Markov chain in all cases with magnitude $M_{thr} \geq 5.5$ and states referring to seismic zones.*

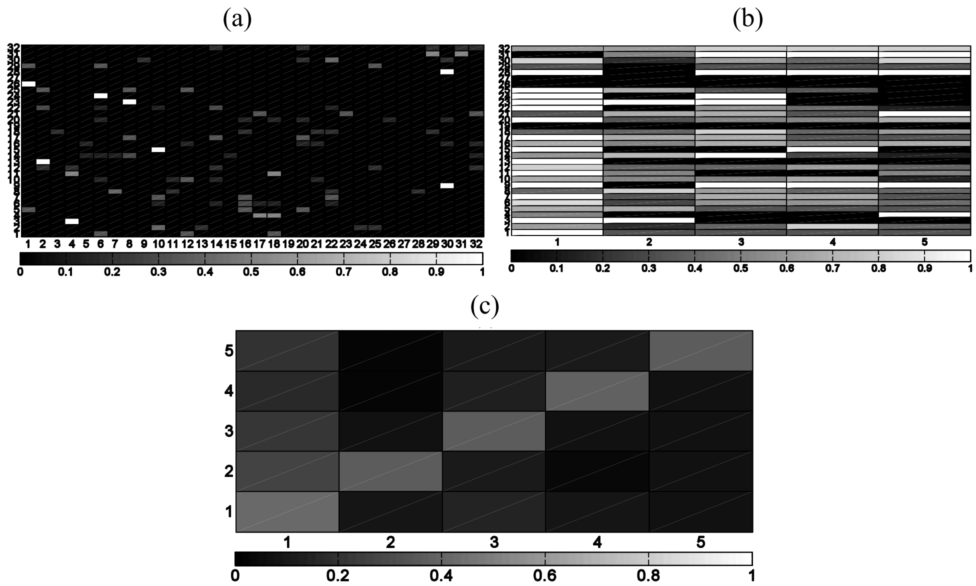
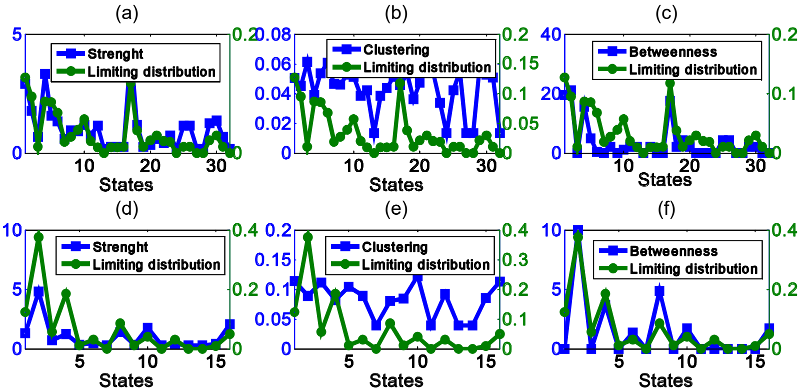


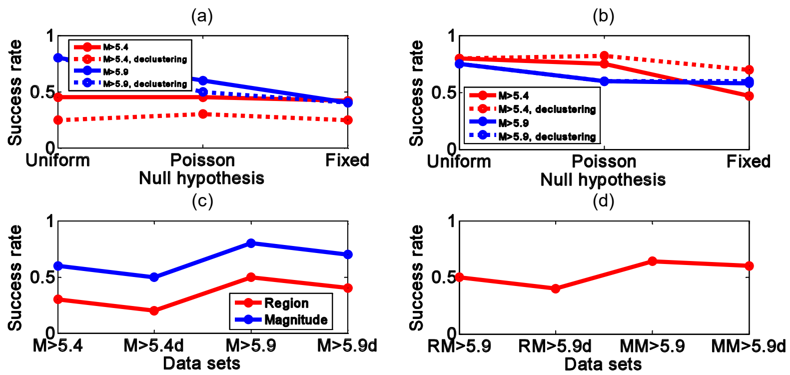
Figure 6. *The comparison of the limiting distribution of Markov chains with $M_{thr} \geq 5.5$ on the basis of the strength in (a) and (d), clustering coefficient in (b) and (e), and betweenness*

centrality in (c) and (f), for the states of seismic zones (2^5) and magnitude ranges (2^4), respectively.



In these 10 years (10 transitions), 14 strong earthquakes with $M_{thr} \geq 6.0$ have occurred, four of them being removed after declustering. Similarly to the previous cases (see Fig. 7a, 7b and 7c) the results of evaluation are better for the magnitude states, the highest success rate being 65%, than the zoning states where the highest success rate is 50% (see Fig. 7d). In particular, the results are better when the entire catalog is used. This remark can be substantiated by the fact that when declustering algorithm is applied, main shocks considered dependent are removed from the catalog, therefore the estimation of the next transition (the next strong earthquake) becomes more unpredictable.

Figure 7. *The performance evaluation in (a) with Markov model for seismic zones, in (b) with Markov model for magnitude ranges both of cases with $\mu = 1$ for the uniform null model, in (c) with semi-Markov model and (d) with new approach. The suffix 'd' in (c) and (d) means that the data come from declustering algorithm and the suffix 'R' and 'M' in (d) means that the data are referred in seismic zones and magnitude ranges, respectively.*



5. CONCLUDING REMARKS

The seismic hazard assessment performed by means of network and analysis and Markov models shows that the occurrence of strong earthquakes could have been predicted to some extent. Application of the models (Markov, semi-Markov, new approach) to the Greek area yields significant results that have negligible probability of being obtained by purely random guessing. The transition probabilities, or for the case of semi-Markov models the transition probability and core matrices, between the states (seismic zones, magnitude ranges) of the system generate useful seismic hazard estimates, on which the forecast of transitions with high probability may be based. The results of model performance evaluation in all cases are better for the estimation of the next magnitude states than the zoning states. In addition, the results of evaluation are better when we do not use the declustering algorithm.

The latter is attributed to the occurrence of dependent strong earthquakes, but given the fact that strong earthquakes cause huge disasters, it is preferable not to exclude these main shocks from the seismic catalog. The network theory can contribute to improve our understanding of these complex systems, such as the seismicity, as the limiting distribution of Markov chain is in agreement with the distribution of some network measures (strength, betweenness centrality). An open issue arising from this study is the application of the same analysis using synthetic seismic catalogs in order to provide more data (simulation data), i.e. more transitions, and thus the results become more robust.

ΠΕΡΙΛΗΨΗ

Ο σκοπός της παρούσας μελέτης είναι η αξιολόγηση της σεισμικής επικινδυνότητας στον χώρο της Ελλάδας με βάση τον υπολογισμό των πιθανοτήτων μετάβασης από ένα Μαρκοβιανό και ημι-Μαρκοβιανό μοντέλο, αντίστοιχα. Θεωρούμε δύο σύνολα δεδομένων τα οποία περιλαμβάνουν ισχυρούς σεισμούς με μέγεθος $M_{thr} \geq 5.5$ και $M_{thr} \geq 6.0$, αντίστοιχα, που συνέβησαν κατά την περίοδο 1911-2015. Η περιοχή μελέτης χωρίζεται σε 5 υποπεριοχές (σεισμικές ζώνες) που είναι ομοιογενείς από την σεισμοτεκτονική άποψη και ο σεισμικός κατάλογος διαιρείται σε υποσύνολα για τέσσερα εύρη μεγέθους. Δύο Μαρκοβιανές αλυσίδες ορίζονται σε σχέση με ένα κατάλληλο επιλεγμένο χρονικό παράθυρο, η μία έχει ως καταστάσεις την εμφάνιση ισχυρών σεισμών σε καμία, μία ή σε περισσότερες υποπεριοχές (2^5=32 καταστάσεις), και η δεύτερη έχει ως καταστάσεις την εμφάνιση ισχυρών σεισμών οπουδήποτε στην Ελλάδα από καθένα από τα τέσσερα εύρη μεγέθους (2^4=16 καταστάσεις). Οι καταστάσεις είναι οι κόμβοι του κατευθυνόμενου δικτύου και οι σταθμισμένες συνδέσεις ορίζονται από τις πιθανότητες μετάβασης της Μαρκοβιανής αλυσίδας οι οποίες εκτιμώνται από τα δεδομένα του σεισμικού καταλόγου. Η εκτίμηση της επόμενης μεταβατικής κατάστασης με την χρήση της Μαρκοβιανής αλυσίδας έχει βρεθεί στατιστικά σημαντική τόσο για τις υποπεριοχές όσο και για τα μεγέθη. Ειδικότερα, η μηδενική υπόθεση ότι η Μαρκοβιανή αλυσίδα δεν έχει μνήμη απορρίπτεται με χρήση ως στατιστικού ελέγχου τρία μοντέλα μνήμης (ομοιόμορφο, Poisson και καθορισμένη αλυσίδα Markov). Επιβεβαιώνεται, ότι η κατανομή του βαθμού (της ισχύος) του δικτύου ταιριάζει με την οριακή κατανομή των καταστάσεων της Μαρκοβιανής αλυσίδας. Σε μια διαφορετική προσέγγιση θεωρούνται δύο ημι-Μαρκοβιανές αλυσίδες, μία για τις υποπεριοχές (5 καταστάσεις) και μία για τα μεγέθη (4 καταστάσεις), για την ακολουθία των ισχυρών σεισμών με χρήση κατάλληλου χρονικού παραθύρου και πινάκων του πυρήνα. Η ημι-Μαρκοβιανή αλυσίδα και για την περίπτωση των υποπεριοχών και για αυτή των μεγεθών έχει βρεθεί να δίνει αξιόπιστη εκτίμηση της επόμενης μετάβασης λαμβάνοντας υπόψη τα δεδομένα μέχρι τη στιγμή της πρόβλεψης. Τέλος, μια νέα προσέγγιση, που συνδυάζει την Μαρκοβιανή και την ημι-Μαρκοβιανή αλυσίδα, επιχειρείται προκειμένου να εκτιμηθεί ο επόμενος ισχυρός σεισμός αν υποτεθεί ότι ο προηγούμενος

ισχυρός σεισμός (από την ημι-Μαρκοβιανή αλυσίδα) και η προηγούμενη κατάσταση (από την Μαρκοβιανή αλυσίδα) δίνονται.

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