ΠΕΠΛΕΓΜΕΝΕΣ Δ.Ε. 1ου ΒΑΘΜΟΥ

I

$$\boxed{x = g(y')}$$

⇒ Χρησιμοποιούμε το $y' = p$ σαν μεταβλητή (αντί του x)

$$x = g(p)$$

$$\frac{dy}{dp} = \frac{dy}{dx}\cdot\frac{dx}{dp} = y'\frac{dx}{dp} = p\,g'(p)$$

$$y = \int p\,g'(p)\,dp + c$$

Παραμετρική έκφραση της τροχιάς

II

$$\boxed{y = g(y')}$$

$$y = g(p)$$

$$\frac{dy}{dp} = y'\frac{dx}{dp} = p\frac{dx}{dp} \rightsquigarrow \frac{dx}{dp} = \frac{g'(p)}{p}$$

$$x = \int \frac{g'(p)}{p}\,dp + c$$

III Clairaut

$$\boxed{y = x y' + g(y')} \qquad y' = p$$

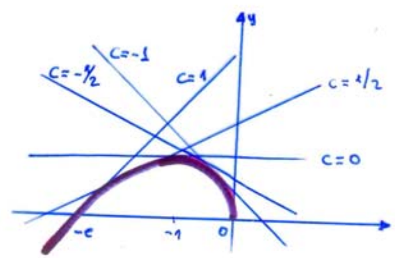
$$\frac{dy}{dp} = y'\frac{dx}{dp} = p\frac{dx}{dp} = $$

$$= \frac{dx}{dp}p + x + g'(p)$$

$$\left.\begin{aligned} x &= -g'(p) \\ y &= -p\,g'(p) + g(y) \end{aligned}\right\} \text{1η Λύση} \leftarrow \text{περιβάλλουσα}$$

$y' = c =$ σταθερά

$y = xc + g(c)$ 2η Λύση ← Ευθείες εφαπτόμενες στην πρώτη λύση!

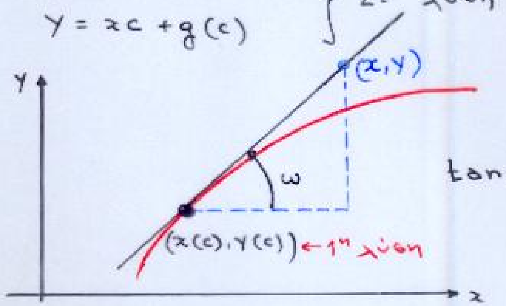


$y = xy' + e^{y'}$

$$y = xy' + g(y')$$

$$\left.\begin{aligned} x &= -g'(p) \\ y &= -pg'(p) + g(p) \end{aligned}\right\} \text{1η λύση}$$

$$\left.\begin{aligned} y' &= c = \text{σταθερά} \\ y &= xc + g(c) \end{aligned}\right\} \text{2η λύση}$$



$\tan\omega = y' = c$

Εξίσωση εφαπτομένης

$$\frac{y - y(c)}{x - x(c)} = c \quad \leftarrow \text{1η λύση}$$

$$\Downarrow$$

$$y = xc + g(c) \quad \leftarrow \text{2η λύση}$$

IV Lagrange ή d'Alembert

$$\boxed{y = x f(y') + g(y')}$$

$$\frac{dy}{dp} = y' \frac{dx}{dp} = p \frac{dx}{dp}$$

$$= \frac{dx}{dp} f(p) + x f'(p) + g'(p)$$

Γραμμική ΔΕ Α' βαθμού

$$\frac{dx}{dp} = \frac{x f'(p)}{p - f(p)} + \frac{g'(p)}{p - f(p)} \rightsquigarrow x = x(p)$$

$$y = x(p) f(p) + g(p)$$