LOGARITHMIC DIMENSION BOUNDS FOR THE MAXIMAL FUNCTION ALONG A POLYNOMIAL CURVE

IOANNIS PARISSIS

Abstract. Let $d\mu$ be a probability measure on $\mathbb{R}^d$ and $d\mu_r$ be appropriate dilations of the measure $d\mu$. The maximal operator associated with the measure $d\mu$ is then defined as

$$\mathcal{M}(f)(x) = \sup_{r>0}(|f| * d\mu_r)(x).$$

The usual maximal operators can be put in this general context. I will discuss an approach in proving L^2 bounds for $\mathcal{M}$ without using the endpoint weak L^1 bounds and interpolation (initiated by Stein, Wainger, Bourgain and others). I will then study in more detail the maximal function along the polynomial curve $(\gamma_1 t, \dots, \gamma_d t^d)$:

$$\mathcal{M}(f)(x) = \sup_{r>0} \frac{1}{2r} \int_{|t|\leq r} |f(x_1 - \gamma_1 t, \dots, x_d - \gamma_d t^d)| dt,$$

and outline the proof of the following estimate:

$$\|\mathcal{M}f\|_{L^2(\mathbb{R}^d)} \leq c \log d \; \|f\|_{L^2(\mathbb{R}^d)},$$

where $c > 0$ is an absolute constant. The proof follows the ideas of Bourgain. The new element is a construction of an appropriate semi-group of operators which is compatible with the anisotropic structure implied by the curve $(\gamma_1 t, \dots, \gamma_d t^d)$.

E-mail address: ioannis.parissis@gmail.com

Centro de Análise Matemática, Geometria e Sistemas Dinâmicos, Departamento de Matemática, Instituto Superior Técnico, Av. Rovisco Pais, 1049-001 Lisboa, PORTUGAL.

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