SZEMERÉDI'S THEOREM, FREQUENT AND MULTIPLE RECURRENCE

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ABSTRACT. In this talk I will describe some recent developments, relating notions in linear dynamics to classical notions in topological dynamics. I will work on a separable Banach space X. A bounded linear operator $T : X \to X$ is called hypercyclic if there exists a vector $x \in X$ such that the set $\{x, Tx, T^2x, \dots\}$ is dense in X, that is, the orbit of x under T is dense in the space.

A relevant notion is that of *frequent hypercyclicity* which examines how often the orbit of a hypercyclic vector visits a non-empty open set of X. I will show how the use of Szemerédi's theorem trivially implies that a frequently hypercyclic operator is *topologically multiply recurrent*. This means that for every positive integer m and for every open set $U \subset X$ there exists a positive integer k such that

$$U \cap T^{-k}U \cap T^{-2k}U \cap \cdots \cap T^{-mk}U \neq \emptyset.$$

I will investigate these notions in the case of *Cesáro hypercyclic* operators and *frequently Cesáro hypercyclic* operators. The operator T is *Cesáro hypercyclic* when the set

$$\left\{ \frac{1}{n} T^n, n = 0, 1, 2, \dots \right\},$$

is dense in X, for some $x \in X$. Note here that the notions of hypercyclicity and Cesáro hypercyclicity are not 'ordered'; none implies the other.

This study was initiated by Costakis and Ruzsa who showed that a *frequently Cesáro hypercyclic* operator is necessarily hypercyclic. In a recent work with George Costakis we show that, under the same assumption, the operator T is topologically multiply recurrent. On the other hand there exists a Cesàro hypercyclic operator (a weighted shift on $\ell^2(\mathbb{Z})$) which is not even recurrent so the hypothesis of frequent Cesáro hypercyclicity cannot be completely removed.

Finally, given time, I will describe how these notions apply in the case that X is a Hilbert space of holomorphic functions (think of $H^2(\mathbb{D})$ or $A^2(\mathbb{D})$) and T is the adjoint of a multiplication operator. Under some mild assumptions on the Hilbert space we can give a complete characterization of topological multiple recurrence in terms of the symbol of the multiplication operator. Some interesting questions arise when one removes some of the hypotheses on the Hilbert space. In particular it is an open question to characterize hypercyclicity, frequent hypercyclicity and topological multiple recurrence for the aforementioned operators in the case of the *Dirichlet space* on the unit disc.

Everything in this talk refers to joint work with George Costakis.

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