

CONTENTS

Detection of Directionality of Information Transfer in Non-linear Dynamical Systems	1
<i>A. Papana and D. Kugiumtzis</i>	
Author Index	15

Detection of Directionality of Information Transfer in Nonlinear Dynamical Systems

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For the quantification of strength and identification of direction of coupling between two sub-systems of a complex dynamical system, observed from bivariate time series, a number of measures have been proposed that can be grouped in measures of phase synchronization, state space and information. We review all these measures and, in particular, for the information measures we examine different estimates of the probability distributions. We propose also a modification of the transfer entropy measure to span larger time windows and thus be more appropriate for flows. Simulations on systems of different types and for varying coupling strengths showed that information measures, and the modified transfer entropy measure in particular, detect best the coupling strength and direction. This is also found when applying the measures to pairs of EEG channels in order to detect the propagation of pre-epileptic brain activity.

Keywords: Information flow; Directionality; Coupling; Time series; Chaos.

1. Introduction

The interaction or coupling between variables or sub-systems of a complex dynamical system is a developing area of nonlinear dynamics and time series analysis.¹⁻³ The recently developed measures of interaction go beyond the standard cross-correlation and exploit nonlinear properties of dynamical systems. Further, indexes have been derived from these interaction measures in order to detect the direction of interaction. Interaction measures can be split in three classes: the state-space based measures that use the inter-distances of reconstructed points from the time series,⁴⁻⁶ the measures making use of the idea of synchronization and quantify the concordance of signal quantities such as phases or events,^{7,8} and the information based measures that quantify information flow through probability distribution

functions.^{9,10} All measures involve specific parameters and their performance depends critically on them.

The information measures make no assumptions on the system dynamics as opposed to phase or event synchronization measures that assume strong oscillatory behavior or distinct event occurrences, respectively, and the state space methods that require local dynamics being preserved in neighborhoods of reconstructed points. For the probability density functions used in information measures, different estimates have been proposed. In a recent work,¹¹ we have compared the histogram-based,¹² kernels,¹³ and k -nearest neighbors¹⁴ estimators for the mutual information on scalar time series and found through simulations that for nonlinear systems they are consistent only when noise is added to the data, meaning that the estimates converge with the time series length. Moreover, the estimation and convergence depends on the method specific parameter, especially for the histogram-based methods, where both equidistant and equiprobable binning have been used.

Here, we extend the simulation study to measures of the information flow that involve joint and conditional probability density estimation from bivariate time series. Information flow measures included in this investigation are the transfer entropy,⁹ the coarse-grained transinformation rate¹⁵ and the conditional mutual information.¹⁶ The information measures are also compared to state space and synchronization directionality measures.

The structure of the paper is as follows. In Section 2, the directional coupling measures are discussed. In Section 3, the evaluation procedure and the simulation systems are presented and in Section 4, the results are discussed. In Section 5, we present an application of the measures to pre-epileptic EEG signals and in Section 6, the main conclusions are drawn.

2. Directional coupling measures

Let us assume a bivariate time series $\{x_t, y_t\}_{t=1}^N$, where x_t and y_t are observed variables from systems that may have unidirectional or bidirectional interaction. For simplicity, we refer to the variables with X and Y , respectively. We are basically concerned here with interaction of nonlinear dynamical systems and therefore we study measures that have the power to detect this type of interaction, skipping correlation and coherence measures used in linear multivariate analysis.^{17,18} We also do not consider methods of mutual nonlinear prediction.¹⁹

2.1. State space measures

This class of interaction measures involves the state space reconstruction from each time series separately using standard delay embedding, i.e. the reconstructed points from each time series are $\mathbf{x}_t = [x_t, x_{t-\tau}, \dots, x_{t-(m-1)\tau}]'$ and $\mathbf{y}_t = [y_t, y_{t-\tau}, \dots, y_{t-(m-1)\tau}]'$ for the same embedding dimension m and delay τ . The measures operate on the neighboring reconstructed points.

The *nonlinear interdependence* measure (NI) assumes the time indices $r_{t,j}$ and $s_{t,j}$, $j = 1, \dots, k$ of the k -nearest neighbors of \mathbf{x}_t and \mathbf{y}_t , respectively.⁴ The mean squared Euclidean distance of the k -nearest neighbors to \mathbf{y}_t is $R_t^{(k)}(Y) = \frac{1}{k} \sum_{j=1}^k \|\mathbf{y}_t - \mathbf{y}_{s_{t,j}}\|^2$, and the X -conditioned mean squared Euclidean distance of \mathbf{y}_t is $R_t^{(k)}(Y|X) = \frac{1}{k} \sum_{j=1}^k \|\mathbf{y}_t - \mathbf{y}_{r_{t,j}}\|^2$. Then the NI measure is defined as

$$S(Y|X) = \frac{1}{N'} \sum_{t=1}^{N'} \frac{R_t^{(k)}(Y)}{R_t^{(k)}(Y|X)}, \quad (1)$$

where $N' = N - (m - 1)\tau$. Values of $S(Y|X)$ close to zero suggest independence of Y and X , while significant positive values of $S(Y|X)$ suggest dependence of Y on X . The measure is non-symmetric, so if $S(Y|X) > S(X|Y)$ then Y depends more on X than vice versa. It has been reported that NI is robust against noise and capable of detecting weak inter-dependence.^{4,20,21}

Recalling that neighboring points correspond to nearby trajectories or equivalently recurrences of a single (chaotic) trajectory at the neighborhood of reference, NI bears a great deal of similarity to the measure of *mean conditional probability of recurrence* (MCR).⁶ According to the visualizing method of recurrence plot,²² the recurrence matrixes of X and Y are respectively $R_{i,j}^X = \Theta(\varepsilon_x - \|\mathbf{x}_i - \mathbf{x}_j\|)$ and $R_{i,j}^Y = \Theta(\varepsilon_y - \|\mathbf{y}_i - \mathbf{y}_j\|)$, where Θ is the Heaviside function and $\varepsilon_x, \varepsilon_y$ are distances (radii when the distance metric is Euclidean). In addition, the joint recurrence matrix of (X, Y) is defined as $J_{i,j}^{X,Y} = \Theta(\varepsilon_x - \|\mathbf{x}_i - \mathbf{x}_j\|)\Theta(\varepsilon_y - \|\mathbf{y}_i - \mathbf{y}_j\|)$. The concept of recurrence has been used to quantify a weaker form of synchronization, whereas MCR is an extension of it that detects the direction of the coupling. Then MCR of Y given X is

$$M_{CR}(Y|X) = \frac{1}{N'} \sum_{i=1}^{N'} \frac{\sum_{j=1}^{N'} J_{i,j}^{X,Y}}{\sum_{j=1}^{N'} R_{i,j}^X}. \quad (2)$$

In loose terms, MCR estimates the probability of recurrence on Y when there is synchronous recurrence on X . The independence and direction of

dependence is defined as for NI, so if $M_{CR}(Y|X) > M_{CR}(X|Y)$ then X drives Y and if $M_{CR}(Y|X) \simeq M_{CR}(X|Y)$ then the coupling is symmetric (same strength of coupling in both directions). The main difference between NI and MCR is that MCR uses counts of neighboring points according to a distance threshold, whereas NI uses the distances for a fixed number of neighboring points.

2.2. Synchronization measures

Reverting from state space dynamics to phase dynamics, the measure of *directionality index* (DI) quantifies the degree the phase dynamics of one oscillator is influenced by the phase dynamics of another oscillator.⁸ To form the DI measure it is assumed that the two time series indeed exhibit oscillating behavior, so that phases $\phi_X(t)$, $t = 1, \dots, N$, can be extracted from $\{X_t\}_{t=1}^N$, typically using the Hilbert transform (the same for Y , also for the following expressions). It is also assumed that the phase increments $\Delta_X(t) = \phi_X(t + \tau) - \phi_X(t)$ are generated by an unknown two-dimensional map $\Delta_X(t) = F_X[\phi_X(t) - \phi_Y(t)]$ fitted by a finite Fourier series $\sum_{m,l} A_{m,l} e^{im\phi_X + il\phi_Y}$. The fitted functions for X and Y are used to quantify the cross dependencies of phase dynamics of the two systems given as

$$c_X^2 = \int_0^{2\pi} \int_0^{2\pi} \frac{\partial F_X}{\partial \phi_X} d\phi_X d\phi_Y, \quad c_Y^2 = \int_0^{2\pi} \int_0^{2\pi} \frac{\partial F_Y}{\partial \phi_Y} d\phi_X d\phi_Y. \quad (3)$$

The directionality index is defined as:

$$d_{X,Y} = \frac{c_Y - c_X}{c_X + c_Y}, \quad (4)$$

where $d_{X,Y}$ close to -1 or 1 suggests unidirectional coupling driven by X or Y , respectively, while $d_{X,Y} \simeq 0$ suggests symmetric bidirectional coupling or no coupling. The main disadvantage of this method is that it is not always possible to extract phases from scalar time series.

Event synchronization is the relative timing of certain “events” in the time series (like spikes, local minima or maxima), i.e. quasi-simultaneous appearances of these events in the two time series.²³ Let $c^\tau(X|Y)$ denote the number of times an event appears in X shortly after it appears in Y (for details see [23]), allowing a time lag τ between two synchronous events. The strength of the coupling, termed as *event synchronization* (ES), is expressed by the normalized total of synchronized events

$$Q_\tau = \frac{c^\tau(Y|X) + c^\tau(X|Y)}{\sqrt{n_x n_y}}, \quad (5)$$

and the coupling direction, termed as *event delay* (EvD), by the normalized difference

$$q_\tau = \frac{c^\tau(Y|X) - c^\tau(X|Y)}{\sqrt{n_x n_y}}, \quad (6)$$

where n_x and n_y are the total number of occurrences of events in X and Y . The measures are normalized so that $0 \leq Q_\tau \leq 1$ and $-1 \leq q_\tau \leq 1$. For $Q_\tau = 1$ the events of the signals are fully synchronized while $Q_\tau = 0$ suggests no synchronization. When q_τ is close to 1 an event in X is likely to precede an event in Y and thus X drives Y , and respectively when q_τ is close to -1 suggests that Y drives X . As also mentioned in [23], τ is not fixed but adapted to the time interval between events at each step (it is half of the minimum of times from the current event to the preceding and to the succeeding event for both X and Y).

2.3. Information measures

The information measures make use of the Shannon entropy for the uncertainty or information in X , $H(X) = -\sum_x p_X(x) \ln p_X(x)$, where $p_X(x)$ is the probability function defined on a proper binning of X . Further, the conditional entropy is $H(X|Y) = -\sum_{x,y} p_{X,Y}(x,y) \ln \frac{p_{X,Y}(x,y)}{p_Y(y)}$ and the mutual information is $I(X,Y) = -\sum_{x,y} p_{X,Y}(x,y) \ln \frac{p_{X,Y}(x,y)}{p_X(x)p_Y(y)}$, where $p_{X,Y}(x,y)$ is the joint probability of (X,Y) (assuming proper binning of Y as well). In the presence of a third variable Z , the conditional mutual information $I(X,Y|Z)$ of the variables X,Y given the variable Z is

$$I(X,Y|Z) = H(X|Z) + H(Y|Z) - H(X,Y|Z). \quad (7)$$

For a scalar time series, mutual information is defined in terms of a delay, $I(X,Y) = I(x_t, x_{t-\tau}) = I(\tau)$ and for bivariate time series a natural extension is the cross mutual information $I(x_t, y_{t-\tau})$. If instead of $y_{t-\tau}$ the difference $\Delta_\tau y_t = y_{t+\tau} - y_t$ is considered, then the (cross) mutual information conditioned on y_t is $I(x_t, \Delta_\tau y_t | y_t)$ and can be found from (7). The average of $I(x_t, \Delta_\tau y_t | y_t)$ for delays up to a maximum delay τ_{\max} is the measure of *mean conditional mutual information* (MCMI)¹⁶

$$i_{X \rightarrow Y} = \frac{1}{\tau_{\max}} \sum_{\tau=1}^{\tau_{\max}} I(x_t, \Delta_\tau y_t | y_t) \quad (8)$$

that quantifies the information transferred from X to Y at a later time conditioning on the current state of Y , i.e. the level of driving of X to Y . $i_{Y \rightarrow X}$

is defined similarly. A different expression for the conditional mutual information is used to define the measure of *Coarse-grained transinformation rate* (CGTR).^{10,24} Averaging on time increments τ as for MCMI, CGTR measures the average rate of the net amount of information transferred from X to Y and is defined as:

$$Ci_{X \rightarrow Y} = \frac{1}{\tau_{\max}} \sum_{\tau=1}^{\tau_{\max}} I(x_t, y_{t+\tau} | y_\tau) - \frac{1}{2\tau_{\max}} \sum_{\tau=-\tau_{\max}, \tau \neq 0}^{\tau_{\max}} I(x_t, y_{t+\tau}). \quad (9)$$

If $Ci_{X \rightarrow Y} \simeq 0$ then there is no information flow, while if $Ci_{X \rightarrow Y} > 0$ then X influences Y . If $Ci_{X \rightarrow Y} > Ci_{Y \rightarrow X}$ then X influences Y more than vice versa.

The states of X and Y in MCMI and CGTR are scalars x_t and y_t , respectively, but can also be given by vectors $\mathbf{x}_t^{m_x} = [x_t, x_{t-1}, \dots, x_{t-(m_x-1)}]'$ and $\mathbf{y}_t^{m_y} = [y_t, y_{t-1}, \dots, y_{t-(m_y-1)}]'$. The latter is used in the measure of *transfer entropy* (TE)⁹

$$T_{X \rightarrow Y} = \sum p(y_{t+1}, \mathbf{y}_t^{m_y}, \mathbf{x}_t^{m_x}) \ln \frac{p(y_{t+1} | \mathbf{y}_t^{m_y}, \mathbf{x}_t^{m_x})}{p(y_{t+1} | \mathbf{y}_t^{m_y})}. \quad (10)$$

TE quantifies the information flow from X to Y by the amount of information explained in Y at one step ahead from the state of X , accounting for the concurrent state of Y . The concept of transfer entropy extends the Shannon entropy for transition probabilities and quantifies how the conditioning on X change the transition probabilities of Y . The inclusion of vectors in the probabilities does not allow for reliable estimation of TE for small data sets, although there are several methods for efficient coarse graining. It has been shown that with proper conditioning, transfer entropy $T_{X \rightarrow Y}$ is the exact equivalent to the conditional mutual information $I(x_t, y_{t+\tau} | y_t)$.^{2,3} In the computation of TE, only one time step (lag) ahead is considered, as opposed to MCMI where all lags up to some τ_{\max} contribute to the estimation.

TE involves the time window lengths m_x and m_y (or embedding dimensions when using delay one). While m_x is selected using standard embedding criteria, m_y has commonly been set either equal to m_x or most frequently to 1. For time series from flows, we propose the states to be defined through separate delay embedding, i.e. $\mathbf{x}_t^{m_x} = [x_t, x_{t-\tau_x}, \dots, x_{t-(m_x-1)\tau_x}]'$ and $\mathbf{y}_t^{m_y} = [y_t, y_{t-\tau_y}, \dots, y_{t-(m_y-1)\tau_y}]'$ for proper selection of the delay parameters. Moreover, there is little information flow to observe for one step ahead when the data are densely sampled and in line with MCMI we use a delay (or better a step ahead) τ instead of 1.

We consider two estimators for the probability functions in all information measures, the one estimator uses equidistant binning (ED)¹² and the other estimator uses correlation sums.²⁵

3. Setup

The evaluation of the measures is assessed by means of Monte Carlo simulation on different types of systems. Directionality coupling measures are computed on coupled systems for increasing coupling strengths in order to evaluate the ability of the measures to detect the degree and direction of coupling. We use 100 realizations from the different simulation systems (linear and nonlinear/ chaotic systems, unidirectional and bidirectional), with time series lengths $n = 1024, 2048, 4096$. Analytically, the simulation systems are:

- Two $AR(1)$ systems with unidirectional coupling:

$$\begin{aligned}x_{t+1} &= 0.5x_t + e_t^x \\ y_{t+1} &= 0.5y_t + cx_t + e_t^y\end{aligned}$$

with coupling strength $c = 0, 0.1, 0.2, 0.3, 0.4, 0.5$ and e_t^x, e_t^y normal iid with variance 1.

- Two unidirectionally coupled Henon maps:

$$\begin{aligned}x_{t+1} &= 1.4 - x_t^2 + 0.3x_{t-1} \\ y_{t+1} &= 1.4 - cx_t y_t + (1 - c)y_t^2 + 0.3y_{t-1}\end{aligned}$$

with coupling strength $c = 0, 0.1, 0.2, 0.3, 0.4, 0.5$.

- A Rössler system (x_1, y_1, z_1) driving a Lorenz system (x_2, y_2, z_2) :

$$\begin{aligned}\dot{x}_1 &= -6(y_1 + z_1) & \dot{x}_2 &= 10(x_2 + y_2) \\ \dot{y}_1 &= -6(x_1 + 0.2y_1) & \dot{y}_2 &= 28x_2 - y_2 - x_2 z_2 + cy_1^2 \\ \dot{z}_1 &= -6(0.2 + z_1(y_1 - 5.7)) & \dot{z}_2 &= x_2 y_2 - \frac{8}{3}z_2\end{aligned}$$

with coupling strength $c = 0, 0.5, 1, 1.5, 2$.

- Two bidirectionally coupled Henon maps:

$$\begin{aligned}x_{t+1} &= 1.4 - x_t^2 + 0.3x_{t-1} + c_2(x_t^2 - y_t^2) \\ y_{t+1} &= 1.4 - y_t^2 + 0.3y_{t-1} + c_1(y_t^2 - x_t^2)\end{aligned}$$

with coupling strengths $(c_1, c_2) = (0.05, 0.05), (0.1, 0.05), (0.1, 0.1), (0.15, 0.05), (0.2, 0.05)$.

The range of the coupling strengths is bounded from zero (independent systems) up to the level of almost complete synchronization. All the time series are first normalized to have mean 0 and standard deviation 1. The values for measure parameters are set as follows: number of bins $b = 8$ for measures requiring coarse graining, radius for finding neighbors $r = 0.15$, number of neighbors $k = 10$, Theiler window $W = 10$ (to exclude time correlated points). For all systems apart from the unidirectionally coupled Rössler–Lorenz system we set: embedding dimension $m = 2$, time lag $\tau = 1$, maximum lag for the average of time increments in information measures $\tau_{\max} = 10$, and for transfer entropy $\tau = 1, m_x = m_y = 1, \tau_x = \tau_y = 1$. For the unidirectionally coupled Rössler–Lorenz system we set: $m = 5, \tau = 5, \tau_{\max} = 20$, and for transfer entropy: $\tau = 5, m_x = m_y = 2, \tau_x = \tau_y = 5$.

4. Results

For systems that are unidirectionally coupled, directionality measures should increase with coupling strength at the direction of coupling and be stable (and close to zero) at the opposite direction and when there is no coupling ($c = 0$). We start with the results from the simulations of unidirectionally coupled $AR(1)$ system. As expected, state space measures and DI do not discriminate between the linear stochastic systems, even for large time series lengths, due to the lack of nonlinear dynamics (in local state space) and phase dynamics, respectively. On the other hand, EvD regarding the local maxima of the time series, seems to discriminate the directionality at a level that increases with the coupling strength, as shown in Fig. 1a. Both estimators of MCMi (with correlation sum and equidistant binning) detect the directionality of the coupling correctly (see Fig. 1b and c), and even for small time series lengths, while CGTR does not seem to detect the directionality of the coupling. Both estimators of TE (from binning and correlation sum) detect the direction of information flow correctly (see Fig. 1d and e).

For the unidirectionally coupled Henon map we observe that MCR detects the increase of coupling in the correct direction, but it wrongly indicates the same but at a lesser degree in the other direction (see Fig. 2a). On the other hand, and as Fig. 2b shows, NI is at the order of 10^{-3} and therefore no information flow is detected. DI detects correctly the strength and the directionality of the coupling but with a large variance, even for large time series lengths, and does not allow for a clear indication of directionality for all cases (see Fig. 2c). EvD also does not detect any information flow. On the other hand, all information measures detect correctly the strength

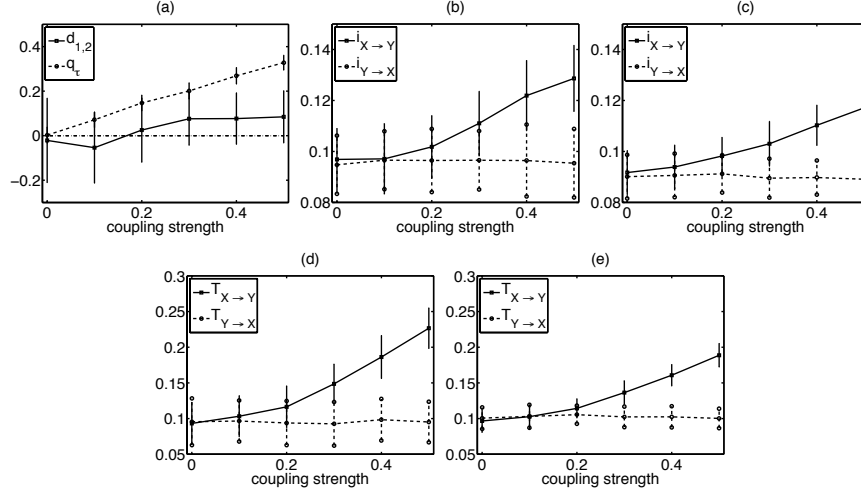


Fig. 1. Average and standard deviation (shown as error bars) of the directionality measures for 100 realizations of unidirectionally coupled $AR(1)$ system. (a) DI and EvD as shown in the legend, for $n = 4096$. (b) MCMI computed by correlation sums, $n = 1024$. (c) As in (b) but for equidistant partition. (d) TE computed by correlation sum, $n = 1024$. (e) As in (d) but for equidistant partition.

and the directionality of the coupling even for small time series lengths (results for CGTR and TE are shown in Fig. 2d-f).

For the unidirectionally coupled Rössler–Lorenz system, we note that the detection of the directionality of the coupling is expected to be more difficult as the two flows are of different type. The simulations showed that EvD, NI, DI and MCMI could not detect the direction and strength of the coupling, whereas MCR and CGTR could both detect it, even for small time series lengths. Using the TE measure with the standard parameters ($\tau = 1, m_x = 2, m_y = 1$ and $\tau_x = \tau_y = 1$) we get only slight difference of the TE on the two directions (somehow larger difference is obtained using the histogram based estimator as shown in Fig. 3a, d). However, estimation of TE using proper reconstruction of state space, detects the coupling only in the correct direction and again better for the histogram based estimate (see Fig. 3b and e). The detection improves with the increase of time series length as shown in Fig. 3c and f. The strong dependence on data size is due to the estimation of probability densities regarding the reconstructed points rather than the samples.

For the bidirectionally coupled Henon system the synchronization measures (for phase and event) fail to detect the direction of stronger coupling,

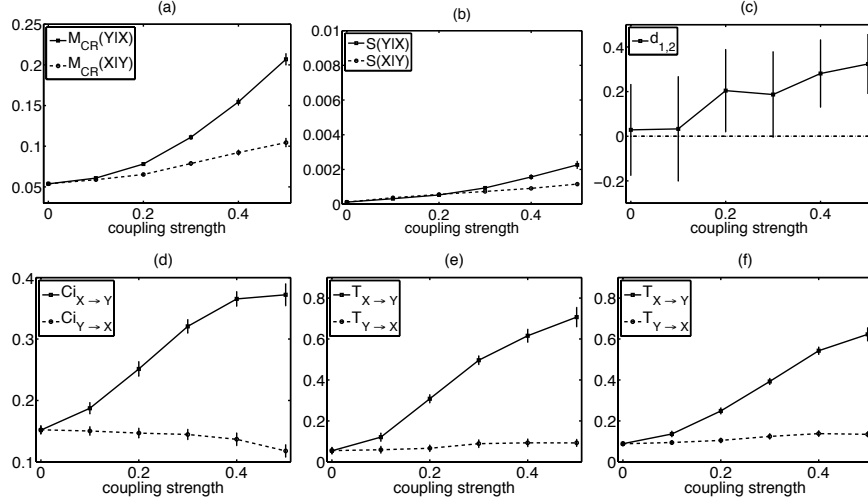


Fig. 2. Average and standard deviation (shown as error bars) of the directionality measures for 100 realizations of unidirectionally coupled Henon system. (a) MCR, for $n = 4096$, (b) NI, for $n = 4096$ and (c) DI, for $n = 4096$. (d) CGTR, for $n = 1024$. (e) TE computed by correlation sums, for $n = 1024$. (f) As in (e) but for equidistant partition.

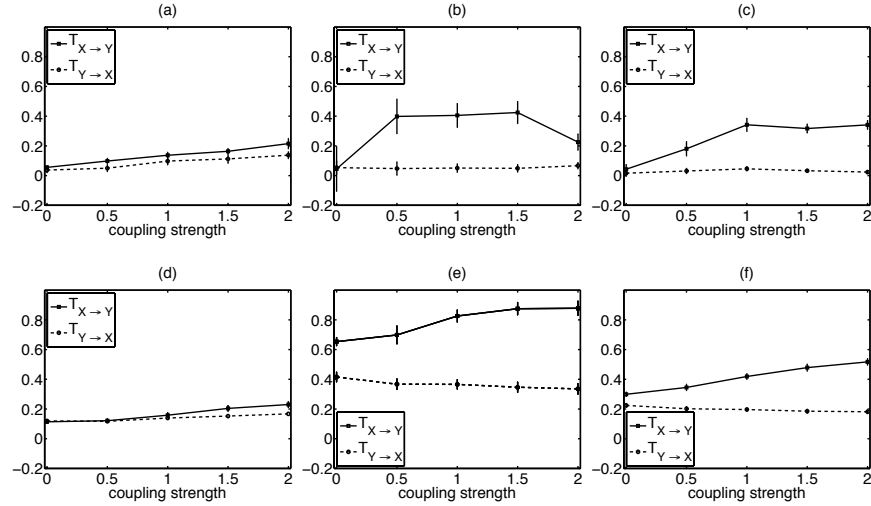


Fig. 3. Average and standard deviation (shown as error bars) of the directionality measures for 100 realizations of unidirectionally coupled Rössler-Lorenz system. (a) TE computed by correlation sums, for $\tau = 1, m_x = 2, m_y = 1, n = 1024$. (b) As in (a) but for $\tau = 5, m_x = m_y = 2, \tau_x = \tau_y = 5, n = 1024$. (c) As in (b) for $n = 2048$. (d), (e), (f) As in (a), (b), (c) respectively, but for equidistant partition.

as there are no distinct phases or events in the time series. Regarding the state space measures, MCR correctly estimates the driver-response interaction only when it is present as shown in Table 1, whereas NI gives always small values independently of the coupling setting. All information mea-

Table 1. Average of the directionality measures for 100 realizations of bidirectionally coupled Henon map for $n = 1024$.

Measures	Coupling strengths				
	(0.05,0.05)	(0.1,0.05)	(0.1,0.1)	(0.15,0.1)	(0.2,0.05)
$M_{CR}(X Y)$	0.0874	0.1018	0.1655	0.1363	0.1873
$M_{CR}(Y X)$	0.0875	0.0940	0.1649	0.1133	0.1453
$S(X Y)$	0.0022	0.0032	0.0075	0.0045	0.0074
$S(Y X)$	0.0022	0.0032	0.0075	0.0051	0.0101
$i_{X \rightarrow Y}$ (Cor. Sum)	0.1519	0.2273	0.3876	0.3159	0.3609
$i_{Y \rightarrow X}$ (Cor. Sum)	0.1473	0.1555	0.3870	0.1820	0.1847
$i_{X \rightarrow Y}$ (hist. based)	0.1398	0.1769	0.2267	0.2172	0.2551
$i_{Y \rightarrow X}$ (hist. based)	0.1386	0.1387	0.2269	0.1503	0.1519
$Ci_{X \rightarrow Y}$	0.1749	0.2036	0.1119	0.2336	0.2629
$Ci_{Y \rightarrow X}$	0.1734	0.1663	0.1124	0.1609	0.1391
$T_{X \rightarrow Y}$ (Cor. Sum)	0.1007	0.1818	0.2787	0.3121	0.3984
$T_{Y \rightarrow X}$ (Cor. Sum)	0.0990	0.1044	0.2804	0.1323	0.1318
$T_{X \rightarrow Y}$ (hist. based)	0.1021	0.1415	0.1660	0.2075	0.3021
$T_{Y \rightarrow X}$ (hist. based)	0.1024	0.0999	0.1658	0.1224	0.1300

asures perform properly (see Table 1). They all give about the same value for both directions when the coupling is the same in both directions. When there is stronger coupling in one direction, TE turns out to give the larger difference for the two directions. For example, for $c_1 = 0.15$ and $c_2 = 0.1$, $T_{X \rightarrow Y}$ increases by about 136% from $T_{Y \rightarrow X}$ for the estimator using correlation sums and 70% for the estimator using histograms, whereas the increase for MCMI is 74% (correlation sums estimator) and 45% (estimator using histograms) and for CGTR is also 45%. Note that these results are for $n = 1024$, and for larger time series the difference in the presence of stronger coupling in one direction is much more clear.

5. Application

As an application, we assess the examined information flow measures in detecting the propagation of the epileptic activity that may be used as a predictor of an impending seizure. The problem of direct and indirect transfer of information among EEG channels is not considered here (for this see e.g. [26,27]). Specifically, we investigate whether changes in directionality of coupling and information flow occur in brain areas of an epileptic patient at

two different periods prior to seizure onset. The data are extra-cranial multichannel recordings of electroencephalographs (EEG) from a patient with back left temporal (LT) lobe epilepsy. We compare the measures on recordings from 60 to 50min before seizure onset (early preictal state) and on recordings from 10min before and up to seizure onset (late preictal state). We use two electrodes from the epileptic focus area (LT1 and LT2) and investigate the information flow to other areas close or farther from it: occipital channels (OC1, OC2), middle channel (MI) and right temporal (RT) channel.

For each of the two periods and for each channel we derive 20 consecutive non-overlapping time series of 30sec each and compute all directionality measures for pairs of LT1 and LT2 to all other selected channels. The parameters of the directionality measures are: $m = 10$, $\tau = 5$, $k = 10$, $W = 10$, $r = 0.15$, $m_x = m_y = 2$, $\tau_x = \tau_y = 5$, $b = 8$, $\tau_{\max} = 20$. We observed a slight decrease in information flow from focus to other areas (mostly from LT to OC and MI) moving from early to late preictal state, but this could be observed only with some measures, as shown in Fig. 4. Although transfer entropy seemed to perform better than other measures in the simulation study, in this application it did not perform any better than the other measures.

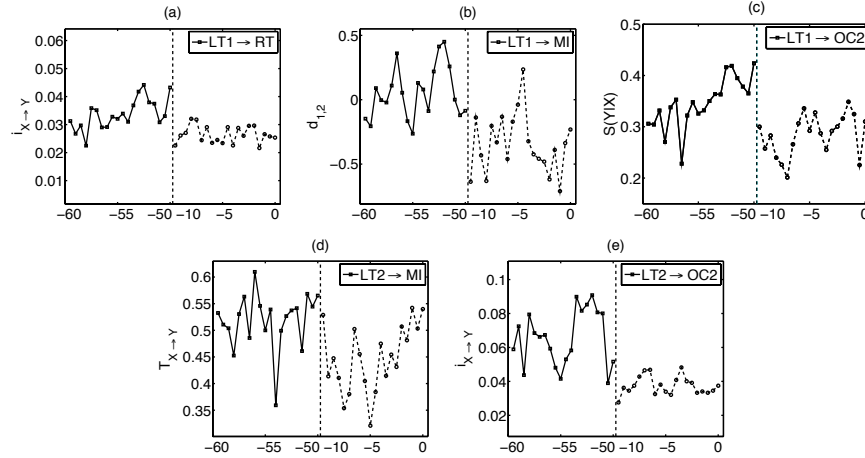


Fig. 4. Estimated values from the two preictal states of (a) MCMI (histogram based) from channel LT1 to RT, (b) DI from LT1 to MI, (c) NI from LT1 to OC2, (d) TE (histogram based) from LT2 to MI, and (e) MCMI (histogram based) from channel LT2 to OC2. The states are indicated by the time in sec, with reference to time 0 at seizure onset and they are separated by a vertical dashed line.

6. Conclusions

From the simulation study we deduced that information measures and specifically transfer entropy were able to detect the directionality in coupled systems independently of the type of the system (linear or non-linear, map or flow). Moreover, information measures seem to perform better for short time series lengths than state space and synchronization measures do. The proposed generalized form of TE introducing reconstructed points and larger time increment seems to perform well, however the selection of the embedding parameters is crucial. In the simulations, we used standard values for measure parameters, as given in the literature, and further investigation with optimized parameters is needed to completely assess the usefulness of the measures. In the EEG application we found that only a few directionality measures could give a slight change in the information flow from the epileptic focus to other brain areas when comparing the period about an hour prior to seizure to the period just before seizure onset.

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14 A. Papana and D. Kugiumtzis

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Papana, A., 1