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Orbits of long-term stability in three-planet systems

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Abstract: Observations of extrasolar systems reveal the existence of a large number of multiplanet systems consisting of two, three, or even more exoplanets, which show orbital features very different of our solar system. By taking into account the mutual planetary interactions, the long term stability is guaranteed only under particular dynamical mechanisms. Such a mechanism is e.g. the phase protection provided by resonant motion. The dynamics of two-planet resonant systems have been extensively studied. In this work we study three-planet systems and we search for orbital configurations that show long-term stability. As basic model we consider the general four-body problem. Our method is based on the planetary migration mechanism provided by the addition of dissipative forces which can drive the system to a stable configuration. Our study showed that the Laplace resonant configuration (1:2:4) is the most likely stable configuration. Also the 1:3:6 resonant configuration appears frequently, as has been also noted in other works. Apart from the above cases, other resonant configurations have been revealed from our numerical simulations providing possible orbital configurations of extrasolar systems, which seem to evolve regularly beyond our numerical integration time that ends at 100My.

1 Introduction

The orbital dynamics of planetary systems is quite complicated when mutual gravitational interactions between planets are taken into account. Although such interactions are small with respect to the gravity between the planet and the star, they are very important for the long-term stability of the system. Such interactions can be modeled by an N-body problem, which, for $N > 2$, is a non-integrable with a complicated phase space topology, where chaotic and regular orbits coexist.

The dynamics for systems consisting of two planets, has been extensively studied [7]. Particular interest has been given for the cases when planets are in mean-motion resonances [9]. Resonances can provide *phase protection* which can generate stable planetary motions even for large eccentricities. Resonances are associated with periodic orbits in a rotating frame of the *three-body problem* [4] or with equilibrium points of the averaged *three-body problem* [3]. Periodic orbits or equilibrium points can be classified as stable or unstable. Generally, unstable orbits are associated with the existence of chaotic regimes that causes slow or fast diffusion and the planetary system is disrupted due to collisions or planetary escapes. Instead, stable periodic orbits or equilibria are surrounded by dense invariant tori in phase space which means that initial conditions in these regions correspond to regular orbits or orbits that their chaotic diffusion is practically negligible.

A lot of studies and numerical simulations of two-planet systems consider the interaction of the planets with the protoplanetary disk. Such an interaction causes the migration of the planets and their possible capture in stable resonant motion [8]. A simple way for modeling the disk–planet interaction is the introduction of a dissipative force in the motion of the planets [1, 5].

Nowadays, about 500 multiplanet systems are known and about one third of them consist of three or more planets (see *http://exoplanets.eu/*). Also, many of them seem to be located in mean motion resonances , e.g. the systems HD 82943, Gliese 876, Kepler 164, Kepler 247, Kepler 288, HD 136352 etc. We remark that the dynamics becomes more and more complicated as the number of interacting planets increases and the determination of initial conditions, which correspond to regular planetary orbits, is not an easy task. In this work we study the dynamics of a multi-planet system consisted of three planets around a single star. Particularly, we are looking for possible resonant stable configurations which provide long-term stability for the planetary system. We use the technique of "planetary migration" which is described in the following section. The results are discussed in section 3.

2 Model and numerical simulations

We consider a planetary four body problem consisted of a star (S) of mass m_0 and three planets P_1, P_2 and P_3 of masses m_1, m_2 and m_3, respectively. In the following the index 0 will refer to the star and the indices 1,2 and 3 will refer to the inner, the middle and the outer planet, respectively. Considering, an inertial frame, fixed in the barycenter of the system, the motion of the three planets under the gravitational forces will be given by the equations

$$\ddot{\mathbf{r}}_i = G \sum_{j=0, j\neq i}^{3} \frac{m_j}{r_{ij}^3}\mathbf{r}_{ij}, \quad \mathbf{r}_{ij} = \mathbf{r}_j - \mathbf{r}_i, \tag{1}$$

where $\mathbf{r}_i$ is the position vector of the planet. In this study we consider the planar problem, $\mathbf{r}_i = (x_i, y_i)$. Also, for normalization of units, we consider $G = 1$ and $m_0 = 1$.

Equations (1) describe the conservative or the purely gravitational model. Now we consider the migrating model by adding the dissipative *Stokes-like* force in the motion of a planet

$$\mathbf{F}_d = -C(\mathbf{v} - \alpha \mathbf{v}_c) \tag{2}$$

where $\mathbf{v}$ is the velocity of the planet and $\mathbf{v}_c$ the velocity of the circular Keplerian orbit that corresponds to the current planetary position neglecting the effect of the other planets. C and α are positive parameters with usual values $0 \leq \alpha \leq 1$ and $C = 10^{-b}$, $6 \leq b \leq 9$. These parameters are related with the decreasing rate of semimajor axis and the eccentricity dumping of the planet [2]. In our study, the dissipative force (2) is applied on the outer planet only.

A similar model has been used in [6], where dissipative force acts on the middle and the outer planet, and the authors study the excitation of eccentricities and inclinations after a capture in a resonance. In our study we focus on the cases where the system is captured in a resonances and is stabilized (at least for a long time) in a constant orbital configuration.

Let n_i, $i = 1, 2, 3$, denotes the mean motion of the planets, then a 3-planet resonance is defined by the commensurability conditions

$$qn_1 - pn_2 = 0, \quad rn_2 - qn_3 = 0, \quad p, q, r \in N^+,$$

and is denoted as $r : q : p$. This means that after r revolutions of the outer planet, the middle planet performs q revolutions and the inner one p revolutions.

Now we define the resonant angles

$$\begin{array}{ll} \theta_1 = p\lambda_2 - q\lambda_1 - (p-q)\varpi_1, & \theta_2 = p\lambda_2 - q\lambda_1 - (p-q)\varpi_2 \\ \theta_3 = q\lambda_3 - r\lambda_2 - (q-r)\varpi_2, & \theta_4 = q\lambda_3 - r\lambda_2 - (q-r)\varpi_3 \end{array} \tag{3}$$

where λ_i denote the mean longitude and ϖ_i the longitude of pericenter. From the above resonant angle we can construct as resonant angles the planetary apsidal differences

$$\Delta\varpi_{ij} = \varpi_j - \varpi_i \tag{4}$$

and, by choosing appropriate integers k_i, the Laplace resonant angle

$$\theta_L = k_1\lambda_1 + k_2\lambda_2 + k_3\lambda_3. \tag{5}$$

Near the exact stable resonance, which is indicated by a stable periodic orbit of the system (1) given in an appropriate rotating frame, all resonant angles librate. Thus, regular libration of resonant angles indicates a long term stability for planetary orbits. When librations take place around 0 or π, the orbital configuration is called *symmetric*, otherwise is called *asymmetric*. In symmetric configurations the lines of apsides of all planetary orbits coincide and the planets can be found in conjunction.

In our numerical simulations we consider always the mass values $m_0 = 1(M_{sun})$, $m_1 = 0.001$ $m_2 = 0.002$ and for the outer planet $m_3 = 0.0005$, 0.001, 0.002 and 0.003. The inner planet starts always with semimajor axis $a_1 = 1$ AU and the outer one with $a_3 = 7AU$ and $e_3 = 0$. For the middle planet we set initially $a_2 = 1.58$, 1.75, 2.1 and 2.3. The initial eccentricities e_1 and e_2 are of small values (about 0.1). Also the migration parameters are $C/m_3 = 10^{-5}$, 10^{-7} and $\alpha = 0.5$, 0.95.

We run the system considering the migration model and we distinguish long time domains where we have a resonant capture and the resonant angles $\Delta\varpi_{ij}$ librate. For a particular time instant in the above time domain, we take the orbital conditions and we evolve them by setting $C = 0$ (i.e. without the effect of the dissipative force). If this evolution is stable, we take the average values of the orbital elements and, by using them as initial conditions, we check again by numerical integration the regularity of the evolution. About 10% of our integrations fulfill the above criteria. The rest cases generally end up in a distabilized system in short time intervals.

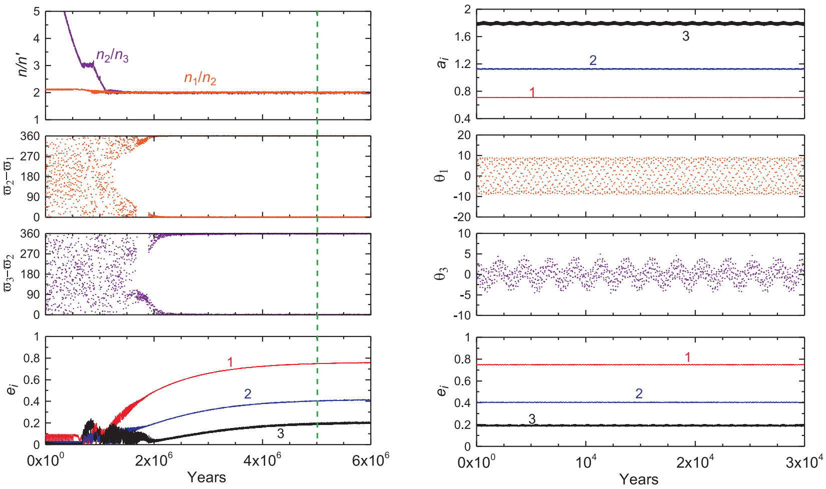

Figure 1: Evolution of the migrating system (left) and the evolution of the system without dissipation (right) with initial conditions at the dashed line. The planetary configuration is symmetric.

3 Results

A typical successful integration is presented in Fig.1a. As the system migrates, it is captured for a long time interval in the Laplace resonance 1:2:4 and $\Delta\varpi_{12}$ and $\Delta\varpi_{23}$ librate around $0°$. By choosing orbital conditions at the time instant indicated by the dashed line, and integrating them without the effect of the dissipative force, we obtain the evolution shown in Fig.1b. All resonant angles librate regularly around $0°$ and the eccentricities are almost constant. Thus we have a stable symmetric 1:2:4 resonant configuration at the particular eccentricity values.

A second example is given in Fig. 2. Now we have an asymmetric 1:2:4 Laplace configuration, where the resonant angles librate around the values

$$\theta_1 = 34°, \quad \theta_2 = 304°, \quad \theta_3 = 30°, \quad \theta_4 = 311°, \quad \Delta\varpi_{12} = 269°, \Delta\varpi_{23} = 280°$$

In Fig. 3 we present the planetary orbits and an initial position of the planets for the above given symmetric and asymmetric configurations. The regular libration of the Laplace resonant angle $\theta_L = \lambda_1 - 3\lambda_2 + 2\lambda_3$ is also presented.

Apart the above cases, symmetric and asymmetric stable configurations have been found for the three-planet resonances 1:3:6, 1:2:6, 1:3:9 and 1:4:12.

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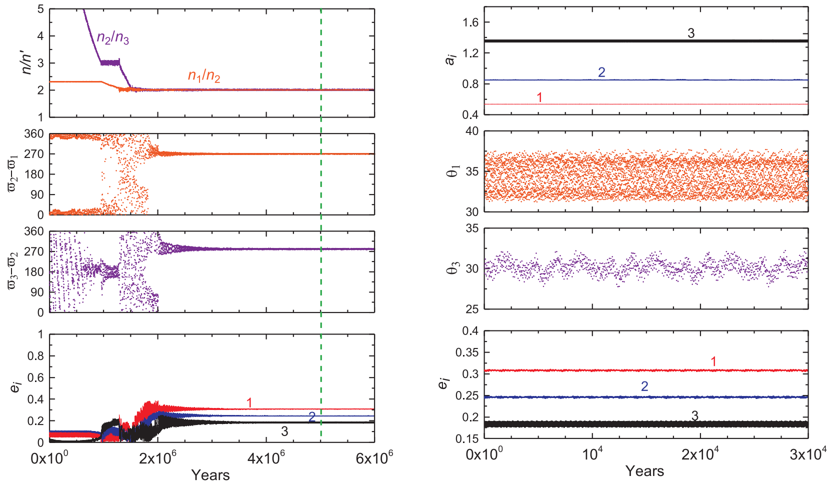

Figure 2: Capture and evolution in an asymmetric Laplace configuration (as in Fig. 1.)

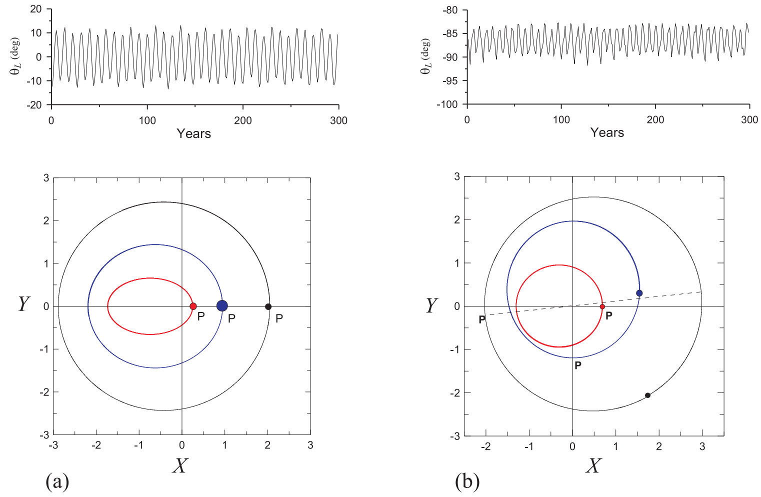

Figure 3: The evolution of the Laplace angle and the planetary orbits for a) the symmetric and b) the asymmetric 1:2:4 configuration. P indicates the pericenter position.

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