# Orbital Evolution in Extra-solar systems

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**Abstract**

Nowadays, extra-solar systems are a hot research topic including various aspects, as observation, formation, composition etc. In this paper, we present the main orbital and dynamical features of these systems and discuss the evolution of planetary orbits in multi-planet systems. Particularly, we consider the dynamics of a two-planet system modeled by the general three body problem. Complicated orbits and chaos due to the mutual planetary interactions occur. However, regions of regular orbits in phase space can host planetary systems with long-term stability.

## 1. Introduction

After the detection of the first three planets orbiting the pulsar *PSR B1257+12* in 1992 and the first planet orbiting the main-sequence star *51 Pegasi* in 1996, a significant growth of detected exoplanets took place, which currently seems to be rather exponential. One of the most reliable catalogs of exoplanets is given by the *Extrasolar Planets Encyclopedia* (EPE) [1]. Very recently, on 6th March 2014, a set of 702 new planet candidates, observed by the Kepler Space Telescope (KST), were added in this catalog, which now includes 1778 planets arranged in 1099 planetary systems[1]. In the blue histogram of figure 1, we present how the number of discovered exoplanets increases year by year and in the red one how the planets are distributed in the planetary systems. A number of 637 systems contains only one planet, while the rest 1141 planets are arranged in 462 multi-planet systems most of them consisting of two planets. The most “populated” system is *Kepler-90* (around the star *KOI-351*), which seems to consist of seven planets[2].

[1] This number changes day by day, including new discoveries or excluding planets for which a confirmation study had failed.
[2] In 2013 it was announced that the system *HD10180* has 10 planets, but today only six planets have been confirmed

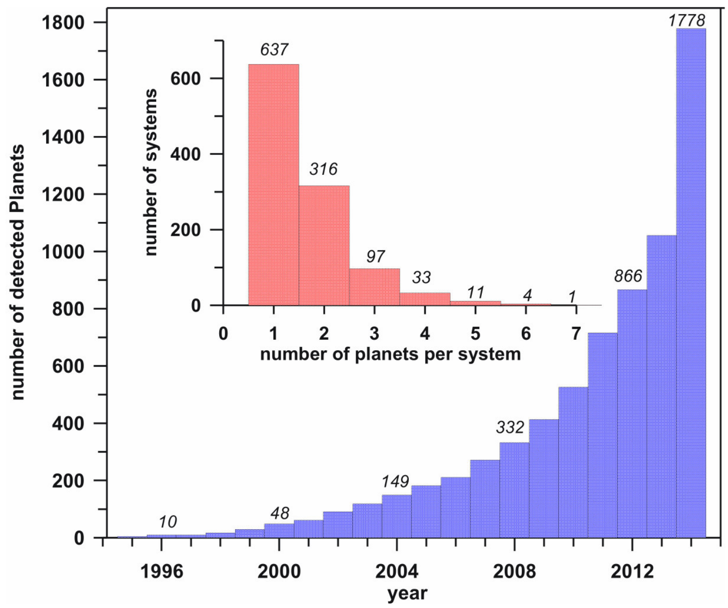


**Figure 1.** The blue histogram shows how many exoplanets have been discovered until the indicated year. The abrupt increase of the number in 2014 (although this number refers to March, $6^{th}$) is due to the addition of a set of 702 planets discovered by the KST. The red histogram shows how the 1778 planets are arranged in the 1099 planetary systems.

The first detection method used was the method of *radial velocity* or *Doppler method* and most of the known exoplanets have been discovered by this method, which is still widely used. The *transit method* has also been proved efficient, mainly after its application by the KST. Gravitational microlencing, reflection/emission methods, polarimetry e.t.c. can also be used as detection methods [2]. *Direct imaging* has given few yet spectacular results, firstly, with the discovery of the planet *Formalhaut b* by the Hubble telescope and, recently with the discovery of *Beta Pictoris b* by the Gemini Planet Imager (see figure 2). The region of observations has a radius of about $10^3$ l.y. and it is estimated that our galaxy contains 400 billion planets. The closest exoplanet to our Solar system is the *Epsilon Eridani b* in a distance of 10 l.y..

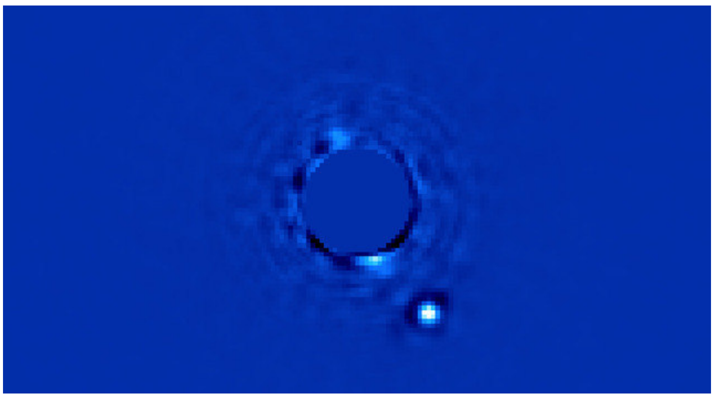

**Figure 2.** The first light image of an exoplanet (Beta Pictoris b) by the Gemini Telescope (announced in 7 January 2014 by NASA JPL, http://planetquest.jpl.nasa.gov/news/144)

Most exoplanets are classified as *hot Jupiters*, namely gas giant planets that are very close to their host star, e.g. *HD 102956 b* which has mass equal to Jupiter's mass (1$M_J$), semimajor axis 0.08*AU* and period 6,5 days. Very massive planets have been found, either close to their host star (e.g. *Kepler-39 b* with mass 20$M_J$, semimajor axis 0.15 AU and period 21 days), or very far from their host star (e.g. *HIP 78530 b* has mass 24$M_J$, semimajor axis 710*AU* and period 12*Ky*). As detection techniques are improved, smaller planets are detected. Many of them have masses equal to 10-20 times the mass of the Earth, are possibly gaseous and are called *Mini-Neptunes*. Nevertheless, planets with mass of the order of the Earth's and of similar radius can be detected by the KST. They are possibly terrestrial (rocky) planets and become very interesting for detailed study, when they are located in a habitable zone, e.g. the *Gliese 667C c* [3].

Studying the general structure of the known extrasolar systems we can certainly argue that our Solar system is an exception. Thus, the proposed mechanisms for formation and evolution of planetary systems should be revised and generalized. In the following, we focus on the orbital characteristics of multi-planet systems, the conditions required for their long-term stability and the role of planetary resonance.

## 2. Orbital and dynamical features

Considering a system consisting of a star and a planet (assumed as point masses) we expect Keplerian elliptic planetary orbits around the star with period $T$, semi-major axis $a$, eccentricity $e$ and inclination $I$. From the observations and after applying particular fitting methods (see e.g. [2]) these orbital parameters can be estimated approximately for each observed planet. In figure 3, we plot the distributions $a$-$e$ and $T$-$e$ of *623* planets for which we have an estimation of the orbital parameters in the list of EPE. The fact that the majority of planets appears with small semi-major axes and periods is possibly caused by an observational selection bias, since large and close to the star planets are more easily detectable. Most of these close-to-star planets have also small orbital eccentricities possibly caused by tidal circulation [4].

An important orbital feature that is clearly seen from the plot of figure 3 is that we obtain the existene of many planets with large eccentricities. E.g. an extreme planet is *HD 20782 b* with $a$=1.38*AU* and $e$=0.97. High eccentric orbits are also observed in multi-planet systems. E.g. in the two-planet system around the star *HD 7449* the inner and the outer planets have eccentricity 0.82 and 0.53, respectively (see figure 4). The formation of such highly eccentric planetary systems is not sufficiently supported by the current theories. However, long-term evolution stability is possible and can be proved as we explain in the last section.

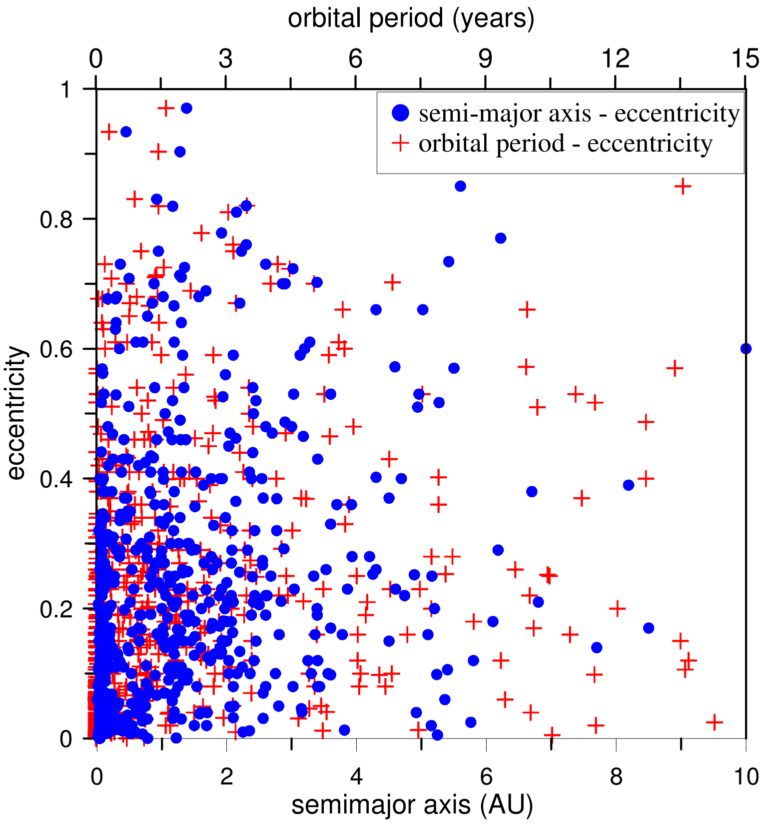


**Figure 3.** The distribution of exoplanets in the planes *a-e* (blue dots) and *T-e* (red crosses). Most of the planets have small semi-major axis (or period) and small eccentricity. There are few more planets, which are located outside the axis' limits of the plot and they are not shown.

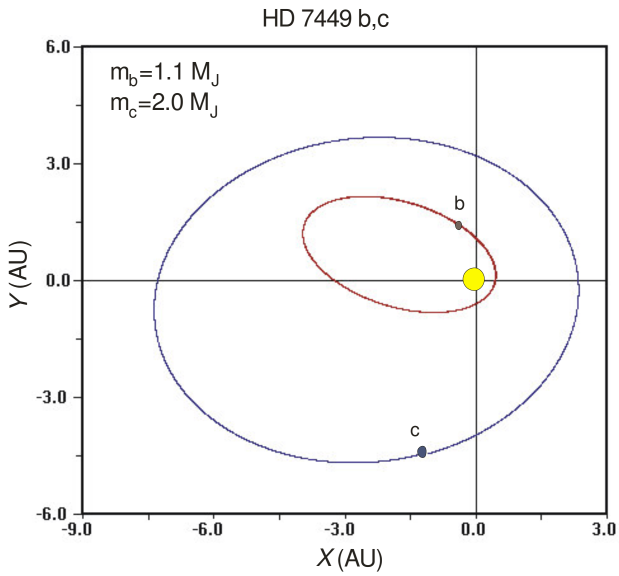


**Figure 4.** The orbits of the giant planets around the star HD 7449. The eccentricities of the inner and the outer planet are 0.82 and 0.53, respectively. The semimajor axes are 2.3*AU* and 4.96*AU*, respectively.

The inclination $I_o$ of the orbital plane with respect to the observer is defined as the angle between the normal to the planet's orbital plane and the line of the observer to the star. Most observational methods, and mainly the transiting method, can observe planets only with inclinations $I_o \approx 90^o$. For radial-velocity observations the inclination is very important for the computation of the planetary mass because the particular measurements provide us with an estimation of the quantity $m \cdot sin\ I_o$.

The inclination $I$ defined between the normal of the orbital plane and the stellar rotational axis is the most important from a dynamical point of view. In this sense, particular studies indicate that most planetary systems are inclined [5]. Cases where $I \approx 180^o$, i.e. planets moving around the star in a direction opposite to the spin of the star, have also been indicated in observations making the puzzle of planetary formation quite complicated. Generally, it seems that in multi-planet systems, planets are almost co-planar, i.e. the *mutual* inclination $\Delta I = I_1 - I_2$ between two planets is close to zero. However, cases of high mutual inclination between planets have also been observed, e.g. the planets c and d around *Upsilon Andromeda* seem to have mutual inclination larger than $30^o$.

Mean motion resonances (MMR) in multi-planet systems appear frequently. Two planets (1 and 2) are in a MMR, when the ratio $\rho = T_1/T_2$ of their orbital period is close to a ratio $r=p/q$ of two (small) integers. Computing the ratio $\rho$ of all planet pairs appearing in the multi-planet systems of the EPE list we obtain a significant number of resonant pairs. Certainly this number depends on the divergence $\delta = (\rho - r)/r$ that we set as a threshold. For divergence less than 5%, 2% or 1% we find respectively 419, 174 or 83 resonant planetary pairs. Their distribution to each value $r$ is shown in figure 5.

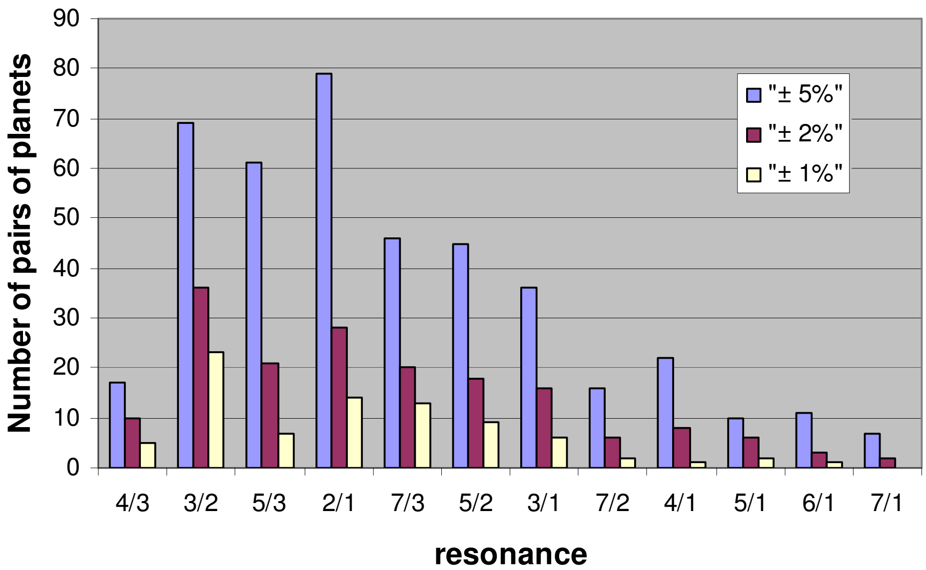


**Figure 5.** The distribution of resonant planetary pairs at each resonance p/q. Different color bars correspond to different threshold values of the difference of the ratio $T_1/T_2$ from the ratio p/q. For each pair we have set as $T_1$ the greater orbital period.

### 3. The three body model and orbital evolution

A two-planet system can be modeled by a system of three point masses $m_0$, $m_1$ and $m_2$ representing the star, the inner and the outer planet, respectively. Thus $m_0 >> m_{1,2}$ and initially it holds $a_1 < a_2$ . Although the planetary masses are very small, the mutual planetary interaction is not negligible if we want to study the orbital evolution for moderate or long time intervals and during such an evolution the inner planet may become outer and vice versa.

Let $X_i$ be the position vector of the three bodies in an inertial frame. Following Poincaré, we can consider for the planets the astrocentric position vectors $r_i=X_i-X_0$ and the barycentric momenta $p_i=m_i(dX_i/dt)$ and write the Hamiltonian of the system in the form $H=H_0+H_1$ , where

$$H_0 = \sum_{k=1}^{2}\left(\frac{p_k^2}{2\beta_k} - \frac{\mu_k \beta_k}{|r_k|}\right)$$

$$H_1 = -\frac{G m_1 m_2}{|r_1 - r_2|} + \frac{p_1 p_2}{m_0}$$

and

$$\mu_k = G(m_0 + m_k), \quad \beta_k = \frac{m_0 m_k}{m_0 + m_k}.$$

The term $H_0$ describes the evolution of the planets in the framework of the two body problem (unperturbed star-planet system). The second term includes the planetary interactions and is a perturbation term for the integrable part $H_0$. Subsequently, we have a nonintegrable model of 6 degrees of freedom and the corresponding canonical equations of motion are integrated numerically. In this system we have the conservation of energy and the three components of the angular momentum vector [6].

A second formalism for the planetary three-body model results if we express the Hamiltonian in orbital elements[3]

$$H = -\sum_{k=1}^{2}\frac{G m_0 m_k}{2a_k} - \frac{G m_1 m_2}{a_2} R(a_i, e_i, I_i, \omega_i, \Omega_i, \lambda_i)$$

where $\omega_i$ is the argument of periastron, $\Omega_i$ is the longitude of ascending node and $\lambda_i$ is the mean longitude (index $i$=1,2 refers to the particular planet) [7]. From the above Hamiltonian simplified models can be constructed by the *method of averaging*.

By averaging that fast motion on the ellipse, given by the angles $\lambda_i$, we obtain the secular model. Up to second order in masses we obtain that the phase space structure depends on the planetary mass ratio $m_1/m_2$. Also, it turns out that the semimajor axes remain almost constant and in a coplanar system the eccentricities $e_1$ and $e_2$ oscillate slowly with opposite phases. Actually, in a 3D system the conservation of angular momentum is expressed as

$$\alpha_1\sqrt{1-e_1^2}\sin I_1 + \alpha_2\sqrt{1-e_2^2}\sin I_2 \approx const.$$

where $\alpha_I$ are almost constants, which depend on the masses and semimajor axes of the planets. Another feature of secular dynamics is also the libration of the difference $\Delta\varpi=\varpi_2-\varpi_1$ of the longitude of pericenter of the two planets [8].

When the system is close to a MMR, the averaging should exclude slow "angle combinations" [7][8]. For a resonance $r=p/q$ we can define the resonant (slow) angles $\sigma_i= q\lambda_1-p\lambda_2+(p-q)\varpi_i$. If $\sigma_1$ or $\sigma_2$ librates we argue that the planetary system involves inside the resonance. The center of a resonant domain or the "exact resonance" is given by the stable stationary solutions i.e. the minima of the averaged Hamiltonian where $\sigma_i$=const. In resonant evolution, the semimajor axes are not invariant (as in the secular evolution), but we can derive the constraint $\beta_1 a_1^2 + \beta_2 a_2^2 = const.$, where $\beta_I$ are almost constants and depend on the masses and the resonance.

[3] More precisely we use canonical Delaunay-like variables

The conservation of the angular momentum can be used for reducing further the degrees of freedom by two. This is achieved by choosing a suitable *rotating frame* Oxyz, where the star and one of the planets (say planet 1) are always located on the plane O*xz* and O*z* axis is chosen to coincide with the vector of angular momentum. In this rotating frame, the planet 2 is given by the components $(x,y,z)=(x_2,y_2,z_2)$ ,but only the component $\chi=x_1$ is required for the determination of the planet 1. Thus the system is described by four degrees of freedom and Hamiltonian

$$H = H(\chi, x, y, z, p_\chi, p_x, p_y, p_z)$$

where $p_\chi$, $p_x$, $p_y$ and $p_z$ are the conjugate momenta [9]. In this formalism, we can obtain directly the well known circular restricted three body problem, if we set $m_2=0$, $\chi$=const. and $n$=const., where $n$ is the angular velocity of the rotating frame.

Hamiltonian, *H,* written for the rotating frame is convenient, when we want to study the dynamics through the periodic orbits of the system. The families of periodic orbits of the restricted three body problem, generally, are continued to the general three body problem and can be computed in a systematic way [10][9]. Periodic orbits are associated with the stationary solutions of the approximate averaged Hamiltonian mentioned above. Families of periodic orbits are either “circular” or “elliptic”. Along a circular family the ratio $\rho=T_2/T_1$ of planetary periods varies. Families of elliptic orbits bifurcate from the circular family, when $\rho$=p/q (i.e. at resonances). Along these bifurcating families $\rho$ remains almost constant. Thus, all elliptic periodic orbits are resonant and, actually, present the “exact” dynamical resonance [11]. Linear stability analysis can be performed classifying the periodic orbits as stable or unstable.

## 4. Orbital Evolution of HD 82943b,c

In this section, we present an example of the dynamical analysis of the orbital evolution of the extra-solar system around the star HD 82943 (with mass $1.18M_{Sun}$). Three planets have been discovered for this star, the planets b,c and d with period 442, 219 and 1078 days, respectively. The first two planets have masses $\sim 4.8M_J$ and the third one is quite smaller with mass $0.29M_J$. So, we can neglect the interaction of the third planet to the other two heavy planets and study the evolution in the framework of the three body model with inner planet ($P_1$) the planet c and outer planet ($P_2$) the planet b. The orbital parameters are the following

| | $m$ ($M_J$) | $a$ (AU) | $e$ | $\varpi$ (°) |
|---|---|---|---|---|
| $P_1$ | 4.78 | 0.75 | 0.425 | 133 |
| $P_2$ | 4.80 | 1.20 | 0.203 | 107 |

No estimation is given for the position of the planets in their elliptic orbits. The two planets are coplanar with $I_0=19^\circ$. Also the system is resonant with $T_2/T_1\approx 2.0$.

For the planetary ratio $m_2/m_1\approx 1$ and for the region of the given eccentricity values, we find that there exists a family of periodic orbits, which is symmetric and corresponds to a planetary configuration with aligned planets, i.e. $\Delta\varpi=0^\circ$, and when the inner planet is at periastron the outer planet could be found at apoastron, i.e. $\sigma_1=0^\circ$. Thus, we will consider for our analysis two planetary initial configurations: *configuration A* with $\Delta\varpi=0^\circ$ and *configuration B* with $\Delta\varpi=-26^\circ$ (the value given in the list). We consider that $P_1$ is initially located at periastron and $P_2$ at apoastron[4].

In Figure 6, we present the evolution of the eccentricities and the apsidal difference $\Delta\varpi$ for configurations A and B. When the planets are aligned (configuration

[4] This is the better configuration with respect to stability

A) the system evolves regularly, $\Delta\varpi$ librates indicating the resonant evolution and the eccentricities show small (anti-phase) oscillations. However, if we consider the configuration B, we see that $\Delta\varpi$ rotates. Although, the evolution initially seems regular, after 1.2*Ky* the system is destabilized. Small planetary encounters occur and the outer planet increases its eccentricity to values almost up to 1 (collision with the star).

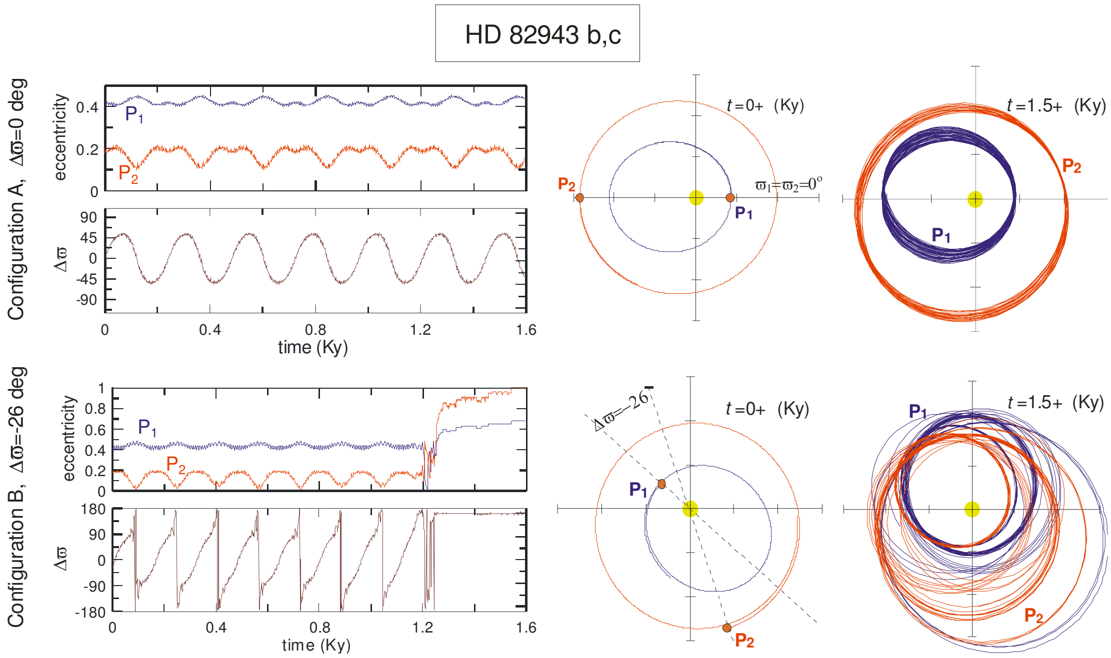


**Figure 6.** The evolution of planets c ($P_1$) and b ($P_2$) of the system HD 82943 for the two different initial configurations A and B (see the text). In the left panels, the time evolution of the eccentricities and the planetary apsidal difference is presented. In the middle, we show the initial configuration of the system and the orbit in its first moments of evolution. In the right, the planetary orbits at about 1.5*Ky* are shown. The destabilization in the configuration B is obvious.

In order to understand the underlying dynamics of the above evolution, we depict the qualitative type of evolution of all orbits in particular domains of the phase space by constructing *dynamical maps of stability*, namely we consider plane grids of initial conditions and for each grid point we evolve the orbits and classify them as regular or chaotic by computing a chaoticity index e.g. the DFLI in our case [12]. In the maps presented below, light (yellow) colors indicate chaotic motion while dark ones (blue-green) corresponds to regular evolution.

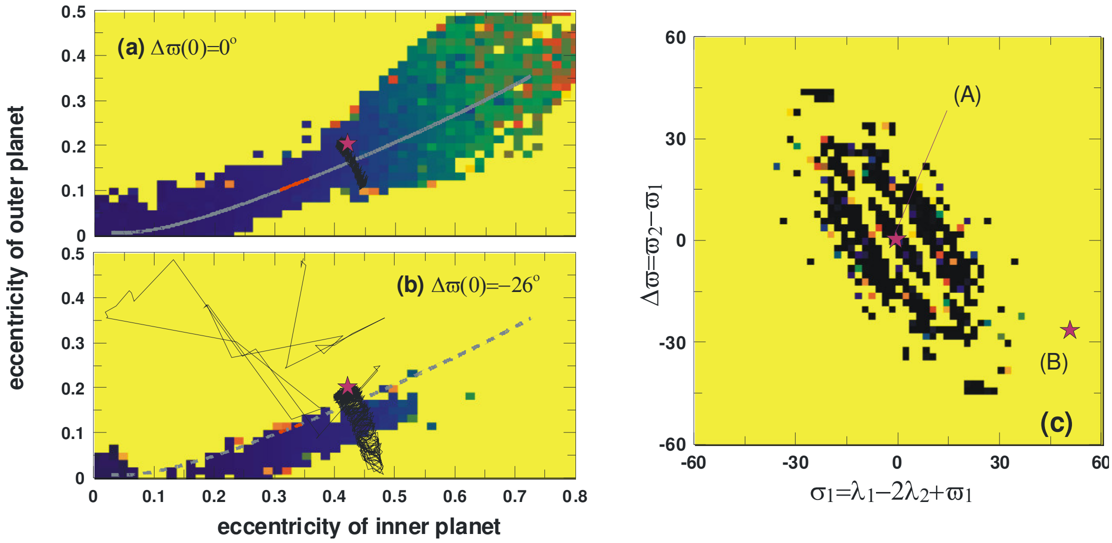


**Figure 7.** Dynamical maps of stability for the system HD 82943. a) map in the plane of initial eccentricities for the configuration A, b) map in the plane of initial eccentricities for the configuration B, c) map in the plane of initial resonant angles ($\sigma_1$,$\Delta\varpi$) at ($e_1$,$e_2$)=(0.425, 0.16). Yellow regions indicate chaos while dark colors correspond to regular orbits. The star shows the starting point of the system.

In panels (a) and (b) of figure 7, we consider all possible eccentricities of the two planets (keeping the rest orbital parameters as it is defined in the two configurations A and B, respectively). We obtain that in configuration A (panel a) there is a strip of regular orbits. The backbone of this region is the stable 2:1 resonant family of periodic orbits for the particular planetary masses (the gray characteristic curve). If we project the planetary evolution in the plane of eccentricities, we obtain oscillations centered at the eccentricity values of a periodic orbit at about ($e_1$,$e_2$)=(0.42, 0.16). When the planets are not initially set to be aligned ($\Delta\varpi\neq0^o$), the regions of regular orbits in the dynamical stability maps shrink and show a shift from the characteristic curve of periodic orbits. Subsequently, for $\Delta\varpi$=-26$^o$ the system seems to be located in the chaotic region (but close to the regular one). Thus, the system destabilizes and shows diffusion in the wide chaotic sea of the phase space, where the evolution is strongly irregular. In these dynamical regions, the system suffers from close encounters, which possibly lead one of the planets to a collision with a star or to escape or to a planet-planet collision.

In panel (c) of figure 7, we consider the eccentricity values (0.42, 0.16), mentioned above, and construct a dynamical map for a grid of initial conditions with all possible planetary alignments $\Delta\varpi$ and initial angle positions (presented by the resonant angle $\sigma_1$). The central regular region is associated with the periodic orbit, located in this map at (0,0). Again, we see that the configuration B locates the system in the chaotic region. We may conclude from the above analysis that the orbital parameters given in the EPE list should be revised. Namely, stability is guaranteed for $|\Delta\varpi|<26^o$. Also, if we assume slightly smaller planetary masses, then the regular region is expanded and the system can be located in the regular region. Possible planetary mutual inclination may also be a stabilization factor.

When two planets have large enough eccentricities, then planetary close encounters (and subsequently destabilization) are, generally, unavoidable unless the system is in resonance. Resonances can offer a phase protection, i.e. although orbits are close (or intersect), the planets cannot be found close to each other. It has been shown that stable resonant periodic orbits can exist for very large eccentricities and therefore, regions of stability can be located. However, as eccentricities increase, the stability domain around the central periodic orbit shrinks. Thus, a real planetary system with large eccentricities may be found only very close to a stable resonant periodic orbit. This, for example, should be the case for the system HD 7449 presented in figure 4.

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