A SYSTEMATIC STUDY OF THE STABILITY OF SYMMETRIC PERIODIC ORBITS IN THE PLANAR, CIRCULAR, RESTRICTED THREE-BODY PROBLEM

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Abstract. We investigate symmetric periodic orbits in the framework of the planar, circular, restricted, three-body problem. Having fixed the mass of the primary equal to that of Jupiter, we determine the linear stability of a number of periodic orbits for different values of the eccentricity. A systematic study of internal resonances, with frequency p/q with $2 \leq p \leq 9$, $1 \leq q \leq 5$ and $4/3 \leq p/q \leq 5$, offers an overall picture of the stability character of inner orbits. For each resonance we compute the stability of the two possible periodic orbits. A similar analysis is performed for some external periodic orbits.

Furthermore, we let the mass of the primary vary and we study the linear stability of the main resonances as a function of the eccentricity and of the mass of the primary. These results lead to interesting conclusions about the stability of exosolar planetary systems. In particular, we study the stability of Earth-like planets in the planetary systems HD168746, GI86, 47UMa,b and HD10697.

Keywords: Periodic orbits, linear stability, three-body problem, exosolar systems

1. Introduction

The three-body problem is a continuous source of study, since the discovery of its non-integrability due to H. Poincaré (1892). Regular and chaotic motions have been widely investigated with any kind of tools, from analytical results to numerical explorations. Within regular dynamics, the phenomenon

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of orbital resonances, i.e. commensurabilities between the angular frequencies of the two small bodies of the system, puzzled scientists and still remains one of the most interesting subjects of Celestial Mechanics. In particular, the explanation of the fact that in the asteroid belt some resonances are empty while others are well populated has been until recently an open problem.

In this paper, we consider the planar, circular, restricted, three-body problem, where we assume that the mass of one body is so small that it does not influence the motion of the other two bodies, called primaries. Let the three bodies be named as S, J, A, later identified with the Sun, Jupiter and an asteroid. We assume that the motion of J around S is circular and that all bodies move on the same plane. Let us denote by n, n' the mean motions of A and J, respectively. We will work in a synodic frame, rotating with the same angular velocity of the primaries S and J, which are assumed to be at rest on the x-axis.

A p/q resonance is a periodic orbit such that the ratio n/n' is approximately equal to p/q for some positive integers p, q. We have exact equality, $n/n' = p/q$, when the mass of the small primary J is equal to zero. Indeed, for any p and q there exist two families of periodic orbits which differ in phase and are generated by a variation of the orbital eccentricity of the small body A.

Our aim is to investigate the stability of these families of periodic orbits (see Ferraz-Mello (1999), Hadjidemetriou and Ichtiaroglou (1984), Hadjidemetriou (1988), (1992), (1996), Henrard (1996), Henrard and Lemaitre (1987), Moons (1997), Morbidelli (1996), Morbidelli and Guzzo (1997), Schubart (1964), Simó (1995), Winter and Murray (1997), (1997), Hénon (1997) and references therein). The linear stability of a periodic orbit can be studied by looking at the eigenvalues of the monodromy matrix along the periodic orbit itself. By computing the trace of the matrix we can conclude whether a periodic orbit is linearly stable or not. The model and the method to study linear stability are presented in §2.

The first part of this work deals with the analysis of the stability as the mass of the primaries is fixed. In particular, we consider the Sun-Jupiter system and we investigate the stability of inner and outer orbits (i.e., interior and exterior resonances). We present results for various values of p/q with $2 \leq p \leq 9$, $1 \leq q \leq 5$ and $4/3 \leq p/q \leq 5$. For each family of periodic orbits with frequency p/q we compute the linear stability as a function of the average eccentricity along the periodic orbit. There are marked differences in the stability behaviour according to the value of p/q; in particular, we noticed that switching between stability and instability occurs more frequently as q gets larger. These results are described in §3.

A study of the stability character as the mass of the primary is varied, is presented in §4. This analysis is particularly suited for the investigation of exosolar planetary systems, where the mass of the primary cannot be determined with accuracy, due to the unknown inclination of the plane of motion. Also in this case we investigate inner and outer orbits, providing a description of the stability character for different values of the masses and eccentricities.

As a concluding result, we apply the analysis to the possibility of existence of stable Earth-like planets in the recently discovered exosolar systems HD168746, GI86, 47UMa,b and HD10697.

2. Model and methods

2.1. Equations of motion

We consider three bodies S, J, A; for later applications we shall identify S with the Sun, J with Jupiter and A with a small body of negligible mass. Let m_S and m_J be the masses of S and J, respectively. We assume that the three bodies move on the same plane and that the orbit of J is circular. We refer to this model as the planar, circular, restricted three-body problem, hereafter, PCR3BP. Let us normalize the units so that the distance between the primaries, the sum of their masses and the gravitational constant are equal to 1.

We consider a synodic reference frame (Oxy), rotating with the same frequency of the primaries. Let the origin coincide with the center of mass of the primaries and let the x-axis correspond to the SJ direction. Denote by (x, y) the coordinates of A and by μ the reduced mass $\mu \equiv m_J/(m_s + m_J)$. The equations of motion of A in the synodic reference frame are given by Szebehely (1967)

$$\begin{aligned}\ddot{x} - 2\dot{y} - x &= -\mu\frac{x+\mu-1}{r_1^3} - (1-\mu)\frac{x+\mu}{r_2^3} \\ \ddot{y} + 2\dot{x} - y &= -\mu\frac{y}{r_1^3} - (1-\mu)\frac{y}{r_2^3},\end{aligned}$$

where $r_1^2 = (x+\mu-1)^2 + y^2$, $r_2^2 = (x+\mu)^2 + y^2$. As is well known, these equations admit the first integral (Jacobian energy)

$$C \equiv \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2\right) - \frac{1}{2}\left[\mu r_1^2 + (1-\mu) r_2^2\right] - \frac{\mu}{r_1} - \frac{1-\mu}{r_2}.$$

2.2. Mirror configurations

In the synodic frame, there exist families of periodic orbits inside the orbit of J that are symmetric with respect to the x-axis (notice that in general these orbits are not symmetric in the inertial frame). If the initial conditions are

TABLE I
Numeric coding of MCs for interior periodic orbits (see the text for an explanation of symbols)

Configuration	MC	l	$\Delta\lambda$
PC-SAJ	00	0	0
PC-ASJ	02	0	π
AC-SAJ	20	π	0
AC-ASJ	22	π	π

such that the periodic orbit intersects perpendicularly the x-axis at $t = 0$ (i.e., $y(0) = 0$, $\dot{x}(0) = 0$), the periodic orbit intersects perpendicularly the x-axis also at a half period (i.e., $y(T/2) = 0$, $\dot{x}(T/2) = 0$, where T denotes the period of the periodic orbit).

These intersections represent particular cases of the so-called *mirror configuration* (MC), which occurs whenever the velocity vectors of a number of gravitationally interacting bodies are perpendicular to their position vectors in a barycentric sidereal frame. It was proven by Roy and Ovenden (1955) that an orbit is periodic if a MC occurs at two different epochs.

Mirror configurations can be classified according to the initial positions of the three bodies; in particular, A could be at $t = 0$ at its pericenter (PC) or at apocenter (AC) and the initial configuration might be $S - A - J$ or $A - S - J$. According to a scheme introduced by Valsecchi et al. (1993), it is possible to code all possible MC's using a pair of integers, indicating the value, in multiples of $\pi/2$, of suitably chosen angles; this is done in Table I, where l denotes the mean anomaly of A and $\Delta\lambda$ is the difference of the mean longitudes of A and J. As an example, consider the MC denoted by 02; then $l = 0$ and $\Delta\lambda = \pi$, which means that A is at its pericenter in the configuration $A - S - J$.

Let $n/n' = p/q$, where n, n' denote the mean motions of A and J, respectively. In a symmetric periodic orbit associated to this resonance, two features are noteworthy:

- l makes p complete revolutions, while $\Delta\lambda$ makes $p - q$;
- two MCs must occur, characterized by different codes.

Following Valsecchi et al. (1993), we note that the parity of p and $p - q$ uniquely determines the codes of the MC's that can appear together in a given symmetric periodic orbit (I and II p.o. in Table II).

We remark that in the following we shall consider only *direct* orbits, which move around S in the same direction of J.

TABLE II
The pairing of MCs within periodic orbits

p, $p-q$	odd,even	even,odd	odd,odd
I p.o.	00,20	00,02	00,22
II p.o.	22,02	22,20	02,20

It will be useful in our analysis to compute an approximate value of the eccentricity which causes collisions or very close encounters with J. An approximate value can be obtained by equating the apocenter distance of A to the SJ distance, which in our units is 1:

$$a(1+e) = 1.$$

By Kepler's third law, we define a collision value e_c as

$$e_c \equiv \left(\frac{p}{q}\right)^{\frac{2}{3}} - 1. \tag{1}$$

2.3. Numerical determination of periodic orbits

Let us briefly describe the method adopted to find a periodic orbit. Let us fix the values of μ, $n/n' = p/q$ and the unperturbed eccentricity e_0.

The initial conditions $x(0)$, $\dot{y}(0)$ (recall that $y(0) = 0$, $\dot{x}(0) = 0$) are found by the two-body approximation. The periodic orbit is computed within a preassigned accuracy, typically 10^{-12}, through a Newton-Raphson method, which provides successive approximations of the initial conditions. Care must be taken at which intersection of the orbit with the x-axis one checks for the perpendicularity condition. As long as the eccentricity is small, the number of intersections is equal to $p - q$; however, as the eccentricity increases, a loop might develop in the rotating frame and the number of intersections increases.

As output, the program provides the initial conditions of the periodic orbit, partial derivatives which are needed for the stability analysis and the average values of the semimajor axis, the eccentricity and the ratio n/n' over half period of the periodic orbit, due to the symmetry assumption. In particular, let us define the average eccentricity as

$$e \equiv \frac{1}{T/2}\int_0^{T/2} e(t)dt, \tag{2}$$

where $T \approx 2\pi q$ is the period of motion. The exact equality holds for $\mu = 0$, and the deviation from the equality is larger for larger values of μ as well as when we are close to a collision.

The numerical program to compute the periodic orbits and the above quantities was kindly made available to all the authors by J. Hadjidemetriou.

2.4. Linear stability

We investigate the linear stability of the families of periodic orbits by looking at the trace of the 4×4 monodromy matrix. In the unperturbed case ($\mu = 0$) of a *resonant periodic orbit*, there exist two pairs of unit eigenvalues.

When the perturbation is switched on ($\mu \neq 0$), one pair of eigenvalues survives, due to the existence of the Jacobian energy integral. However, the other unit pair has two possibilities to evolve: the two eigenvalues can move *on the unit circle*, and form a complex conjugate pair of the form $e^{\pm i\phi}$, or move *on the real axis* and form a real pair λ and λ^{-1}.

In the case of a complex conjugate pair on the unit circle we have stability, while in the case of a pair of reciprocal real numbers we have instability. In some particular cases, the second unperturbed unit pair also survives as a second unit pair. This is a critical case as far as stability is concerned and it is a transition point from stability to instability or vice versa.

The type of stability can be obtained from the trace of the monodromy matrix. As is well known, as far as the trace is in modulus less than 4 one has stability, while if it is greater than 4 the periodic orbit is unstable. We refer to *indifferent equilibrium* whenever the trace is exactly equal to 4.

Instead of computing the 4×4 monodromy matrix, the stability can be obtained by investigating the Poincaré map, where the periodic orbit is a fixed point. More precisely, fix the Jacobian energy and take the surface of section corresponding to $y = 0$. Then the map is in a two dimensional space $(x, \dot{x})$. The linear stability character can be found by a 2×2 matrix and it can be proven that its eigenvalues coincide with the two non unit eigenvalues of the monodromy matrix.

Finally, let us remark that nonlinear stability can be investigated by means of KAM theory (Kolmogorov (1954), Arnol'd (1963), Moser (1962)), but such an analysis goes beyond the aims of the present work. However, the nonlinear stability can be obtained numerically by computing the Poincaré map on the above surface of section. If around the fixed point of the map, i.e. around the periodic orbit, we have islands, then the motion is stable, trapped inside the island. The size of the largest island gives the nonlinear stability region. If, on the other hand, the consecutive points of the map do not define closed curves around the fixed point, the motion is not bounded close the the fixed point, and in this case the periodic orbit is unstable.

3. Stability of Sun-Jupiter-Asteroid resonances

3.1. Interior resonances

We consider the three body problem composed by the Sun (S), Jupiter (J) and an asteroid (A), which moves in the region interior to the orbit of Jupiter. Families of periodic orbits in the PCR3BP are generated as the eccentricity is varied. We showed in §2 that for each frequency p/q there exist two families depending on the initial MC. The stability character changes as p/q is varied and the eccentricity is increased. In what follows we fix μ equal to the mass of Jupiter, i.e. $\mu = 0.0009552$. Moreover, we analyze all resonances with $2 \leq p \leq 9$ and $1 \leq q \leq 5$ and we add the further constraint that $4/3 \leq p/q \leq 5$.

For each of the two families of periodic orbits with frequency p/q we compute the linear stability through the trace of the monodromy matrix projected on the Poincaré surface of section, as explained in subsection 2.4. The linear stability is analyzed with respect to the average eccentricity defined in Eq. 2. Marked differences can be observed as a function of the frequency p/q.

Let us examine in more detail the stability results. The two families associated to $n/n' = 2/1$, $4/1$, $3/2$, $5/2$, $7/2$, $9/2$ show a similar behaviour, since one periodic orbit (the one denoted I p.o. in Table II) is stable, while the other (II p.o.) is unstable. This kind of result is observed for all values of the average eccentricity.

The opposite behaviour is observed for the $3/1$ and $5/1$ families, where I p.o. is unstable, while II p.o. is stable. We remark that in some cases we noticed very small regions of the eccentricity where the stability character was the opposite of the overall behaviour observed in the interval $e \in (0, 1)$. In particular we refer to the families associated to $p/q = 5/1$, where a small region of stability was found for I p.o. for $e \in (0.766, 0.787)$, while instability was observed for II p.o. in the interval $e \in (0.769, 0.778)$.

As we study higher order resonances, with $q \geq 3$, the analysis of the stability becomes much more complicated and stable regions alternate with unstable regions as the eccentricity is varied. In Fig. 1 we report a schematic picture of the stability character of the families of periodic orbits with resonance $4/3$, $5/3$, $7/3$, $8/3$, $7/4$, $9/4$, $7/5$, $8/5$, $9/5$. In Fig. 1, S stands for stable motion, U for unstable motion and ND for the fact that *no data* were available, in the sense that the computer program was not able to find the periodic orbit within the preassigned accuracy (10^{-12}). In most cases, the ND regions can be explained by invoking close encounters with Jupiter. An approximate value of the eccentricity leading to collision is provided in Table III according to Eq. 1.

TABLE III
Approximate eccentricity leading to collision

resonance	e_c
4/3	0.21
5/3	0.41
7/3	0.76
8/3	0.92
7/4	0.45
9/4	0.72
7/5	0.25
8/5	0.37
9/5	0.48

A diagram showing the trace of the monodromy matrix of the Poincaré mapping versus the mean motion ratio n/n' is reported in Fig. 2 for various values of the average eccentricity. We remark that the values of the trace tend to flatten to 2 as n/n' gets larger.

3.2. Exterior resonances

In the framework of the PCR3BP, we consider the Sun (S), Jupiter (J) and a small body (A) moving outside the circular orbit defined by Jupiter. We fix $\mu = 0.0009552$.

Exterior resonances are characterized by a mean motion ratio less than 1; in particular, we study the families of periodic orbits associated to $p/q = 1/2$, $1/3$, $1/4$, $2/3$, $3/4$. MCs can again be coded by the initial position of the small body at pericenter (PC) or at apocenter (AC) and by the initial configurations $S - J - A$ or $A - S - J$. We give in Table IV the codes of the MCs.

As for interior resonances, we can identify two families of periodic orbits according to the values of p and $p - q$, as described in Table II.

As a summary of our results, we report in Fig. 3 the value of the trace as a function of the mean motion n/n' corresponding to the above mentioned choice of p/q, for different values of the average eccentricity. The value $e = 0.01$ was not considered since very few data are available for small eccentricities.

Upon examination of Fig. 3 we note that stable and unstable regions alternate more often than for interior resonances. In many cases it was not possible to draw the behaviour of the stability index of one of the two

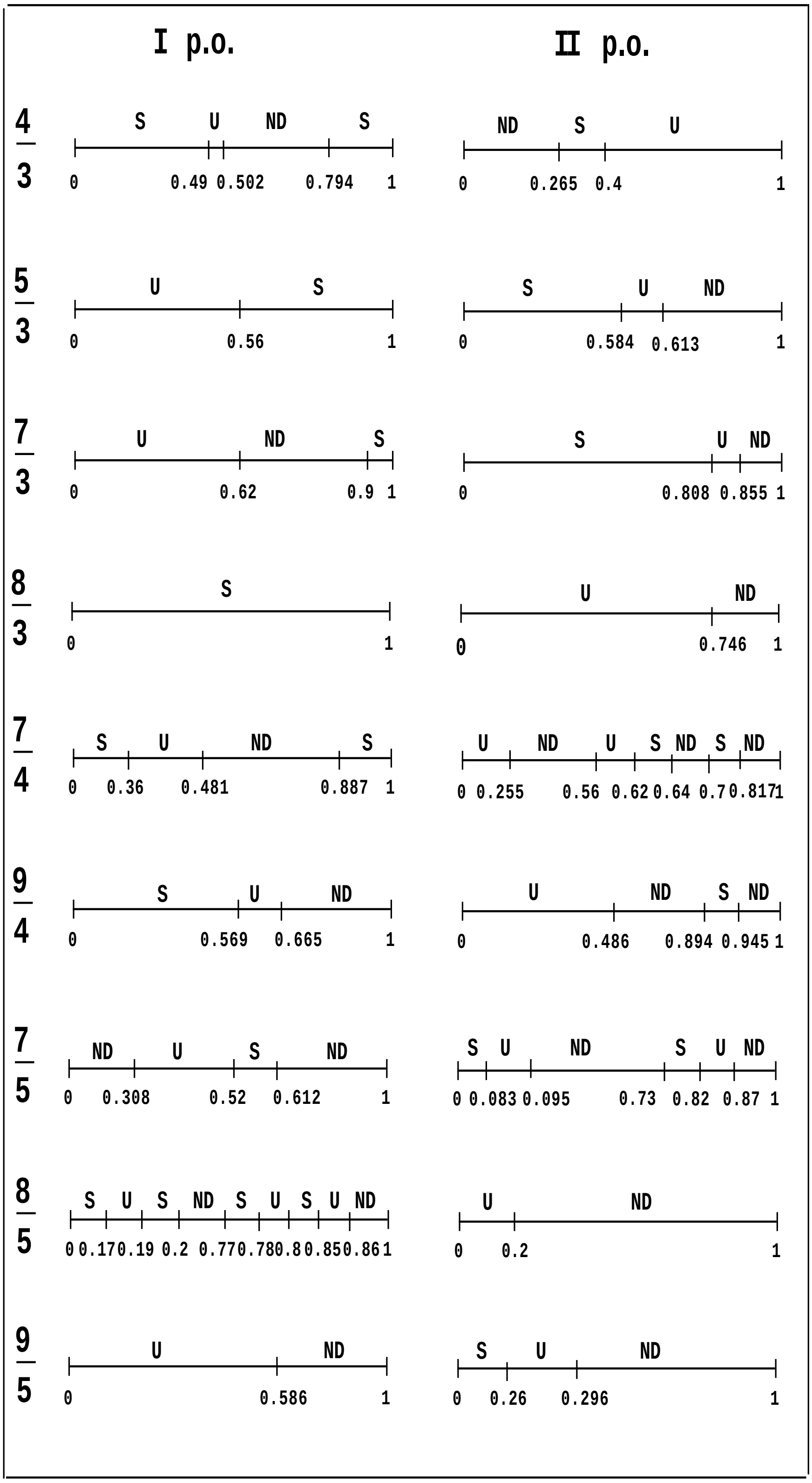

Figure 1. Stability character of the two families of periodic orbits with frequency p/q ($q \geq 3$) as a function of the average eccentricity in the interval $(0,1)$. Here S denotes a stable region, U an unstable region and ND indicates that no data were available (see the text).

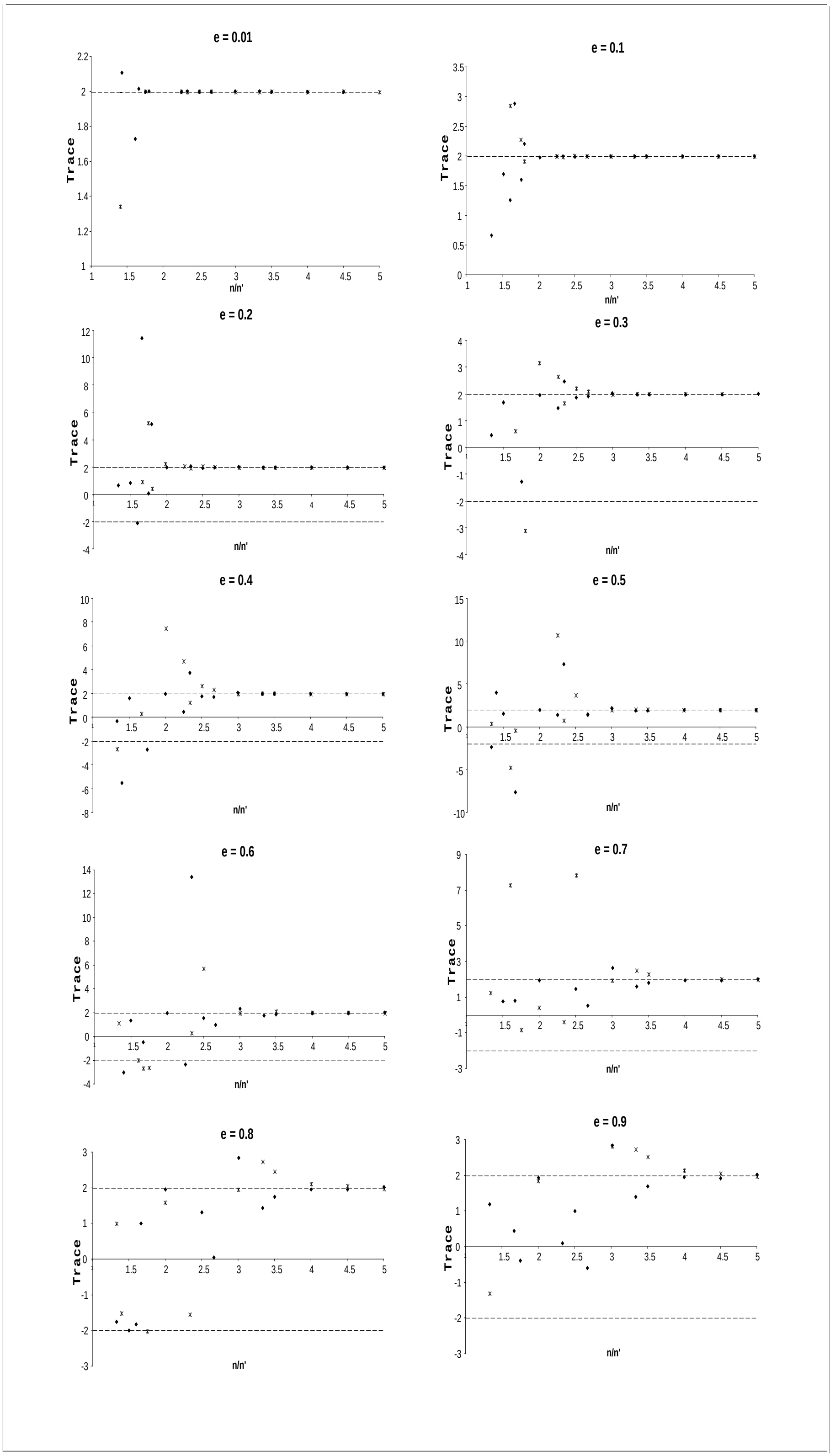

Figure 2. Inner orbits: the trace of the monodromy matrix versus n/n' for various values of the average eccentricity. Dashed straight lines at ± 2 indicate the boundary of the stability/instability region. Diamonds correspond to I p.o., while stars denote II p.o.

TABLE IV
Numeric coding of MCs for exterior periodic orbits

Configuration	MC	l	$\Delta\lambda$
PC-SJA	00	0	0
PC-ASJ	02	0	π
AC-SJA	20	π	0
AC-ASJ	22	π	π

periodic orbits (typically I p.o.), due to the highly unstable trace that would contribute to shrink the graph to a smaller picture.

4. Varying the mass of the primary

In the previous section we have considered only the case of μ corresponding to the mass of Jupiter, as it was our goal to study the stability of resonances of the solar system. In the last years many other planetary systems have been discovered around stars of different types and sizes. The ratio of the mass of the planet to that of the central star is usually uncertain, due to the unknown value of the planetary orbital inclination. A lower bound on such value can be given, but the actual mass could be larger.

As a consequence of this, it is useful to examine the stability of a three-body problem, which we still assume as restricted, planar and circular, as the mass of the planet varies.

4.1. Effects of the increase of the mass

We will study the stability of exosolar planetary systems with a large planet and a small planet. The particular problem that we set is the following: we keep the semimajor axis of the planet fixed to 1 AU (Earth-like planets) and vary the semimajor axis a' of Jupiter. We will find the range of values of a' (or, equivalently, the range of values of n/n') for which the system is unstable.

We focus our attention to *nearly circular* orbits of the small planet. The reason is that it is more likely that life would develop in such a planet, because the variations of the distance from the Sun will be small, implying that the variations of the climate will be also small during one "year".

Two different cases will be investigated: $a' > 1$ AU and $a' < 1$ AU. In the first case the orbit of the planet ($a = 1$ AU) is an inner orbit and in the

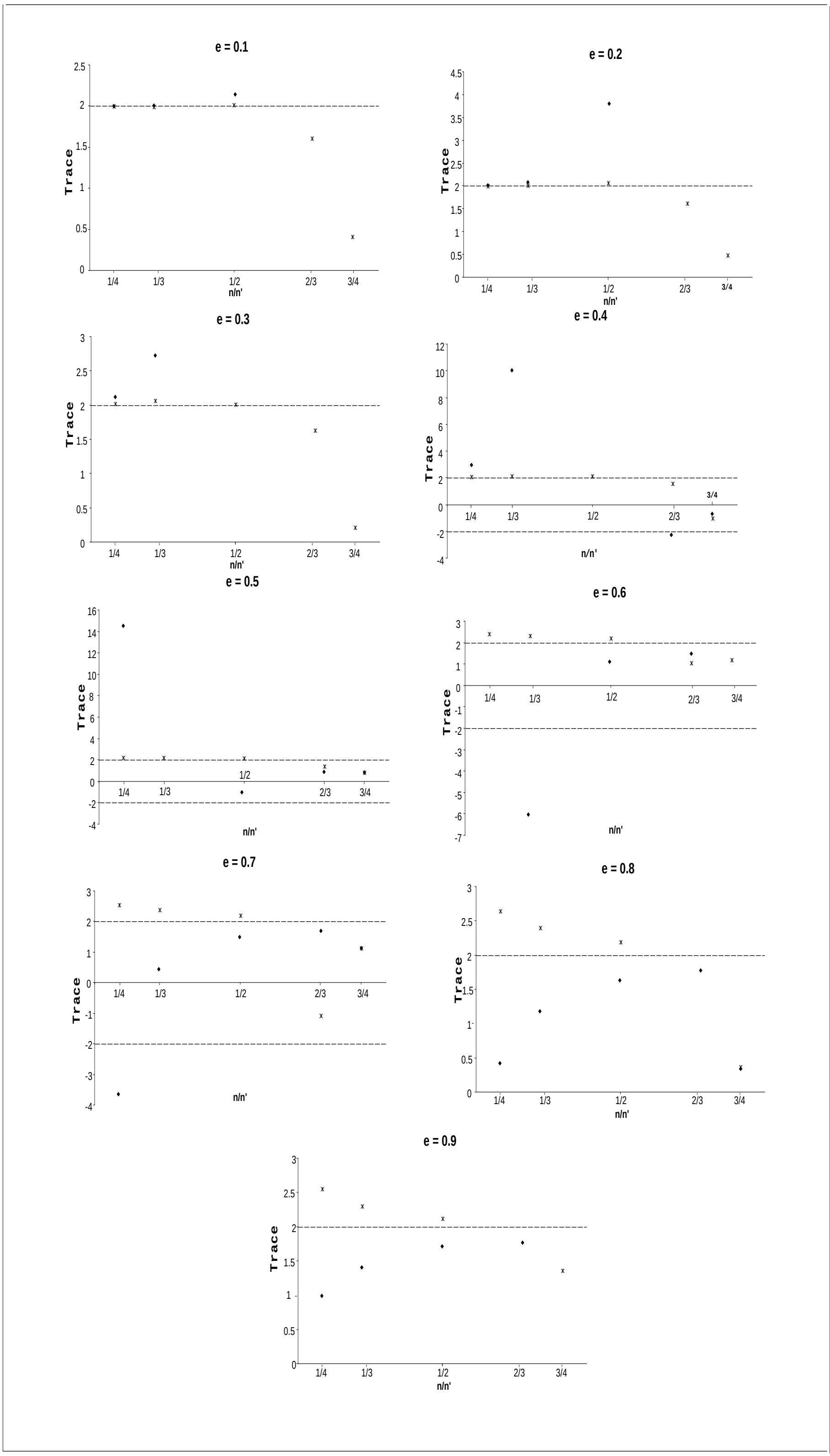

Figure 3. Outer orbits: plot of the trace versus the mean motion for different values of the average eccentricity. Dashed straight lines at ± 2 indicate the boundary of the stability/instability region. Diamonds correspond to I p.o., while stars denote II p.o.

second case it is an outer orbit. We will study separately these two cases, although they are similar in many aspects.

We remark that a model with a circular orbit of Jupiter may not be realistic, as a more realistic model would have an elliptic orbit of Jupiter. However, as far as *instability* is concerned, if the circular model is unstable, then the elliptic model is also unstable. Consequently, the results obtained with a circular model are also valid for more complicated models.

4.2. Inner orbits, $a' > 1$ AU

Let us start with the unperturbed problem ($m_J = 0$), with Jupiter describing a circular orbit with radius a'. There exists a family of circular orbits of the small planet, with semimajor axis $a < a'$, along which the radius a of the planet varies (we keep the value of a' fixed). Equivalently, the ratio n/n' varies along the family, approximately between values 1 (close to Jupiter) and ∞ (very far from Jupiter and close to the Sun). When m_J is nonzero, the above circular orbits of the small planet are continued as nearly circular periodic orbits in a *synodic system*. The continuation is not possible only at the resonances 2/1, 3/2 ...

In this study we will consider nearly circular orbits of the planet, for a range of values of n/n' larger than 2/1. For smaller values of n/n' the planet is closer to Jupiter, and the region is more unstable. In the above resonance region, $2/1 < n/n' < \infty$, the continuation of an unperturbed circular orbit of the planet to the case $m_J > 0$ is possible. Therefore, there exists a branch of nearly circular orbits of the small planet in the above range of n/n'.

It can be proven that instability develops only in a region close to the 3/1 resonance. For small values of m_J the unstable region is small, but extends and covers a large part of the phase space as m_J increases. From the numerical computations we found also that as m_J increases, the osculating eccentricity of the "circular" periodic orbits is not very small. This means that even if the orbit is stable, the variations of its eccentricity are quite large, implying large variations of the climate during a "year". This may not be favorable for the development of life.

In Fig. 4 we present the stability index of various families of nearly circular periodic orbits of the small planet, in the rotating frame, for different values of $\mu \equiv m_J/(m_S + m_J)$. They correspond to the range of n/n' from 2 to 3.4 . The range of the values of μ is between 0.01 and 0.25. We have instability if the stability index is smaller than -2.

From Fig. 4 we can also deduce that the *Kirkwood gaps* in an exosolar planetary system with a large "Jupiter" should be much larger than that in the asteroid belt of our Solar System.

In Fig. 5 we plot the minimum and the maximum value of n/n', for each value μ, as obtained from Fig. 4. We also indicate the mean values of

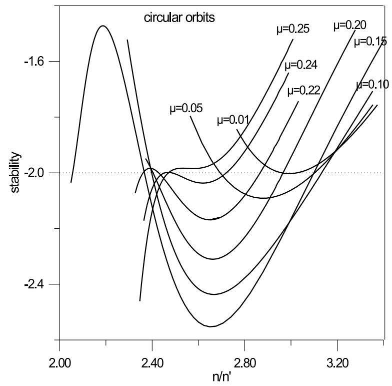

Figure 4. The stability index of inner nearly circular periodic orbits, as a function of the mean value of the ratio n/n'. As μ increases, the unstable region increases as well, and moves towards smaller values of n/n', i.e. closer to Jupiter.

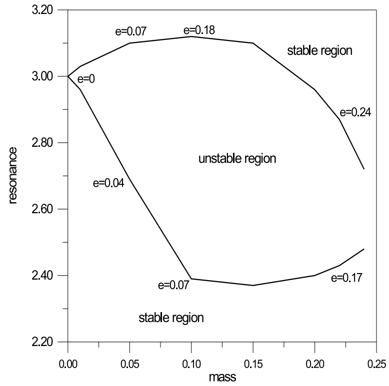

Figure 5. The unstable region as a function of μ, for inner orbits. The values of the mean eccentricity are also indicated at the boundaries.

the osculating eccentricities, at the boundaries of the unstable region, for different values of μ. Note that the eccentricity is quite large for large values of μ.

4.3. Outer orbits, $a' < 1$ *AU*

The situation is similar to the previous case $a' > 1$ AU. We also have a continuous branch of nearly circular orbits of the small planet, in the synodic frame for the range of values of the semimajor axis of the planet between $a = \infty$, and $a = 1.587a'$, so that n/n' varies between the values of 0 and 1/2. The continuation from zero mass of Jupiter is possible from $n/n' = 0$ up to

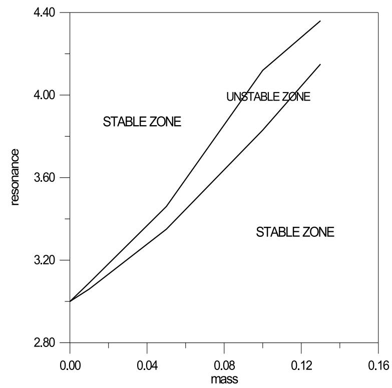

Figure 6. The unstable region as a function of μ for outer orbits.

$n/n' = 1/2$. We will not consider systems beyond this frequency, because the planet is close to Jupiter and large regions of instability develop.

As in the case of the inner orbits, it can be proven that instability develops only close to the resonance 1/3. The unstable region extends as the value of the mass of Jupiter increases. In Fig. 6 we plot the boundaries of the unstable region as a function of the mean value n/n' along the periodic orbit. This figure is similar to Fig. 5 for the inner orbits.

Remark: From the above analysis we can derive that a planetary system with a large planet may have a stable Earth-like planet. More precisely, let the semimajor axis of Jupiter be equal to a' AU. For an Earth-like planet with $a = 1$ AU the ratio n/n' of the frequencies will be $n/n' = (a/a')^{3/2}$. Let μ be the value of the mass of Jupiter. From Fig. 5 or Fig. 6 we can find whether or not an Earth-like planet can exist in this system.

4.4. Applications to exosolar systems

We will apply now the previous study to the possibility of existence of Earth-like planets in some recently discovered exosolar planetary systems. We have selected four such systems, where the eccentricity of the major planet, which we shall call "Jupiter", is close to zero. We define as *Earth-like planet* a planet whose distance from the star, which we shall call "Sun", is equal to 1 AU. In two of the above systems the semimajor axis of "Jupiter" is smaller than 1 AU, which means that the Earth-like planet is an outer planet. In the other three cases the semimajor axis of "Jupiter" is larger than 1 AU, which means that the Earth-like planet is an inner planet.

– DH168746

  We have $M \sin i = 0.24 M_J$, P=6.409 d, $a = 0.066$ AU, e=0, where P is the orbital period. The corresponding resonance of a possible Earth-like planet would be equal to $n/n' = 57$, and the value of μ is larger than

0.0003 (assuming the mass of the "Sun" is equal to 0.92 Solar mass). From Fig. 6 we see that it is well outside the unstable region. So, as far as the stability of the planet is concerned, an outer Earth-like planet could exist.

- GI86(HD13455)
  We have $M \sin i = 4M_J$, P=15.78 d, a=0.11 AU, e=0.04 (Han et al. (2000)). The corresponding resonance of a possible Earth-like planet would be equal to $n/n' = 23$, and the value of μ is larger than 0.005 (assuming the mass of the "Sun" is equal to 0.79 Solar mass). From Fig. 6 we see that it is well outside the unstable region. So, as far as the stability of the planet is concerned, an outer Earth-like planet could exist.
- 47UMab
  We have $M \sin i = 2.54M_J$, P=1089.0 d, a=2.09 AU, e=0.06 (Butler and Marcy (1996)). The corresponding resonance of a possible inner Earth-like planet would be equal to $n/n' = 2.98$, i.e. very close to the unstable resonance 3/1 and the value of μ is larger than 0.003 (assuming the mass of the "Sun" is equal to 1 Solar mass). From Fig. 5 we see that an Earth-like planet would be in the 3/1 unstable zone, which in fact is the corresponding 3/1 Kirkwood gap. Since the mass of "Jupiter" is larger than that of the Jupiter of our Solar System, the instability is stronger. Consequently, it is not likely that an Earth-like planet could survive in this exosolar system.
- HD10697
  We have $M \sin i = 6.59M_J$, P=1093.0 d, a=2.0 AU, e=0.12 (Han et al. (2000), Zucker and Mazeh (2000)). The corresponding resonance of a possible Earth-like planet would be equal to $n/n' = 2.99$, i.e. very close to the unstable resonance 3/1 and the value of μ is larger than 0.006 (assuming the mass of the "Sun" is equal to 1.1 Solar mass). From Fig. 5 we see that an Earth-like planet would be in the 3/1 unstable zone, which in fact is the corresponding 3/1 Kirkwood gap. Since the mass of "Jupiter" is larger than that of our Solar System, the instability is stronger. On the other hand, the eccentricity of "Jupiter" is not very close to zero, but this is expected to make stronger the instability obtained by the circular restricted three-body problem approximation. Consequently, it is not likelythat an Earth-like planet could survive in this exosolar system.

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References

Arnol'd V.I.: *Proof of a Theorem by A.N. Kolmogorov on the invariance of quasi-periodic motions under small perturbations of the Hamiltonian*, Russ. Math. Surveys **18**, 9 (1963)

Butler R.P., Marcy G.W.: *A planet orbiting 47Uma*, Astrophys. J. **464**, L153-L156 (1996)

Ferraz-Mello, S.: *Slow and fast diffusion in asteroid-belt resonances: a review*, Celest. Mech. & Dyn. Astron. **73**, 25-37 (1999)

Hadjidemetriou J.D.: *Algebraic mappings near a resonance with an application to asteroid motion*, in: Long-term dynamical behaviour of natural and artificial N-body systems, A.E. Roy ed., pp. 257-276 (1988)

Hadjidemetriou J.D.: *The elliptic restricted problem at the 3:1 resonance*, Celest. Mech. & Dyn. Astron. **53**, 151-183 (1992)

Hadjidemetriou J.D.: *Symplectic mappings*, in: Dynamics, Ephemerids and Astrometry of the Solar System, IAU Symp. 172, S. Ferraz-Mello, B. Morando and J.-E. Arlot eds., pp. 255-266 (1996)

Hadjidemetriou J.D., Ichtiaroglou S.: *A qualitative study of the Kirkwood gaps in the asteroids*, Astron. Astrophys. **131**, 20-31 (1984)

Han I., Black D., Gatewood G.: *Preliminary astrometric masses for proposed extrasolar panetary companions*, Astrophys. J. **548**, L57-L60 (2000)

Hénon, M.: *Generating Families in the Restricted Three-Body Problem*, Springer-Verlag, Berlin, Heidelberg and New York (1997)

Henrard, J.: *A note concerning the 2:1 and the 3:2 resonances in the asteroid belt*, Celest. Mech. & Dyn. Astron. **64**, 107-114 (1996)

Henrard, J. and Lemaitre, A.: *A perturbative treatment of the 2/1 Jovian resonance*, Icarus **69**, 266-279 (1987)

Kolmogorov A.N.: *On the conservation of conditionally periodic motions under small perturbation of the Hamiltonian*, Dokl. Akad. Nauk. SSR **98**, 469 (1954)

Moons, M.: *Review of the dynamics in the Kirkwood gaps*, Celest. Mec. & Dyn. Astron. **65**, 175-204 (1997)

Morbidelli, A.: *On the Kirkwood gap at the 2/1 commensurability with Jupiter: numerical results*, Astron. J. **111**, 2453-2461 (1996)

Morbidelli, A. and Guzzo, M.: *The Nekhoroshev theorem and the asteroid belt dynamical system*, Celest. Mech. & Dyn. Astron. **65**, 107-136 (1997)

Moser J.: *On invariant curves of area-preserving mappings of an annulus*, Nach. Akad. Wiss. Göttingen, Math. Phys. Kl. II **1**, 1 (1962)

Poincaré H.: *Les Méthodes Nouvelles de la Méchanique Céleste*, Gauthier Villars, Paris (1892)

Roy A.E., Ovenden M.W.: *On the occurrence of commensurable mean motions in the Solar System. II The mirror theorem*, Mon. Not. R. astron. Soc. **115**, 296 (1955)

Schubart, J.: *Long-period effects in nearly commensurable cases of the restricted three-body problem*, Smithsonian Astrophys. Obs. report No 149 (1964).

Simó, C.: *The bicircular model near the triangular libration points in the RTBP*, in: From Newton to Chaos: Modern techniques for understanding and coping with chaos in N-body Dynamical Systems, A.E Roy ed., Plenum Press, pp. 343-370 (1995)

Szebehely V.: *Theory of orbits*, Academic Press, New York and London (1967)

Valsecchi G.B., Perozzi E., Roy A.E., Steves B.A.: *Periodic orbits close to that of the Moon*, Astron. Astrophys. **271**, 308-314 (1993)

Winter O.C., Murray C.D.: *Resonance and Chaos. I. First-order interior resonances*, Astron. Astrophys. **319**, 290-304 (1997)

Winter O.C., Murray C.D.: *Resonance and Chaos. II. Exterior resonances and asymmetric libration*, Astron. Astrophys. **328**, 399-408 (1997)

Zucker S., Mazeh T.: *Analysis of the Hipparcos measurments of HD10697 - A mass determination of a brown-dwarf secondary*, Astrophys. J. **531**, L67-L69 (2000)