Lienard Dynamical systems

A linear oscillator with damping is described by the differential equation $\ddot{x}+b\dot{x}+kx=0$ where *b,k* are positive constants. Lienard generalized this system in the form

$\ddot{x}+f(x)\dot{x}+g(x)=0$ or $\begin{aligned}\dot{x}&=y\\ \dot{y}&=-g(x)-f(x)y\end{aligned}$ (Lienard system)

The following theorem holds

Lienard Theorem: The Lienard system possesses a unique periodic solution (limit cycle) if

i) f(x) is an even function with f(0)<0 an f(x)>0 for x>a (where a is a positive constant)
ii) g(x) is an odd function and g(x)>0 for x>0.

Exercise in Mathematica:

Choose two particular functions f(x) and g(x) that satisfy the criteria of the Lienard theorem and

α) Plot the functions f(x) and g(x) as well as the potential function $V=\int g(x)dx$

β) Plot the phase space portrait of the system

γ) Plot the time evolution of x=x(t) by choosing some initial conditions

δ) The system must possess a limit cycle. Determine the amplitude and the period of the oscillations of the limit cycle solution

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Avoid to use the Lienard systems given in the book of Stephen Lynch (choose different functions f(x) and g(x)).