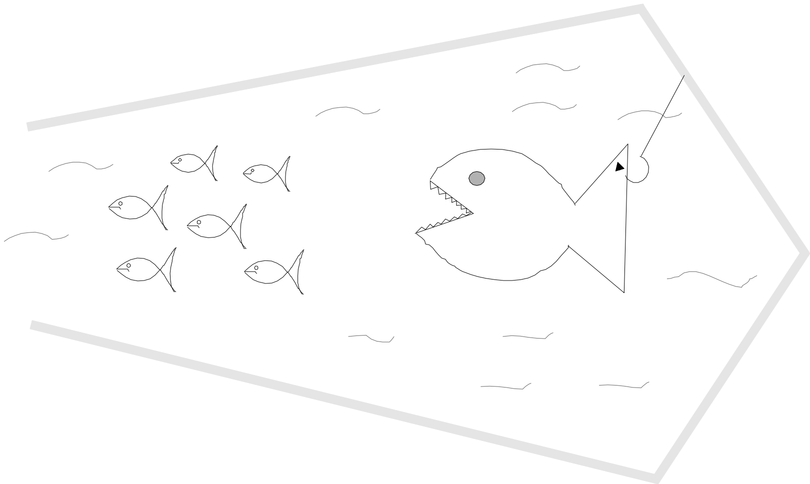
Simple Applications of Nonlinear systems

Systems Lotka-Volterra

competition in closed ecosystems

LITTLE and BIG fishes



X : Population of big fishes in the ecosystem
Y : Population of little fishes

- Big fishes eat little fishes
- Little fishes eat plankton (unlimited quantity in the eco system)

Construction of the model

- Step 1 : There is no interaction between species

Little fish are continuously reproduced proportionally to their number (there is plenty of Plankton)

$$\dot{Y} = aY, \quad a > 0$$

The number of big fishes decreases quickly (big fishes do not eat little fishes since we assume no interaction)

$$\dot{X} = -bX\ , \quad b > 0$$

- Step 2 : The species interact

The coefficient *a* of the reproduction of little fishes decreases as the number of big fishes increases

$$a \quad \to \quad a - r_1 X \qquad (r_1 > 0)$$

The coefficient *b* of the decreasing rate of big fishes decreases as the number of little fishes increases (there is sufficient food)

$$b \quad \to \quad b - r_2 Y \qquad (r_2 > 0)$$

- Equations of the system

$$\dot{X} = -bX + r_2 X Y$$
$$\dot{Y} = aY - r_1 X Y$$

- Reduction of the number of parameters by suitable choice of units (unit normalization)

1) We change the time scale as $t' = bt$

$$\dot{X} = -X + \frac{r_2}{b} X\,Y$$

$$\dot{Y} = \frac{a}{b} Y - \frac{r_1}{b} X\,Y$$

*derivatives with respect to *t'*

2) Since $(r_2/b)>0$ we normalize the number of fishes with the new variable

$x = \left(\frac{r_2}{b}\right) X,\qquad y = \left(\frac{r_2}{b}\right) Y$, *i.e. we set* $(X,Y) = \left(\frac{b}{r_2}\right)(x, y),$

Then the system of equations takes the form

$$\dot{x} = -x + x\,y$$

$$\dot{y} = \frac{a}{b} y - \frac{r_1}{r_2} x\,y$$

MODEL 1

$$\dot{x} = -x + xy$$

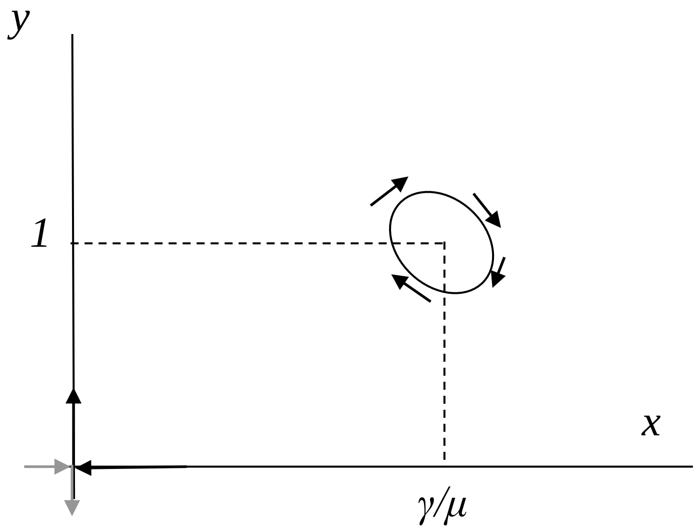
$$\dot{y} = \gamma y - \mu x y$$

where

$\gamma = \frac{a}{b} > 0$: coefficient of ratio «increase-decay» in the absence of species interaction

$\mu = \frac{r_1}{r_2} > 0$: coefficient of the ratio of coupling coefficients

Fishes_model1



MODEL 2

Model 1 + saturation

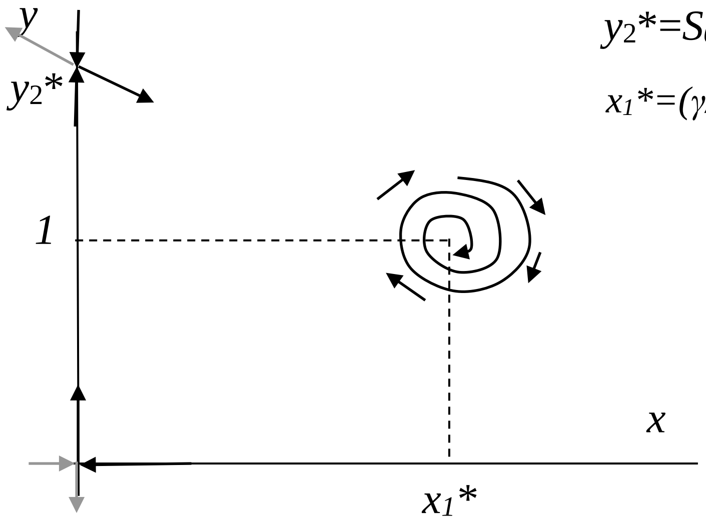
The increasing number y of little fishes can't exceed a maximum number S_0 (no plenty of food or limited capacity of the ecosystem)

$$\gamma \rightarrow \gamma s, \quad s = \frac{S_0 - y}{S_0}, \qquad \lim_{y \to 0} s = 1 \qquad \lim_{y \to S_0} s = 0$$

$S_0 > 1$: maximum number (normalized) of little fishes that can be supported by the ecosystem.

$$\dot{x} = -x + xy$$
$$\dot{y} = \gamma \frac{S_0 - y}{S_0} y - \mu x y$$

Fishes_model2



$y_2^* = S_0$

$x_1^* = (\gamma/\mu)(1 - 1/S_0)$