Wave Propagation Characteristics of Overhead Conductors Above Imperfect Stratified Earth for a Wide Frequency Range

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The influence of stratified earth on the wave propagation characteristics of overhead conductors is analyzed, using a generalized two-layer earth model of varying electromagnetic characteristics. A systematic comparison to simpler stratified earth models is presented, showing the differences in the propagation characteristics.

***Index Terms*—Earth return admittance, earth return impedance, multiconductor lines, nonhomogenous earth.**

I. INTRODUCTION

THE calculation of the transient responses for overhead power lines requires the detailed representation of the influence of the imperfect earth. For problems including the calculation of voltages and currents along the line or crosstalk phenomena, the Quasi-transverse electromagnetic (TEM) field propagation is a satisfactory approximation, neglecting the contribution of additional modes [1].

In the Quasi-TEM approach, the propagation constants are determined from the per-unit-length (pul) impedance and admittance parameters, which result from the solution of the electromagnetic field (EM) equations [2]. The EM problem can be solved rigorously, introducing accurate models, suitable to be applied for a wide frequency range. Such accurate models have been reported in the literature [3], but they are based on the assumption of the semi-infinite, homogeneous earth topology. In practice however, earth is composed of several layers of different electromagnetic properties.

Many efforts to develop models for the stratified earth structure are reported in the literature. They are mainly focused on the calculation of the earth return impedances and each of them uses different concepts and approximations. Nakagawa's multilayered earth model [4] can be assumed as the most generalized, since it allows different EM properties for each earth layer.

However, the accuracy of these models is limited to the low frequency range, especially for cases of high earth resistivity, since they all neglect the influence of the imperfect earth on the shunt admittances. Recently, Ametani [5] proposed formulas for the shunt admittances in the case of a two-layer earth.

The scope of this paper is to present generalized expressions for the series impedances and admittances of overhead conductors above a two-layer earth, following the quasi-TEM field propagation theory, described in [2]. The wave propagation characteristics of an overhead arrangement are analyzed, considering the frequency dependent behavior of the stratified earth. The proposed model is suitable for transmission line (TL) modeling over a wide frequency range, covering most practical power engineering problems.

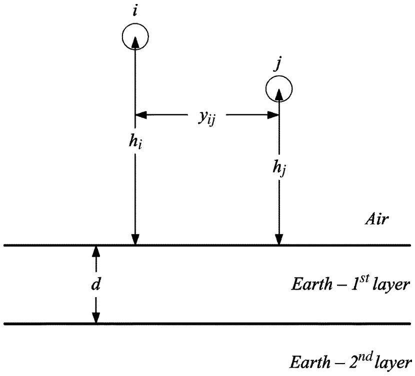

Fig. 1. Geometric configuration of two thin conductors above a two-layered earth.

II. PROBLEM FORMULATION

A. Self and Mutual Impedances and Admittances

The general overhead line configuration of Fig. 1 is considered. It consists of two uniform, electrically thin perfect conductors of infinite length, located in the air above a two-layer earth structure. The heights of the two conductors from the earth surface are h_i and h_j, respectively, and their horizontal distance is y_{ij}.

The first layer has a depth d, while the second layer extends to infinity. The permittivity and permeability of air are ε_0 and μ_0, respectively. The permittivity of the first layer is ε_1, the permeability and conductivity are μ_1 and σ_1, respectively, while the corresponding characteristics of the second layer are ε_2, μ_2, and σ_2.

Assuming a quasi-TEM field propagation, the EM field equations are solved using the Herzian Vector approach [2]. The influence of the proposed earth model on the series impedances and shunt admittances of the conductors is expressed through correction terms both for the self and mutual components.

These terms are calculated following the generalized methodology proposed by Sunde [2] and take the form of (1) and (2), respectively.

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Earth return impedance correction term

$$Z'_{g_{ij}} = \frac{j\omega\mu_0}{\pi}\int_0^\infty F(u)\cdot e^{-u(h_i+h_j)}\cos(y_{ij}u)\cdot du \quad \text{(1a)}$$

$$F(u) = \mu_1\frac{s_{12}+d_{12}e^{-2\alpha_1 d}}{s_{01}s_{12}+d_{01}d_{12}e^{-2\alpha_1 d}}. \quad \text{(1b)}$$

Earth return admittance correction term

$$Y'_{g_{ij}} = j\omega P_{g_{ij}}^{-1} \quad \text{(2a)}$$

$$P_{g_{ij}} = \frac{1}{\pi\varepsilon_0}\int_0^\infty \left[F(u)+G(u)\right]e^{-u(h_i+h_j)}\cos(y_{ij}u)du \quad \text{(2b)}$$

and 2(c) shown at the bottom of the page.

Where j is the imaginary unit, $\omega = 2\pi f$ is the angular velocity, and u is the integral argument.

The generalized functions $F(u)$ and $G(u)$, have the form shown in (1b) and (2c), respectively, while their terms are presented analytically in the Appendix. The corresponding earth correction terms for the pul self-impedance and the self-admittance of conductor i of Fig. 1 are derived from (1) and (2), respectively, by replacing y_{ij} with the conductor's outer radius and h_j with h_i.

The $G(u)$ function is due to the radial displacement currents in the earth. Ignoring $G(u)$, results in a purely imaginary propagation constant and a lossless propagation above imperfect earth [1].

The semi-infinite integrals of (1) and (2) are evaluated numerically, using the integration scheme of [6], which is a combination of different methods and it proved to be very efficient numerically for the type of the integrands involved.

B. Comparison With Other Stratified Earth Models

The previous equations have a generalized form, since the EM properties of each of the two earth layers can take arbitrary values. In this way, other stratified earth models may be derived by applying the corresponding assumptions in (1) and (2). Therefore, it follows.

Impedance Earth Correction Terms:

- For $\mu_i = \mu_0$, $\varepsilon_0 = 0$, $\varepsilon_i \neq 0$, (1) reduces to Sunde's two-layer earth formula [2].
- For $\mu_i \neq \mu_0$, $\varepsilon_i \neq \varepsilon_0$, the approach of [4] is expressed by (1), for the case of the two-layer earth.

A systematic comparison of the numerical results obtained by the above approximations is presented in [6].

Admittance Earth Correction Terms: Equation (2) is similar to the two-layered earth model for the calculation of the earth return admittance, proposed by Ametani [5].

C. Homogenous Earth Model

Equations (1) and (2) of the two-layer earth model are transformed to the corresponding expressions for the homogeneous earth case, simply by replacing a_2 with a_1 and γ_2 with γ_1. In this case, the proposed model reduces to the generalized model of Kikuchi [2].

III. REMARKS ON THE PROPOSED MODEL

The proposed model includes the influence of the displacement currents in all media and the influence of the imperfect earth on the pul admittances for a generalized two-layer earth model. These parameters are important for the accurate modeling of a TL and their significance is further analyzed.

A. Frequency Dependent Behavior of the Earth Layers

According to [7], the frequency dependent behavior of an earth layer can be classified, adopting the corresponding definition of the critical frequency f_{cr}

$$f_{cr} = \frac{\sigma_1}{2\pi\varepsilon_0\varepsilon_{r1}} \quad (\text{Hz}). \quad (3)$$

- If $f < 0.1\cdot f_{cr} = f_{cr,\min}$ the displacement currents are negligible and the earth layer behaves as a conductor.
- If $f_{cr,\min} < f < 2\cdot f_{cr}$ the displacement and resistive currents are comparable and the earth layer behaves both as a conductor and as an insulator.
- If $f > 2\cdot f_{cr}$ the displacement currents are predominant and the earth layer behaves as an insulator.

For frequencies above $f_{cr,\min}$ of an earth layer, the influence of the displacement currents on both impedances and admittances is significant and the generalized expressions of (1) and (2) must be used.

B. Earth Stratification

In the literature, significant differences to the homogenous earth case have been recorded for the earth return parameters [4], [5], due to the different electric characteristics of the earth layers, in the kilohertz frequency range.

At higher frequencies, the influence of earth stratification is more significant, due to the simultaneous effect of displacement currents in both earth layers. This is better explained, considering the field penetration depth [7], given by (4). As shown in Table I for the case where earth permittivity is assumed equal to 10, the penetration depth tends asymptotically to nonzero values even at high frequencies (HF), especially for cases of high earth resistivity. Therefore, the stratification of the earth must be taken into account

$$\delta = \left(\omega\sqrt{\varepsilon_1\mu_1/2\left(\sqrt{1+(\sigma_1^2/\omega^2\cdot\varepsilon_1^2)}-1\right)}\right)^{-1}. \quad (4)$$

$$G(u) = u\frac{\mu_0\mu_1\left(\gamma_0^2-\gamma_1^2\right)\left(s_{12}+d_{12}e^{-2\alpha_1 d}\right)\left(S_{12}+D_{12}e^{-2\alpha_1 d}\right)-4\mu_0\mu_1^2\mu_2\alpha_1^2\gamma_0^2\left(\gamma_2^2-\gamma_1^2\right)e^{-2\alpha_1 d}}{\Delta_2\cdot\Delta} \quad \text{(2c)}$$

TABLE I
PENETRATION DEPTH IN METERS

Frequencies	*Earth resistivities*		
	10 Ωm	500 Ωm	1000 Ωm
500 kHz	2.25	17.06	25.82
1 MHz	1.60	12.91	20.76
5 MHz	0.72	8.87	17.05
10MHz	0.51	8.52	16.86

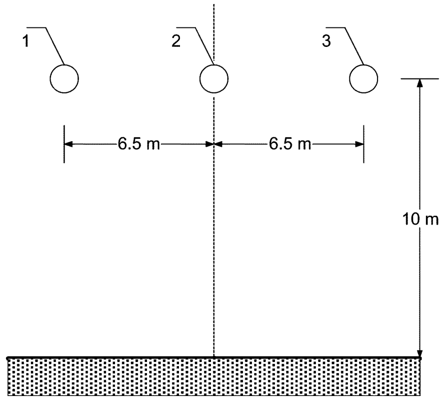

Fig. 2. 150 kV single-circuit 3-phase overhead transmission line configuration.

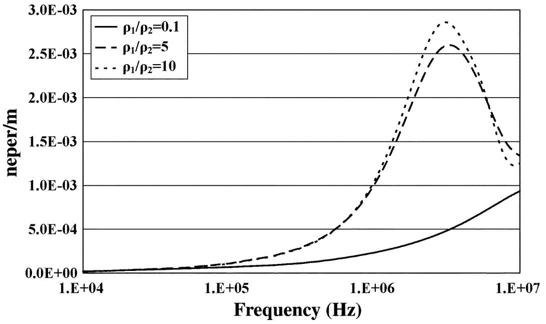

Fig. 3. Ground mode attenuation constant of the two-layer earth case for different ratios of earth resistivities.

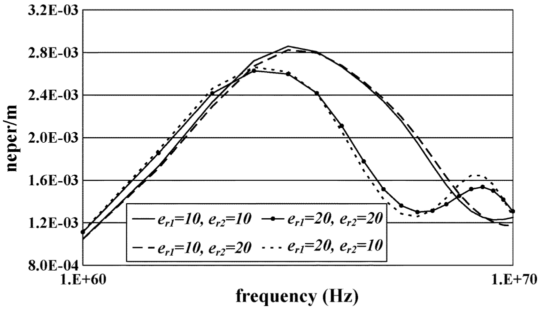

Fig. 4. Ground mode attenuation constant for the two-layer earth case for different earth permittivities.

IV. SYSTEM UNDER STUDY

A typical horizontal single-circuit 150 kV overhead TL with ACSR phase conductors and no ground wires is used in the analysis as shown in Fig. 2.

The examined two-layer earth topologies include cases where the ratio of the resistivities of the first to the second layer was $\rho_1/\rho_2 = 10, 5, 0.1, 0.2$, with $\rho_2 = 10, 100, 1000$ Ωm. Different relative permittivities for the two layers are assumed taking values from 1 to 20. The depth of the first layer is assumed to be variable, ranging between 5 to 20 m, while the second layer is of infinite extent. The relative permeability of both layers is considered to be equal to unity, since most soil types are nonmagnetic.

V. NUMERICAL RESULTS

A. *Influence of Earth Characteristics*

The wave propagation characteristics of the overhead line configuration of Fig. 2 are calculated using (1) and (2) and proper modal decomposition [8].

First, the ground mode attenuation is plotted in Fig. 3 for cases with different ratios of resistivities of the first to the second layer. The resistivity of the second layer is $\rho_2 = 100$ Ωm, the relative permittivities of both layers are equal to 10 and the depth of the first layer is $d = 5$ m.

The most interesting case is that, where the first to second layer earth resistivity ratio equals to 10. As shown in Table I, the penetration depth of the EM field is significantly higher than the depth of the first layer and so the influence of the earth stratification is maximized.

This specific two-layer earth topology has been selected for the examination of the different cases of earth permittivities.

Fig. 4 shows the ground mode attenuation constant against the frequency. The curves are increasing with frequency up to some megahertz, showing a maximum point. In the megahertz frequency region the penetration depth of the EM field is still higher than the depth of the first layer. In this case, the displacement currents in the second layer with a better conductivity become significant. Due to the simultaneous influence of the displacement currents in both layers, resonant oscillations occur, especially when the permittivity of the first layer is high. Similar remarks can be also concluded for the other modal attenuation constants and the magnitude of the ground mode characteristic impedance.

B. *Comparison With Other Two-Layer Earth Models*

Next, the modal propagation characteristics of the proposed model are compared to those obtained by other approaches, using the same stratified earth topology.

First, a simplified model is assumed, where both the influence of the permittivity of the earth and the influence of the admittance earth correction terms are neglected. This model is characterized as "*low frequency (LF) stratified Earth model.*"

The second model is a slight improvement to the latter in the HF range, since it takes into account the effect of earth permittivities only on the earth impedances. This model is equivalent to the two-layer earth of Nakagawa [4].

The ground mode attenuation constant, calculated by the three models is presented in Fig. 5. The corresponding curves of the LF Stratified model are monotonically increasing functions with frequency, due to the simplifications described previously. Significant differences are recorded for frequencies higher than the minimum critical frequency of the first layer. In the HF region, the ground mode characteristics by the two models show a completely different behavior.

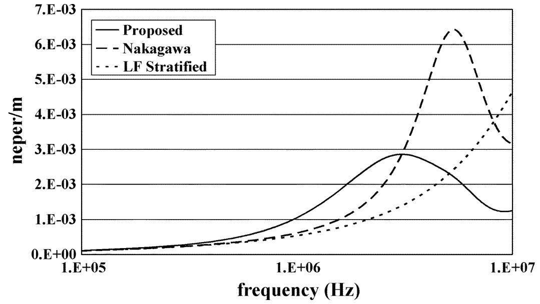

Fig. 5. Ground mode attenuation constant for different two-layer methods.

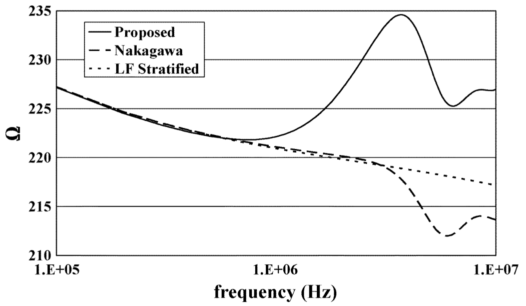

Fig. 6. Magnitude of ground mode characteristic impedance for different two-layer methods.

Although Nakagawa's model is an improvement to the LF model, the differences with the proposed model still remain significant, especially in the HF region. This is due to the increasing influence of the admittance earth correction terms. The ground mode attenuation curve presents almost the same behavior as the corresponding of the LF Stratified model up to several megahertz. However, in the upper frequency range a maximum peak point is recorded, due to the influence of earth permittivity on the series impedances.

The magnitude of the ground mode characteristic impedance is shown in Fig. 6. The different models show a significantly different behavior, leading to considerable differences in the results that cannot be ignored in HF applications.

VI. CONCLUSION

A new model for the calculation of the wave propagation characteristics for overhead power transmission lines over a two layer earth is presented. The rigorous solution of the EM field equations offers generalized expressions suitable for the calculation of both impedance and admittance earth correction terms.

The comparative analysis of the results by different approaches shows that the influence of the earth permittivity and the influence of the imperfect earth must be taken into account for both impedance and admittance correction terms. This is most evident in cases where the penetration depth of the EM field exceeds the depth of the first layer, especially in the frequency range where the earth layers behave both as a conductor and as an insulator.

The proposed theoretical model together with the numerical integration scheme can be used for any type of overhead line configuration, thus offering a useful tool in the calculation of transient response of arbitrary overhead power line configurations.

APPENDIX

The terms appearing in (1b) and (2c) are defined as

$$s_{mn} = (a_m\mu_n + a_n\mu_m) \tag{A.1}$$

$$d_{mn} = (a_m\mu_n - a_n\mu_m) \tag{A.2}$$

$$S_{mn} = \left(\mu_m\gamma_n^2 a_m + \mu_n\gamma_m^2 a_n\right) \tag{A.3}$$

$$D_{mn} = \left(\mu_m\gamma_n^2 a_m - \mu_n\gamma_m^2 a_n\right) \tag{A.4}$$

$$\Delta = s_{01}s_{12} + d_{01}d_{12}e^{-2a_1' d} \tag{A.5}$$

$$\Delta_2 = S_{01}S_{12} + D_{01}D_{12}e^{-2a_1 d} \tag{A.6}$$

where $a_m = \sqrt{u^2 + \gamma_m^2 - \gamma_0^2}$, $\gamma_m^2 = j\omega\mu_m(\sigma_m + j\omega\varepsilon_m)$. The m, n indices, take the values 0, 1, 2, corresponding to the air and the two earth layers, respectively.

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