A One-Step Finite Element Formulation for the Modeling of Single and Double-Circuit Transmission Lines

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***Abstract*—A method is presented, by which a Finite Element Method (FEM) formulation is used for the direct computation of overhead transmission line series and sequence impedances. The method is applied in single and double circuit line configurations of arbitrary geometry, giving results in perfect agreement with those available from classical calculation methods. The new method can easily handle cases of nonhomogenous and/or irregular terrain, where classical methods may fail.**

***Index Terms*—Finite element methods, impedance matrix, power transmission lines, symmetrical components.**

I. Introduction

OVERHEAD transmission lines are a vital link in power transmission systems. For the proper overhead transmission line modeling, in either steady state or transient calculations, the knowledge of the per unit length series impedances and shunt admittances of the line is necessary.

Transmission line modeling may be complicated, due to the asymmetrical, multiconductor physical arrangement of the conductors as well as to the lossy nature of the conductors and of the earth return path. This results in the frequency dependence of the distributed transmission line parameters.

For transient simulations, where detailed transmission line modeling is required, general methods for the computation of overhead transmission line parameters are available. The well-known Electromagnetic Transients Program (EMTP) [1] includes such a supporting tool, dedicated to the computation of overhead transmission line parameters. Line parameters are computed taking into account the geometric configuration of the line. Conductor impedances are calculated using skin effect formulas for solid or tubular conductors [2], [3] The influence of the imperfect earth is included through correction terms, calculated using Carson's formulas [4]. Multiple conductors in bundles are properly combined and ground wires are mathematically eliminated in order to obtain reduced matrices for equivalent phase conductors [5]. Positive and zero sequence parameters may be obtained by applying symmetrical component transformations [6].

A closed-form approximation, leading to much simpler formulas for the ground return impedance calculation has been recently proposed [7]. This approach gives results in excellent agreement with the corresponding results from Carson's formulas and may replace these formulas in the future [5].

A major drawback of the above approaches is that they are unable to handle cases where certain terrain irregularities or discontinuities of soil characteristics are present in the vicinity of the line.

The Finite Element Method (FEM) is a numerical method widely used for the solution of electromagnetic field problems, regardless of their geometric complexity. In this paper, the application of FEM in the direct calculation of overhead transmission line series impedances is presented. By the proposed method, electromagnetic field variables are properly linked with the transmission line equivalent circuit parameters. The method can handle cases of arbitrary terrain irregularities and nonhomogenous ground.

II. Problem Formulation

A system of N parallel conductors, carrying rms currents $I_i (i = 1, 2, \cdots, N)$ over an imperfect earth of resistivity ρ is considered. The nonuniform current distribution inside the conductors and in the ground influences the effective impedance of the conductors at a given frequency. The following matrix equation links voltages and currents in any conductor of the line,

$$\frac{\partial}{\partial z} \boldsymbol{V} = -\boldsymbol{Z}(\omega)\boldsymbol{I} \tag{1}$$

where $\boldsymbol{V}$ is the voltage vector with respect to a reference, $\boldsymbol{I}$ is the current vector, and z is the longitudinal direction along the transmission fine. The elements of matrix $\boldsymbol{Z}(\omega)$ are the frequency dependent per unit length series impedances, depending on the geometric configuration, skin and proximity effect, eddy currents and the influence of the imperfect earth. The calculation of each of the above contributions is complex. Closed form solutions may be obtained only for relatively simple cases.

The problem itself could be greatly simplified, assuming that the per unit length voltage drop $\overline{V}_i$, on every conductor is known for a specific current excitation. The mutual complex impedance $\overline{Z}_{ij}$ between conductor i and another conductor j carrying current $\overline{I}_j$, where all other conductors are forced to carry zero currents, is then given by:

$$\overline{Z}_{ij} = \frac{\overline{V}_i}{\overline{I}_j} \quad (i, j = 1, 2, \cdots, N). \tag{2}$$

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The self impedance of a conductor may also be calculated from (2), by setting $i = j$. In such a case, the following procedure may be used for the calculation of the transmission line impedance matrix $Z(\omega)$:

- A sinusoidal current excitation of arbitrary magnitude is applied sequentially to each conductor, while the remaining conductors are forced to carry zero currents. The corresponding voltages are recorded.
- Using (2), the jth column of $Z(\omega)$ may be calculated. This procedure is repeated N times, in order to calculate the N columns of $Z(\omega)$.

Therefore, the problem is reduced to that of calculating the actual per unit length voltage drops, when a current excitation is applied to the conductors. This may be achieved by a suitable FEM formulation of the electromagnetic diffusion equation.

III. Electromagnetic Field Equations and Circuit Parameters

In the previously described system of N conductors above lossy ground, conductors are assumed to be long enough to ignore end effects or possible discontinuities due to grounded towers. Furthermore, if the current density vector is supposed to be in the z direction, the problem becomes two-dimensional, confined in the x–y plane, in which the conductors cross sections lie. The linear electromagnetic diffusion equation is described by the following system of equations [8]

$$\frac{1}{\mu_0 \mu_r}\left[\frac{\partial^2 \overline{A}_z}{\partial x^2} + \frac{\partial^2 \overline{A}_z}{\partial y^2}\right] - j\omega\sigma\overline{A}_z + \overline{J}_{sz} = 0 \qquad (3)$$

$$-j\omega\sigma\overline{A}_z + \overline{J}_{sz} = \overline{J}_z \qquad (4)$$

$$\iint_{S_i} \overline{J}_z dS = \overline{I}_i, \qquad i = 1, 2, \cdots, N \qquad (5)$$

where $\overline{A}_z$ is the z direction component of the magnetic vector potential (MVP).

In (4) the total current density $\overline{J}_z$ is decomposed in two components [8],[9],

$$\overline{J}_z = \overline{J}_{ez} + \overline{J}_{sz} \qquad (6)$$

where $\overline{J}_{ez}$ is the eddy current density and $\overline{J}_{ez}$ the source current density, given by (7) and (8), respectively.

$$\overline{J}_{ez} = -j\omega\sigma\overline{A}_z \qquad (7)$$

$$\overline{J}_{sz} = -\sigma \nabla \Phi. \qquad (8)$$

FEM is applied for the solution of (3) and (4) with the boundary conditions of (5), by imposing an homogenous Dirichlet boundary condition for the MVP on the boundary. Values for $\overline{J}_{sz_i}$, on each conductor i of conductivity σ_i are then obtained and (2) takes the form [10]

$$\overline{Z}_{ij} = \frac{\overline{V}_i}{\overline{I}_j} = \frac{\overline{J}_{sz_i}/\sigma_i}{\overline{I}_j} \quad (i, j = 1, 2, \cdots, N) \qquad (9)$$

linking properly electromagnetic field variables and equivalent circuit parameters. Finally, positive, negative, and zero sequence impedances may be easily obtained, either by simple transformations from matrix Z [6], or even directly by applying in (5) sequentially unitary positive, negative, and zero sequence currents to the conductors.

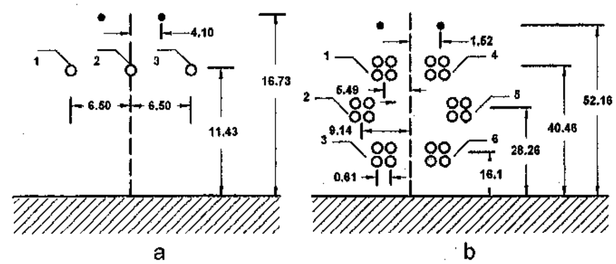

Fig. 1. Transmission line configurations: (a) 150 kV single circuit transmission line and (b) 735 kV double circuit transmission line.

IV. Finite Element Formulation

The electromagnetic field of an overhead transmission line may be considered unbounded. The FEM has been used to solve unbounded field problems using several approaches, such as the extension of the discretization area (direct solution), the use of integral equations (Green's function) [11], the window frame technique [12], the boundary element method [13], the infinitesimal scaling [14], the hybrid harmonic/finite element method [15]. For reasons explained in [16], the first method was adopted here. The discretization area is a square 10 km × 10 km, with the transmission line located in its center.

Following considerations apply for the FEM computation:

- The discretization area is subdivided in first order triangular finite elements.
- A Delaunay based [17] adaptive mesh generation algorithm has been developed.
- An iteratively adaptive mesh generation algorithm has been used, based on the continuity requirement for the magnetic field on the interface between neighboring elements.
- Bundled conductors are treated as a single conductor of arbitrary shape, by assigning the same material identity to all conductors in the bundle.
- ACSR conductors are treated as tubular conductors.
- Overhead ground wires are treated as individual conductors with no excitation current applied to them.
- Existing symmetries in the geometry of the problem are properly utilized to improve the computational efficiency of the method.

V. Numerical Results

The proposed method has been applied for the calculation of overhead transmission line impedance matrices. Two line configurations are examined, namely a single circuit 150 kV transmission line with two ground wires shown in Fig. 1(a) and a double circuit 735 kV line with a 4-conductor bundle per phase and two ground wires shown in Fig. 1(b). Data for the first line were obtained from Public Power Corporation of Greece and for the second line from [18], with all dimensions converted to

TABLE I
CONDUCTORS AND GROUND WIRES DATA

	150kV single circuit line (Fig. 1a)	735kV double circuit line (Fig. 1b)
Phase conductors.		
Conductor type	ACSR	ACSR
Outside diameter (mm)	25.146	35.103
Inside diameter (mm)	9.71	23,364
dc resistance (Ω/km)	0.09136	0.04965
Ground wires.		
Conductor type	solid St conductor	Alumoweld strand
Outside diameter (mm)	0.9525	0.9779
dc resistance (Ω/km)	3.4431	1.4913

SI units. All phase conductors and ground wires data appear in Table I.

First, the proposed method is checked against classical methods, as they are implemented in EMTP, for the calculation of overhead transmission line impedances. Both line configurations were considered to run over a flat semi-infinite homogenous earth with resistivity $\rho = 5, 50, 500$, and 5000 Ωm. Series impedances are calculated by FEM over a frequency range from 50 Hz up to 1 MHz. Results were compared with the corresponding from EMTP. The percent difference, defined by

$$\text{Difference}(\%) = \frac{|Z_{\text{EMTP}}| - |Z_{\text{FEM}}|}{|Z_{\text{EMTP}}|} \cdot 100 \qquad (15)$$

is shown in Fig. 2(a) and (b) for the magnitude of the zero sequence impedance of the single circuit line and of the left circuit of the double circuit fine respectively. In all above cases the calculated differences were less than 2%. The new method allows the direct calculation of the actual return current distribution between ground wires and earth, being together with its magnitude the two critical factors in transmission line modeling [19]. Fig. 3(a) and (b) show the percentage of the return current flowing in each of the symmetrical ground wires for the same frequency and earth resistivity range. As expected, ground wire currents are increased in cases of high earth resistivities, with all curves showing a maximum in the kHz frequency region. All currents tend to smaller and almost constant values in the MHz frequency region, as in these high frequencies eddy currents are restricted by the effect of their own field.

Table II presents the changes in % difference between EMTP and FEM results as a function of the number of finite elements. Data for phase conductors, ground wires, earth and surrounding air is shown for the transmission line of Fig. 1(a). Frequency is 50 Hz and earth resistivity is 5000 Ωm. The continuity requirement of the flux density on the interface between neighboring elements has been used as the criterion for the iterations. The final FEM iteration for this case led to a discretization mesh consisting of 21 134 first order triangular elements and 10 597 nodes.

Next the method is used in the case of a nonhomogenous and irregular terrain. More specifically, the configuration of Fig. 4 is

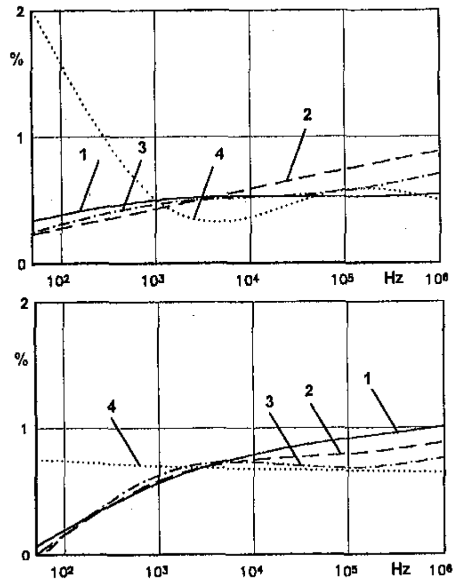

Fig. 2. EMTP–FEM differences (in %) for zero sequence impedance calculation versus frequency. Curves 1–4 correspond to earth resistivities 5, 50, 500, and 5000 Ωm, respectively: (a) single circuit line and (b) double circuit line.

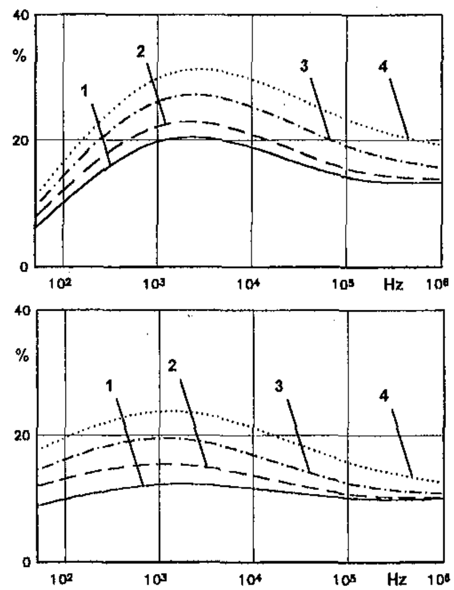

Fig. 3. Percentage of the return current in each ground wire versus frequency. Curves 1–4 correspond to earth resistivities 5, 50, 500 and 5000 Ωm respectively: (a) Single circuit line and (b) Double circuit line.

considered. The single circuit line of Fig. 1(a) is assumed to run parallel to a water region of variable length, consisting of 4 individual segments, each having a length of 25 m and a constant depth of 5 m. Specific resistivity is $\rho_1 = 0.25$ Ωm for all segments. The water region is surrounded by two 100 m long and 5 m deep segments having an earth resistivity of $\rho_2 = 50$ Ωm. The remaining semi-infinite ground to the limits of the discretization area is assumed to have $\rho_3 = 500$ Ωm. In cases A–D, the length of the water region is gradually reduced, by replacing

TABLE II
EMTP–FEM DIFFERENCE (%) AS A FUNCTION OF FINITE ELEMENTS PER ITERATION

Iteration	Phase 1	Phase 2	Phase 3	Left GW	Right GW	Earth	Air	Total elem.	Total Nodes	Differ. %
1	67	67	67	36	36	62	1306	1641	827	13.19
2	67	67	67	36	36	194	2177	2644	1332	6.32
3	67	67	67	36	40	520	4071	4868	2451	3.43
4	79	71	73	71	78	1354	8173	9899	4975	2.36
5	121	134	140	141	134	3135	17329	21134	10597	2.00

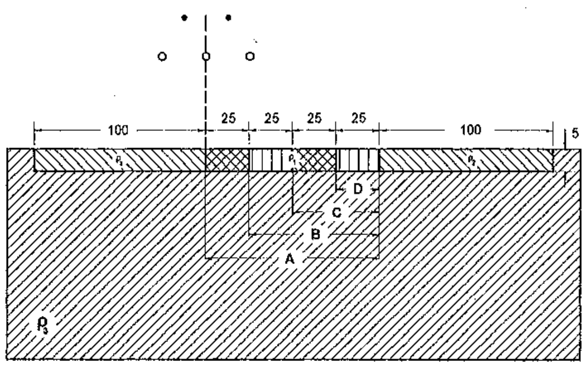

Fig. 4. Transmission line parallel to a variable length and constant depth water region.

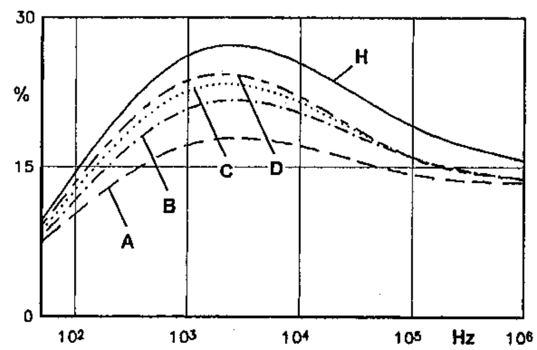

Fig. 6. Percentage of the return current in each ground wire for a transmission line parallel to a variable length and constant depth water region. Curves A–D correspond to the water segments shown in Fig. 5. Curve H is for homogenous semi-infinite earth.

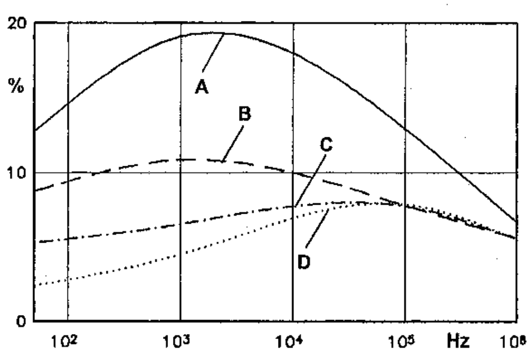

Fig. 5. Divergence (%) in transmission line zero sequence impedance from homogenous semi-infinite earth solution. Curves A–D correspond to the water segments shown in Fig. 4.

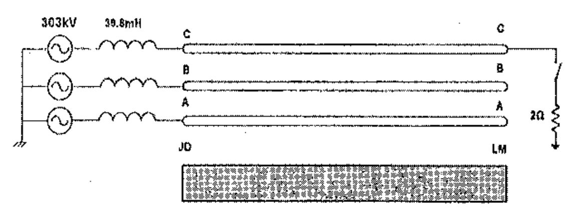

Fig. 7. Test configuration for transient simulation.

each 25 m water segment with a soil segment of equal dimensions, having an earth resistivity of $\rho_2 = 50$ Ωm. Such a case cannot be handled by methods based on Carson's correction terms. Transmission line series impedance matrices are computed, using the proposed method, for all cases A–D. Results are compared to the corresponding results obtained by FEM for the case of homogenous semi-infinite earth, having a resistivity $\rho = 500$ Ωm and for a frequency range from 50 Hz to 1 MHz. Fig. 5 shows the percent divergence of the magnitude of the line zero sequence impedance as a function of frequency for the various water segment lengths, reaching almost 20% in the case of full water length. In Fig. 6 the fraction of the return current on each ground wire of the transmission line is shown for the above cases A–D. Curve H corresponds to the case of semi-infinite homogenous earth having $\rho = 500$ Ωm. Results show that the presence of the water region may lead, for certain frequencies, to a considerable reduction in ground wire current depending on line configuration and ground resistivities.

The results obtained by FEM were used in a transient simulation, in order to demonstrate the effect of ignoring soil irregularities. The BPA field test configuration, described in [20], was adapted as shown in Fig. 7. The transmission line arrangement of Fig. 4 was considered with a total length of 10 km. A single line to ground fault through a 2 Ω resistance is applied on the open-ended phase c at $t = 10.15$ ms.

A time domain transmission line model was used with distributed parameters obtained by EMTP, considering semi-infinite homogenous ground with $\rho = 50$ Ωm and by FEM for the actual line arrangement. In both cases line parameters were calculated at 5 kHz. The frequency dependent line model by J. Marti [21] with $\rho = 50$ Ωm was also used as reference. Integration step was 50 μs in all cases. Fig. 8 presents the results of the simulation for the phase b open end voltage. Peak voltage

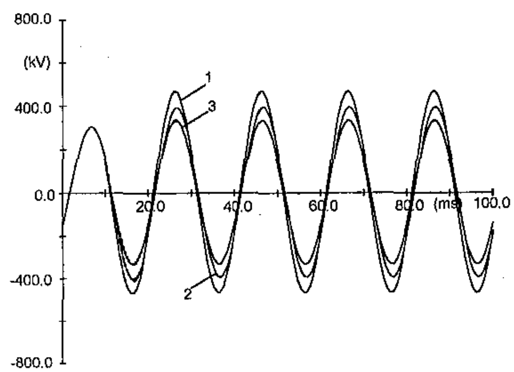

Fig. 8. Phase b open end voltage. Curves 1 and 2: Time domain transmission line model using parameters obtained by EMPT and FEM, respectively. Curve 3: J. Marti frequency domain line model.

TABLE III
TOTAL EXECUTION TIMES OF FEM CALCULATIONS AND MAXIMUM MEMORY ALLOCATED BY THE FEM SOLVER, FOR THREE DIFFERENT FREQUENCIES

Frequency	Execution time [min:sec]	Memory [Mbytes]
50 Hz	48:55	43
5000 Hz	31:19	30
1 MHz	25:54	16

values, corresponding to EMTP and FEM parameter calculations, differ 39.4 and 14.2% from the J. Marti model results, respectively.

Finally, Table III shows the computational cost of the developed FEM program. Total execution times as well as memory required are reported for three different cases of the problem shown in Fig. 7, for line parameters calculation at 50 Hz, 5 kHz and 1 MHz respectively. Computations have been made on a Pentium 166 MHz PC with 96 Mb RAM.

VI. CONCLUSIONS

A new method, allowing the computation of the series impedance matrix and of the sequence impedances of an overhead transmission line directly from the Finite Element Method output, is presented. The method can handle physical overhead transmission line arrangements of arbitrary geometry, with ACSR or solid conductors, single or bundled, as well as ground wires.

Results obtained by the proposed method for single and double circuit lines were checked against those obtained by classical computation methods and showed excellent agreement over a wide frequency and earth-resistivity range.

The strong advantage of the new method is its ability to handle cases of terrain surface irregularities and/or nonhomogenous, multi-layered, stratified soil, where classical methods may fail. Such an example is examined in this contribution, with results showing divergences of up to 20% in the magnitude of the zero sequence impedance of the line compared to the case of homogenous semi-infinite earth.

Implementation of the results in a short circuit simulation showed a considerable improvement in transient responses compared to corresponding results from frequency domain models. Furthermore, the method may be used to estimate the actual current distribution in phase conductors, ground wires and in the earth for any arbitrary excitation of the transmission line.

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