Calculation of Overhead Transmission Line Impedances A Finite Element Approach

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Abstract: In this paper, the finite element method (FEM) is used to calculate the frequency dependent series impedance matrix of an overhead transmission line. A novel approach is proposed, leading from FEM results to the direct computation of the symmetrical components impedance matrix of any single or double circuit transmission line. Results show excellent agreement with those obtained by classical computation methods. Test cases examined include impedance calculations in the presence of certain terrain irregularities in the line neighborhood, such as line by a mountain side of variable slope, line inside a canyon or line near a water region.

Keywords: power transmission lines, impedance matrix, symmetrical components, finite element methods.

I. Introduction

The parameters required to describe an overhead power transmission line in power system transient analysis are series impedances and shunt admittances per unit length.

The present paper deals with the calculation of the frequency dependent series impedances of a transmission line. Originally, these calculations were based exclusively on the geometry. Later, skin and proximity effect were taken into account [1], [2], [3]. In 1926 Carson [4] first proposed a method of calculating the influence of imperfect earth on transmission lines. These formulae have been used for decades, before new approaches for these calculations were made available to the scientific community [5], [6], [7], [8], [9].

An implementation of skin effect and Carson formulae can be found in Electromagnetic Transient Program (EMTP) [10], [11], as a supporting tool dedicated to the calculation of line parameters. Line parameters calculated by EMTP have been used in this paper as a reference.

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Although a variety of methods is available today, none of them is able to calculate line parameters, when certain terrain irregularities are present in the vicinity of the line.

The Finite Element Method (FEM) is a numerical method, which may be used to solve the electromagnetic field equations in a region, regardless of geometric complexity. In this paper, a methodology using FEM for the calculation of transmission line series impedances is proposed. By the new method, electromagnetic field variables are linked to the symmetrical components impedance matrix of a power transmission line.

II. Transmission Line Modelling

A transmission line is described by the two matrix equations (1) and (2), linking the voltages and currents of the line,

$$\frac{\partial}{\partial z}\mathbf{V} = -\mathbf{Z}(\omega)\mathbf{I} \tag{1}$$

$$\frac{\partial}{\partial z}\mathbf{I} = -\mathbf{Y}(\omega)\mathbf{V} \tag{2}$$

where $\mathbf{V}$ is the voltage vector with respect to a reference conductor, $\mathbf{I}$ is the current vector and z is the longitudinal direction along the transmission line. Matrices $\mathbf{Z}(\omega)$ and $\mathbf{Y}(\omega)$ are the frequency dependent series impedance and shunt admittance matrix per unit length, respectively.

The proposed method deals with the calculation of $\mathbf{Z}(\omega)$ of an overhead transmission line using the Finite Element Method. $\mathbf{Z}(\omega)$ consists of four components,

$$\mathbf{Z}(\omega) = \mathbf{Z}_{geom} + \mathbf{Z}_{skin}(\omega) + \mathbf{Z}_{prox}(\omega) + \mathbf{Z}_{earth}(\omega) \tag{3}$$

where $\mathbf{Z}_{geom}$ depends on the geometric configuration of the transmission line, $\mathbf{Z}_{skin}(\omega)$ and $\mathbf{Z}_{prox}(\omega)$ express skin and proximity effect respectively and $\mathbf{Z}_{earth}(\omega)$ accounts for the influence of imperfect earth.

Carson [4] proposed an infinite series approach in order to calculate correction terms for the per unit length resistance and reactance of a transmission line, due to the existence of lossy ground. Different series apply, depending on the value of parameter k,

$$k = 4\pi\sqrt{5}\cdot 10^{-4}\cdot D\cdot\sqrt{\frac{f}{\rho}} \tag{4}$$

where D depends on the geometrical configuration of the line, f is the excitation frequency and ρ is the resistivity of the ground, which is considered semi-infinite and homogeneous.

III. Field Equations And Equivalent Circuits

A system of N infinitely long conductors, carrying currents I_i $(i=1,2,\ldots,N)$ over imperfect earth is considered. If the conductor cross sections lie on the x-y plane, the linear two-dimensional electromagnetic diffusion problem for the magnetic vector potential (MVP) A_z and the total current density vector J_z in the longitudinal direction z is described by the system of equations (5), (6) and (7) [12]

$$\frac{1}{\mu_0\mu_r}\left[\frac{\partial^2 A_z}{\partial x^2}+\frac{\partial^2 A_z}{\partial y^2}\right]-j\omega\sigma A_z + J_{sz} = 0 \tag{5}$$

$$-j\omega\sigma A_z + J_{sz} = J_z \tag{6}$$

$$\iint_{S_i} J_z dS = I_i\ ,\quad i = 1,\ 2,\ \ldots,\ N. \tag{7}$$

The total current density J_z can be decomposed in two components

$$J_z = J_{ez} + J_{sz} \tag{8}$$

as shown in [13]. In (8) J_{ez} is the eddy current density given by (9) and J_{sz} is the source current density, which is given by (10).

$$J_{ez} = -j\omega\sigma A_z \tag{9}$$

$$J_{sz} = -\sigma\nabla\phi \tag{10}$$

Thus, considering a system of N conductors of arbitrary shape over lossy ground, the mutual complex impedance between a conductor i of conductivity σ_i and another conductor j carrying a current I_j is given by [15]

$$Z_{ij} = \frac{V_i}{I_j} = \frac{J_{s_i}/\sigma_i}{I_j}\quad (i,j = 1,\ 2,\ \ldots,\ N). \tag{11}$$

When $i=j$, the self impedance of a conductor is calculated by (11). The impedance matrix $\mathbf{Z}$, representing the equivalent circuit of the transmission line described in (1), may be calculated as follows [16]:

- A current is applied sequentially to each conductor, while the remaining conductors are forced to carry zero currents.
- Using (11), the jth column of $\mathbf{Z}$ can be calculated.

This procedure has to be repeated N times in order to calculate the N columns of $\mathbf{Z}$.

A common practice in power engineering analysis is the use of symmetrical components. However, mainly because of the uncertainty as to the actual current distribution, the zero-sequence impedance of transmission lines is one of the most approximate parameters in system studies. In this aspect, a novel FEM approach is proposed, allowing the direct computation of the symmetrical components impedance matrix of any single or double circuit power transmission line, taking into account a more realistic distribution of the return current between overhead ground wires and earth. This is accomplished by applying a positive, a negative and a zero sequence system of currents successively to the line, leading to (12), as shown in Appendix A.

$$\mathbf{Z}_{012} = \mathbf{A}^{-1}\cdot\frac{1}{|I|}\mathbf{V}_{012\text{FEM}} \tag{12}$$

In the above equation $\mathbf{Z_{012}}$ is the symmetrical components impedance matrix of the line, $\mathbf{V_{012FEM}}$ is the matrix containing the voltages across line conductors, as calculated by FEM and $\mathbf{A}$ is the symmetrical components transformation matrix [17].

IV. Finite Element Formulation

The electromagnetic field associated with an overhead transmission line may be considered unbounded. The FEM has been used to solve unbounded field problems using several approaches, such as the extension of the discretization area (direct solution), the use of integral equations (Green's function) [18], the "window frame technique" [19], the boundary element method [20], the "infinitesimal scaling" [21] as well as the newer "hybrid harmonic/finite element method" [22]. For the same reasons explained in [23], the first method was adopted here. The discretization area was a square 10 km x 10 km, with the transmission line located in its center. An homogeneous Dirichlet boundary condition for the MVP is imposed on the perimeter of this square.

The proposed method was used for the FEM computation of overhead transmission line impedances under the following considerations:

- The discretization area is subdivided in first order triangular finite elements.
- A Delaunay based [24] adaptive mesh generation algorithm has been developed for the original discretization.
- An iteratively adaptive mesh generation algorithm [25] has been used, based on the continuity requirement for the magnetic field on the interface between neighboring elements.
- Bundled conductors are treated as a single conductor of arbitrary shape, by assigning the same material identity to all conductors in the bundle.
- ACSR conductors are treated as tubular conductors.
- Overhead ground wires are assumed to be segmented, in order to eliminate the losses associated with circulating

currents magnetically induced to them [10], [11]. Therefore, these wires are treated as individual conductors with no current applied to them. This results in a zero voltage drop per unit length, which is the case for all conductors with no current applied, in two-dimensional problems.

- Existing symmetries in the geometry of the problem are properly utilized to improve the computational efficiency of the method.

V. Numerical Results

Two line configurations have been investigated, namely a single circuit medium voltage distribution line (Fig. 1a) and a double circuit high voltage transmission line (Fig. 1b), taken from [14] with all dimensions converted to SI units. For the double circuit line the following test cases have been considered:

1. Single solid conductor per phase, no ground wires.
2. Single solid conductor per phase, two ground wires.
3. Four solid conductor bundle per phase, two ground wires.
4. Single tubular conductor per phase, two ground wires.
5. Single tubular conductor per phase, two ground wires, line positioned next to mountain or in a canyon.
6. Single tubular conductor per phase, two ground wires, line positioned parallel to a water region.

(a)

(b)

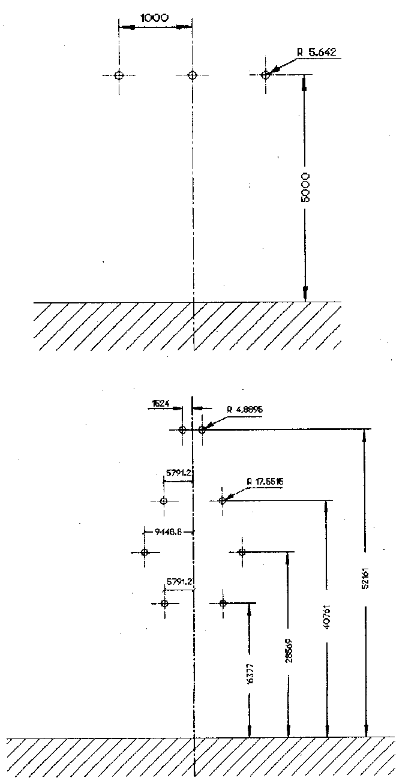

Figure 1: Single (a) and double (b) circuit transmission line.

The double circuit line of test case #4 was examined over homogeneous earth with resistivities ρ=1, 10, 100, 1000, 10000 Ωm respectively and over a frequency range from 50 Hz to 1 MHz. Figures 2 and 3 show the % difference defined in (13), between FEM and EMTP results, concerning the magnitude of the zero and positive sequence impedances Z_{00} and Z_{11} respectively, for the left circuit of the line.

In Table 1 a comparison between FEM and EMTP results is shown, using the % difference defined in (13), as a function of the number of finite elements. The details given correspond to a case shown already in Fig.2, i.e. to the % difference concerning the zero sequence impedance Z_{00} of the left circuit of the line, for a frequency equal to 5000 Hz and earth resistivity equal to 100 Ωm. The EMTP result for this case is Z_{00}=6.001+j74.865 Ω, leading to a magnitude equal to 75.105 Ω.

$$\text{Difference}(\%) = \frac{|Z_{EMTP}|-|Z_{FEM}|}{|Z_{EMTP}|}\cdot 100 \qquad (13)$$

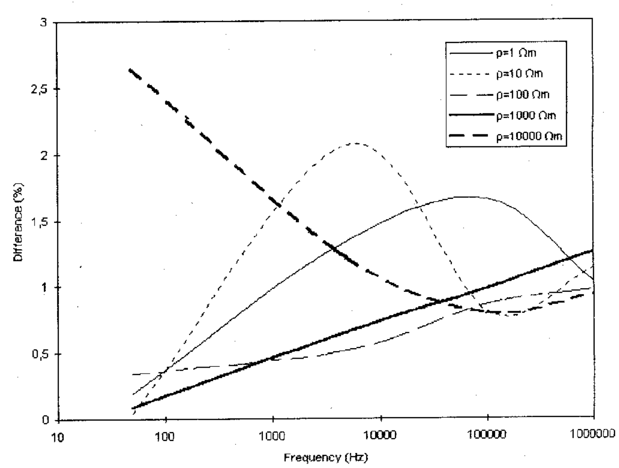

Figure 2: EMTP-FEM differences for $|Z_{00}|$.

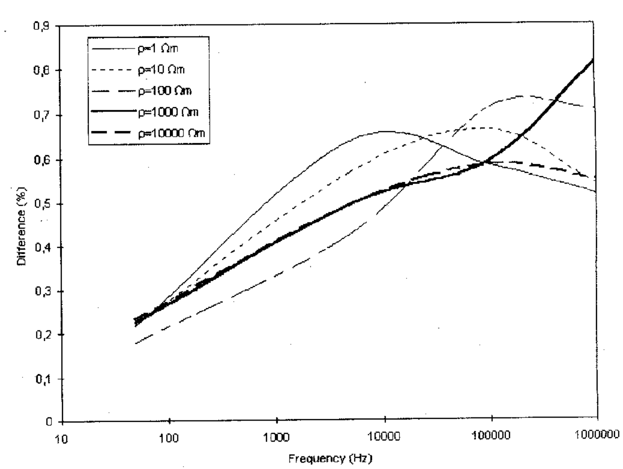

Figure 3: EMTP-FEM differences for $|Z_{11}|$.

TABLE 1

Comparison between FEM and EMTP results, as a function of the number of finite elements. Data are shown for the phase conductors of the left circuit of test case #4, as well as for the two ground wires and the earth. The continuity requirement of the magnetic field on the interface between neighboring elements has been used as the criterion for the iterations. Frequency is 5 kHz and earth resistivity 100 Ωm. The EMTP result for this case is Z_{00}=6.001+j74.865 Ω. The fourth FEM iteration led to a final discretization mesh (including all regions) consisting of 26623 first order triangular elements and 13347 nodes.

	Number of elements								
Iteration	Phase 1 conductor	Phase 2 conductor	Phase 3 conductor	Left ground wire	Right ground wire	Earth	Z_{00} [Ω]	$\lvert Z_{00}\rvert$ [Ω]	difference %
1	108	108	108	108	108	2376	6.225+j68.449	68.731	8.486
2	176	176	176	109	109	2557	6.002+j72.986	73.232	2.493
3	326	328	328	124	118	3046	6.006+j73.956	74.199	1.205
4	670	674	674	258	248	4266	6.015+j74.474	74.717	0.516

Additionally, the limiting case of a single conductor transmission line with earth return (Fig. 4) has been examined, for various combinations of earth resistivities and excitation frequencies. In Fig. 5 the corresponding FEM-EMTP differences for the line impedance are shown, for a wide range of values of Carson's parameter *k*.

The results of all above investigations, i.e. of the single circuit line of Fig. 1a, of test cases #1, #2, #3, #4 of the double circuit line of Fig. 1b, as well as of the single conductor line of Fig. 4, ensure that FEM and EMTP calculations show insignificant differences for similar test cases.

Next, the remaining test cases #5 and #6 were considered, in order to examine the influence of certain terrain irregularities on transmission line impedances. These cases can not be handled by the method implemented in EMTP, which is based on Carson's correction terms.

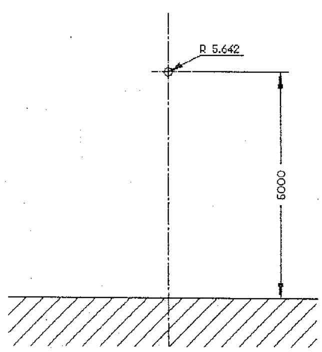

Figure 4: Single conductor line with earth return.

First the line is assumed to run parallel to the right of a mountain side of variable slope, as shown in Fig. 6a. FEM results for elements of the symmetrical components impedance matrix are compared to FEM results corresponding to semi-infinite earth. The % divergence, as a function of the mountain side slope, is shown in Fig. 7. Fig. 8 shows the corresponding results for the special case of a line inside a canyon, as in Fig. 6b. In both test cases earth resistivity was ρ=100 Ωm and excitation frequency f=50 Hz.

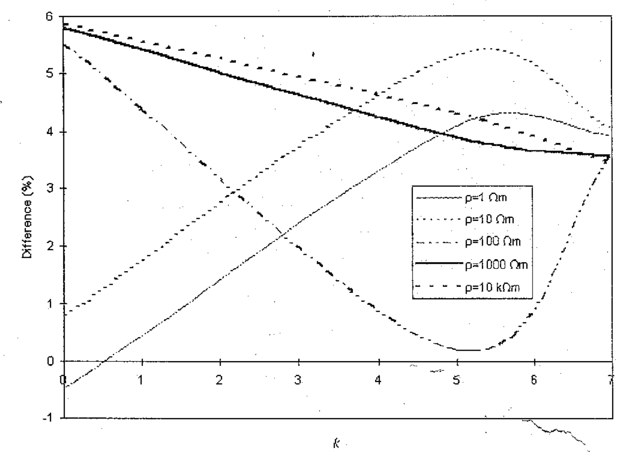

Figure 5: EMTP-FEM differences for various values of parameter *k*.

Finally, the same line is examined, parallel to a water region of variable depth. The first water region, starting at a distance of 250 m from the tower axis, was considered to have a depth of 150 m and a length equal to 750 m, while the second region has a depth equal to 1000 m and an infinite length. Fig. 9 presents the equipotential lines (A=const) when the above transmission line is energized by a zero sequence system of currents, in cases of homogeneous ground and variable depth water. Earth and water resistivities were taken ρ=100 and ρ=0.25 Ωm respectively. The excitation frequency was f=50 Hz. Results for the elements of the symmetrical components matrix for the above case are compared to those corresponding to semi-infinite earth. The % divergence for certain elements is shown in Fig. 10.

Both test cases revealed that terrain irregularities have negligible influence on positive, while they may affect up to 11% the zero sequence impedances.

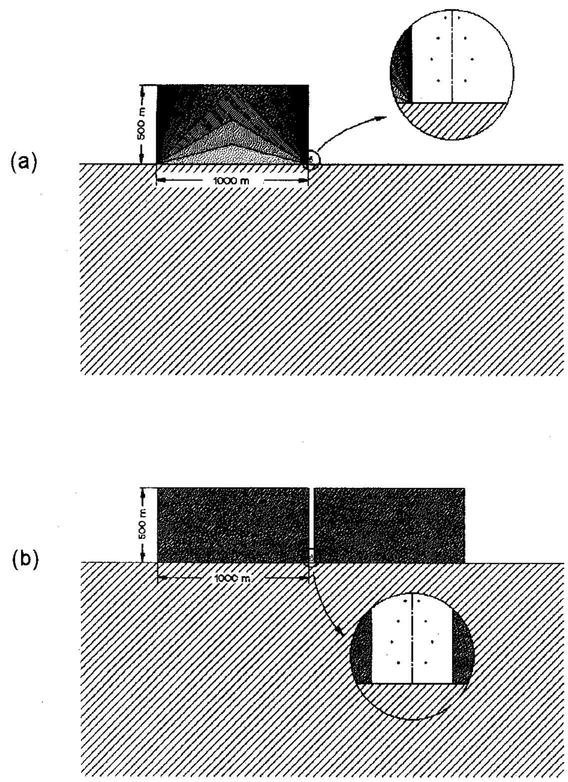

Figure 6: Transmission line located (a) parallel to mountain side and (b) inside a canyon.

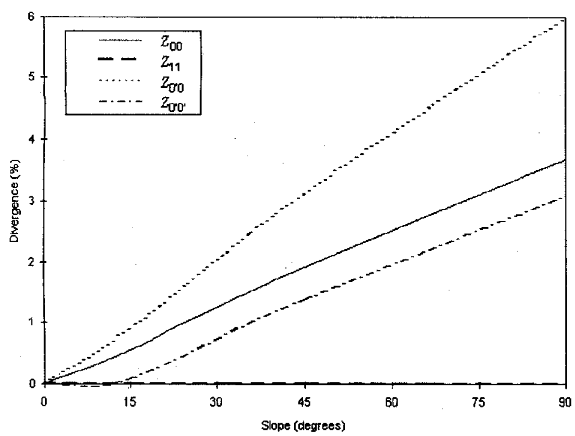

Figure 7: Matrix element divergence from semi-infinite earth solution vs mountain slope as in Fig. 6a.

VI. Conclusions

The scope of this paper is to present a new technique by which the output of the Finite Element Method (FEM) may be used for the direct calculation of the symmetrical components impedance matrix of overhead transmission lines.

The proposed method was applied in cases of single and double circuit lines, consisting of single or bundled conduc-

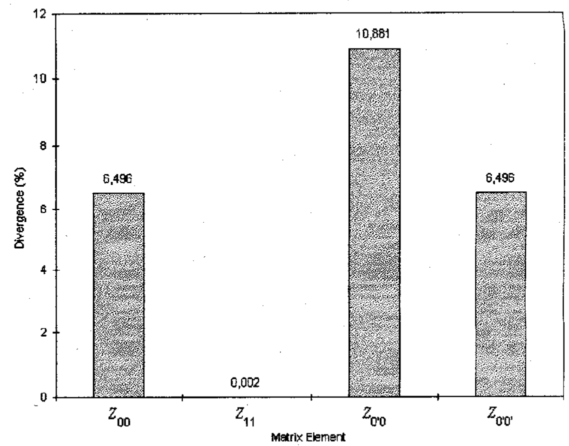

Figure 8: Matrix element divergences for of a transmission line in a canyon.

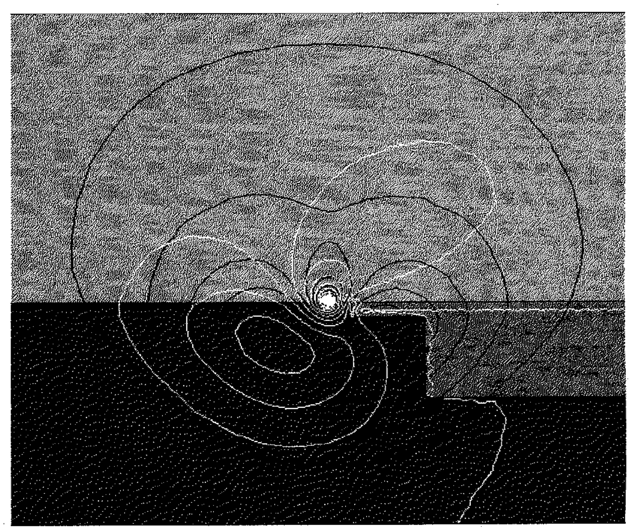

Figure 9: Equipotentials of transmission line. Black lines correspond to homogeneous earth and white lines to the case of neighbouring water, respectively.

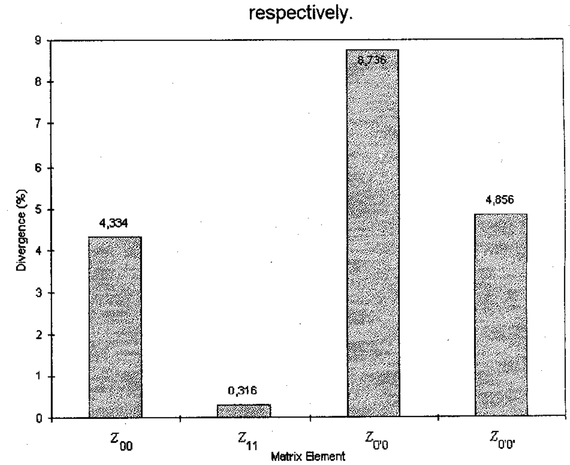

Figure 10: Matrix element divergences for of a transmission line next to sea.

tors, either solid or ACSR, with or without ground wires. Results show excellent agreement with those obtained by classical computation methods over a wide frequency range and for varying earth resistivities.

Furthermore, the new method is able of handling successfully cases of terrain irregularities, where classical methods can not be applied, taking into account a more realistic current distribution for the return current. The authors finally believe that an analysis showing the variations of the line parameters with terrain irregularities in the direction of the line, i.e. a three-dimensional analysis, must be a next and necessary step in this area.

VII. Appendix

Considering a three phase *a*, *b*, *c* transmission line, if V_{b1} is the complex voltage across the conductor of phase *b*, when a positive sequence of currents is applied to the line, then

$$V_{b1} = Z_{ba} \cdot I_{a1} + Z_{bb} \cdot I_{b1} + Z_{bc} \cdot I_{c1} \qquad (A.1)$$

In the above equation subscript letters indicate the referring phase, while subscript numbers indicate the type of current system applied (1 for positive, 2 for negative and 0 for zero sequence system of currents).

If all currents are of equal magnitude $|I|$, then dividing (A.1) with $|I|$ yields:

$$\frac{V_{b1}}{|I|} = Z_{ba} \cdot \frac{I_{a1}}{|I|} + Z_{bb} \cdot \frac{I_{b1}}{|I|} + Z_{bc} \cdot \frac{I_{c1}}{|I|} \qquad (A.2)$$

$$\frac{V_{b1}}{|I|} = Z_{ba} \cdot 1 + Z_{bb} \cdot 1 < -120^0 + Z_{bc} \cdot 1 < -240^0 \qquad (A.3)$$

$$\frac{V_{b1}}{|I|} = Z_{ba} + Z_{bb} \cdot a^2 + Z_{bc} \cdot a \qquad (A.4)$$

where $a=1<120^0$.

Similar equations may be derived for all phase voltages by applying the positive, negative and zero sequence currents. The matrix equation linking the voltages resulting from FEM and the impedance matrix of the line may be written

$$\begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} = \frac{1}{|I|} \begin{bmatrix} V_{a0} & V_{a1} & V_{a2} \\ V_{b0} & V_{b1} & V_{b2} \\ V_{c0} & V_{c1} & V_{c2} \end{bmatrix} \qquad (A.5)$$

or

$$\mathbf{Z} \cdot \mathbf{A} = \frac{1}{|I|} \mathbf{V}_{012FEM} \qquad (A.6)$$

where $\mathbf{A}$ is the symmetrical components transformation matrix [17] and $\mathbf{V}_{012FEM}$ is the matrix containing the voltages across line conductors, as calculated by FEM.

Applying the inverse transformation to equation (A.6) yields

$$\mathbf{Z}_{012} = \mathbf{A}^{-1} \cdot \frac{1}{|I|} \mathbf{V}_{012FEM} \qquad (A.7)$$

where $\mathbf{Z}_{012}$ is the symmetrical components impedance matrix of the line.

In the case of a double circuit line consisting of phases *a*, *b*, *c* and *a'*, *b'*, *c'* similar equations apply

$$\begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} & Z_{15} & Z_{16} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} & Z_{25} & Z_{26} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} & Z_{35} & Z_{36} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} & Z_{45} & Z_{46} \\ Z_{51} & Z_{52} & Z_{53} & Z_{54} & Z_{55} & Z_{56} \\ Z_{61} & Z_{62} & Z_{63} & Z_{64} & Z_{65} & Z_{66} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & a^2 & a & 0 & 0 & 0 \\ 1 & a & a^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & a^2 & a \\ 0 & 0 & 0 & 1 & a & a^2 \end{bmatrix} =$$

$$= \frac{1}{|I|} \begin{bmatrix} V_{a0} & V_{a1} & V_{a2} & V_{a0'} & V_{a1'} & V_{a2'} \\ V_{b0} & V_{b1} & V_{b2} & V_{b0'} & V_{b1'} & V_{b2'} \\ V_{c0} & V_{c1} & V_{c2} & V_{c0'} & V_{c1'} & V_{c2'} \\ V_{a'0} & V_{a'1} & V_{a'2} & V_{a'0'} & V_{a'1'} & V_{a'2'} \\ V_{b'0} & V_{b'1} & V_{b'2} & V_{b'0'} & V_{b'1'} & V_{b'2'} \\ V_{c'0} & V_{c'1} & V_{c'2} & V_{c'0'} & V_{c'1'} & V_{c'2'} \end{bmatrix} \qquad (A.8)$$

$$\mathbf{Z} \cdot \mathbf{A}_2 = \frac{1}{|I|} \mathbf{V}_{012FEM} \qquad (A.9)$$

or

$$\mathbf{Z}_{012} = \mathbf{A}_2^{-1} \cdot \frac{1}{|I|} \mathbf{V}_{012FEM} \qquad (A.10)$$

In this case, $\mathbf{A}_2$ is the extension of $\mathbf{A}$ for a double circuit line. Matrix $\mathbf{V}_{012FEM}$ consists of FEM results, which are obtained by applying sequentially a positive, zero and negative system of currents to each circuit of the line.

Assuming unit currents, the above results may be further simplified leading to

$$\mathbf{Z}_{012} = \mathbf{A}^{-1} \cdot \mathbf{V}_{012FEM} \qquad (A.11.a)$$

$$\mathbf{Z}_{012} = \mathbf{A}_2^{-1} \cdot \mathbf{V}_{012FEM} \qquad (A.11.b)$$

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IX. Biographies

Dimitrios G. Triantafyllidis was born in Stuttgart, Germany, on September 25, 1972. He received the Dipl. Eng. degree from the Department of Electrical and Computer Engineering at the Aristotle University of Thessaloniki in 1996. Since 1996 he is a Ph.D. student in the Department of Electrical and Computer Engineering at the Aristotle University of Thessaloniki. His research interests are finite elements and power systems engineering. Mr. Triantafyllidis is a student member of IEEE and the Society of Professional Engineers of Greece.

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Dimitris P. Labridis (S' 88-M' 90) was born in Thessaloniki, Greece, on July 26, 1958. He received the Dipl.-Eng. degree and the Ph.D. degree from the Department of Electrical Engineering at the Aristotle University of Thessaloniki, in 1981 and 1989 respectively.

During 1982-1993 he has been working, at first as a research assistant and later as a Lecturer, at the Department of Electrical Engineering at the Aristotle University of Thessaloniki, Greece. Since 1994 he has been an Assistant Professor at the same Department. His special interests are power system analysis with special emphasis on the simulation of transmission and distribution systems, electromagnetic and thermal field analysis, numerical methods in engineering and artificial intelligence applications in power systems.