Finite Element Computation of Field and Eddy Currents of a System Consisting of a Power Transmission Line Above Conductors Buried in Nonhomogeneous Earth

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Abstract: The present work investigates the electromagnetic field and the eddy currents of a system consisting of a faulted power transmission line above parallel conductors buried in nonhomogeneous earth. The electromagnetic field diffusion equation has been numerically solved, using finite element method (FEM). Using FEM results, magnetic vector potential distribution in the cross-section of the parallel exposure, as well as eddy currents induced in all conductive parts, are calculated for various nonhomogeneous earth models. An analysis concerning the most important operational parameters in determining power transmission line influence is presented. Such parameters are the depth of the first earth layer, the resistivities of the different earth layers and the number of the mitigation wires.

Keywords: power transmission lines, eddy currents, finite element methods, nonhomogeneous earth.

I. INTRODUCTION

World-wide tendency is to use a common band of land, which is usually a straight narrow corridor, for utilities such as power transmission lines, pipelines, railroads and communication lines. This policy minimizes the amount of land used. An energized power transmission line in such a corridor causes significant interferences to nearby parallel buried conductors, especially in cases of single-phase faults. These interferences were first studied using the widely-known Carson’s relations [1]. Various other approximating formulae have also been proposed, leading to technical recommendations [2-3].

Progress in computing power led to advanced analytical models [4-9]. Recently, two extensive research projects of Electrical Power Research Institute (EPRI) and American Gas Association (AGA) introduced:


i) practical analytical expressions, which could be programmed on hand-held calculators [10-12], and ii) computerized techniques [13], for the analysis of power transmission line inductive interference to gas pipelines.

More recently, EPRI and AGA in a joint research developed the ECCAPP program [10-16]. Equivalent circuits with concentrated or distributed elements are used in ECCAPP and the self and mutual impedances are calculated using formulae from Carson [1], Pollaczek [17] and Sunde [4]. The program combines an input data preprocessor with a computation algorithm, which evaluate effects, defined there as conductive and inductive interference, for arbitrarily positioned conductors. Combinations of parallel and non-parallel constructions, power line discontinuities, as well as grouping pipeline sections according to their electrical length proposed by [10-12] are also included.

Although the mechanisms by which inductive and conductive interferences arise are well understood, the determination of these effects is a complex procedure. This procedure requires not only a good knowledge of conductors layout, power transmission line and pipeline electrical and geometrical properties, but also an accurate representation of the earth.

The earth model is of particular importance for the determination of interference levels and of the performance of mitigation systems. Without an adequate modeling of the earth structure, the mitigation systems have to be conservatively estimated, in order to ensure a sufficient protection. When the earth structure is accurately modeled, effective and low-cost mitigation designs may be evaluated.

Therefore, this paper addresses the influence of nonhomogeneous earth on the electromagnetic field and on the eddy currents induced in all conductive parts, i.e. in overhead ground wires, mitigation wires, buried pipeline and earth layers. The use of finite elements method (FEM) for the solution of Maxwell’s equations leads always to useful conclusions, even in the case of complex electromagnetic field problems. Therefore, a procedure based on FEM has been proposed to solve the two dimensional electromagnetic field problem of a faulted power transmission line, in the presence of buried conductors and nonhomogeneous earth.
II. THE ELECTROMAGNETIC FIELD PROBLEM

A. System arrangement and assumptions justification

To investigate the influence of nonhomogeneous earth on the electromagnetic field of a power transmission line, a typical system shown in Fig. 1 has been chosen. This system consists of a power transmission line running parallel for some kilometers to a buried pipeline, as shown in Fig. 1c. The assumption that the power line is parallel to the buried pipeline for some kilometers is valid for many practical applications, when both are located in straight narrow corridors.

The power transmission line consists of an ACSR (HAWK) two conductors bundle per phase. The buried pipeline, the cross section of which is shown in Fig. 1b, is wrapped in an electrically insulating coating and has been dimensioned according to characteristics given in Appendix of [16]. The separation distance d between the power transmission line and the parallel buried pipeline is shown in Fig. 1a.

Pipeline's metal and overhead ground wires (OHGW) have conductivities \( \sigma_m = \sigma_g = 7.0 \times 10^6 \text{ S/m} \) and relative permeabilities \( \mu_m = \mu_g = 250 \) respectively. For these ferromagnetic metal components of the system, i.e. the steel used in the ground wires and in the pipeline, the non-linear effects are not considered. This may be easily justified, because the maximum occurring magnetic field values are low and therefore cannot force these materials into saturation. For example, the magnetic flux density on OHGW #1 under the fault conditions examined in chapter III of this paper is lower than 0.45 T, while the corresponding magnetic flux density on the pipeline cannot exceed 0.052 T. Both are obtained from the most pessimistic case examined, i.e. when \( d = 25 \text{ m} \) and \( \rho_1 = \rho_2 = \rho_3 = 1000 \Omega \text{m} \).

Pipeline's coating has a conductivity \( \sigma_c = 1.0 \times 10^8 \text{ S/m} \), which is a common accepted value for synthetic coatings, and a relative permeability \( \mu_c = 1 \). Aluminum mitigation wires shown in Fig. 1a are bare. In order to avoid possible corrosion problems, a system to protect these bare mitigation wires against corrosion may be required. The earth finally is subdivided in layers with different resistivities, while their relative permeabilities are \( \mu_{12} = \mu_{13} = \mu_{23} = 1 \). These layers, shown in Fig. 1a, are assumed to be homogeneous across the x-dimension, although this is not a necessity for the method developed.

A standard power frequency of 60.0 Hz has been used to simulate a single-phase fault. This fault is happening outside of the parallel exposure, as shown in Fig. 1c. Therefore, conductive interference is negligible and only the inductive interference, due to the magnetic field, exists. Consequently eddy currents are induced in buried pipeline, mitigation wires, OHGW and earth layers. End effects are neglected for the inductive interference calculations, leading to a two dimensional problem. The assumption concerning the ignorance of the end effects is valid for the inductive interference calculations and for the lengths of parallel exposures encountered in practical applications.

B. Field equations and FEM formulation

The previous assumption lead to a linear two-dimensional electromagnetic diffusion problem for the z-direction components of the magnetic vector potential (MVP) \( A_z \) and of the total current density vector \( J_z \). This problem is described [18] by the system of equations

\[
\frac{1}{\mu_0 \rho \sigma} \left[ \frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} \right] - j \omega \sigma A_z + J_{sz} = 0 \tag{1a}
\]

\[
- j \omega A_z + J_{sz} = J_z \tag{1b}
\]
layer has the same resistivity as that of the third earth layer, leading to a model with two earth layers. The depth of the first earth layer is equal to $l_1$, while second earth layer thickness is equal to $l_2 = 2 \text{ km} - l_1$. The layers extend across x-axis up to 2 km from the power transmission line axis of symmetry, in both directions. Initially it is assumed that no mitigation wires are present. The influence of each earth layer resistivity on the electromagnetic field and on the inductive interference depends on the depth of the first earth layer. It may be interesting to estimate the depth $l_1$ from which the first earth layer resistivity determines almost exclusively the electromagnetic field and the inductive interference.

First and second earth layers are initially assumed to have resistivities $\rho_1 = 100 \ \Omega \text{m (dry earth)}$ and $\rho_2 = 1000 \ \Omega \text{m (rocky earth)}$ respectively. The parametric analysis for $l_1$, concerning the pipeline eddy current $I_p$, is shown in Fig. 2. The application of the proposed method for the same separation distances of Fig. 2 but for homogeneous earth (i.e., when the resistivities of the earth layers of Fig. 1a are $\rho = \rho_1 = \rho_2 = \rho_3$) having resistivity $\rho = 100 \ \Omega \text{m}$ or $\rho = 1000 \ \Omega \text{m}$ has given the results shown in Table 1. Comparing eddy current values of Table 1 with the corresponding values obtained from Fig. 2 it is evident that:

- when the depth $l_1$ of the first earth layer varies between 10 m and 50 m, the electromagnetic field and the inductive interference is determined properly from the resistivity of the second earth layer.
- for depth $l_1 = 400$ m and greater the resistivity of the first earth layer plays the dominant role in the electromagnetic field and inductive interference influence.

The above conclusions may be also explained with the help of Fig. 3, since when $l_1$ is equal to 50 m, the eddy current $I_{SEL}$ induced in the second earth layer is three times bigger than the eddy current $I_{SEL}$ induced in the first earth layer. The contrary condition holds when $l_1$ is equal to 400 m.

The influence of the depth of the first earth layer may be easily understood from Fig. 4-5, in which flux lines are shown for two cases with $l_1 = 50$ m and $l_1 = 400$ m respectively. From the MVP distribution of Fig. 4, which is valid for a first earth layer depth $l_1 = 50$, it is clear that the influence of the resistivity of the first earth layer on the electromagnetic field is negligible. In this case, the field is almost exclusively influenced by the resistivity of the second earth layer. On the other hand, from the MVP distribution of Fig. 5, which is valid for a first earth layer depth $l_1 = 400$, the influence on the electromagnetic field due to the resistivity of the first earth layer is evident, since the field lines are now compressed in that layer.

Finally, the first and second earth layers are assumed to have $\rho_1 = 1000 \ \Omega \text{m}$ and $\rho_2 = 1000 \ \Omega \text{m}$ respectively. The corresponding parametric analysis for the depth of the first earth layer, concerning the pipeline eddy current $I_p$, is now shown in Fig. 6. Comparing the results shown in Fig. 6 and in Table 1, it can be concluded that when the depth of the first
earth layer is greater than $l_1 = 900$ m, the electromagnetic field and the inductive interference are determined solely from the first earth layer.

![Graph of eddy current $I_p$ vs. first earth layer depth $l_1$]

Fig. 2. Pipeline eddy current $I_p$ as a function of the depth of the first earth layer of Fig. 1a, for a two earth layer model with resistivities $\rho_1 = 100$ $\Omega$ and $\rho_2 = 1000$ $\Omega$. Curve A corresponds to separation distance $20$ m, curve B to $100$ m and curve C to $500$ m.

B. Two earth layers with mitigation wires

The previous examined model is further extended to include mitigation wires near the pipeline, as shown in Fig. 1a. The reduction of pipeline eddy current $I_p$ obtained by installing progressively more bare mitigation wires, made of a low resistivity and permeability material such as aluminum, is shown in Table 2. From this Table it is clear that buried mitigation wires are very effective. This Table also shows that if the depth of the first earth layer is equal to $250$ m, the resistivities of the two earth layers have almost the same influence on the electromagnetic field and on the inductive interference.

Table 3 shows the eddy currents induced in OHGW, earth layers, buried pipeline and mitigation wires by a current $I_F = 1000$ A of the faulted phase $a$. These currents are shown for both cases, i.e. with and without the three aluminum mitigation wires shown in Fig. 1a. It can be observed that in the absence of mitigation wires, the eddy currents induced in the pipeline and in the two earth layers are bigger compared to the corresponding values obtained when the three aluminum mitigation wires have been installed.

Finally, flux lines of the electromagnetic field near the power transmission line are shown in Fig. 7, when the three aluminum wires are located near to the pipeline. The effect of mitigation of inductive interference level due to the aluminum wires is here easily understood, since the electromagnetic field is concentrated towards the faulted power line. In the absence of mitigation wires, the magnetic field would have a $y$-axis symmetry and flux lines in the pipeline region would be similar to those of the left part of $y$-axis of Fig. 7.

![Graph of eddy currents $I_{FEL}$ and $I_{SEL}$ vs. first earth layer depth $l_1$]

Fig. 3. Eddy currents induced in the earth layers as a function of the depth of the first earth layer, for a two earth layer model with resistivities $\rho_1 = 100$ $\Omega$ and $\rho_2 = 1000$ $\Omega$. The separation distance $d$ is equal to $100$ m. $I_{FEL}$ is the eddy current induced in the first earth layer and $I_{SEL}$ the eddy current induced in the second earth layer.

C. Three earth layers

Consider now a model with three earth layers, as shown in Fig. 1a, having layer thicknesses $l_1 = 10$ m, $l_2 = 190$ m and $l_3 = 1800$ m respectively. The separation distance $d$ between the power transmission line and the buried pipeline is equal to $100$ m and no mitigation wires are present. A parametric analysis for various resistivities of the three earth layers, concerning pipeline eddy current $I_p$, is presented in Table 4. This Table, in the case of $\rho_1 = \rho_2 = \rho_3 = 100$ $\Omega$m shows pipeline eddy current equal to $234.77$ A, while in the case of $\rho_1 = \rho_2 = \rho_3 = 1000$ $\Omega$m equal to $307.7$ A. These two cases lead to the minimum and maximum pipeline eddy current values and correspond to the minimum and maximum inductive interference levels respectively. From the other
results of Table 4 it can be concluded that $I_P$ is mainly influenced by the resistivities of the second and the third earth layer. The corresponding influence of the first earth layer is negligible, because $I_T$ is only 10 m.

All the above calculations have been made for a fault current equal to 1000 A. However, MVP distribution and therefore eddy current values are proportional to the fault current, so the presented results may be easily used for any given fault current $I_F$.

From all the previous results it is evident that resistivity measurements in relative small depths, in an attempt to simulate the inductive interference problem using a homogeneous earth, may lead to an erroneous earth model. This model may lead to an inaccurate determination of the inductive interference problem.

### TABLE 2
Effect of the buried mitigation wires of Fig. 1a on pipeline eddy current $I_P$, for a two earth layer model. The depth of the first earth layer $I_I$ is equal to 250 m, the separation distance $d$ is equal to 200 m, the bare mitigation wires have a radius equal to 5 mm and they are located at a distance equal to 1 m right to the pipeline axis.

<table>
<thead>
<tr>
<th>Number of Aluminum Mitigation Wires</th>
<th>$I_P$ [A]</th>
<th>$I_P$ [A]</th>
</tr>
</thead>
<tbody>
<tr>
<td>none</td>
<td>216.02</td>
<td>210.35</td>
</tr>
<tr>
<td>1</td>
<td>173.10</td>
<td>168.07</td>
</tr>
<tr>
<td>2</td>
<td>142.80</td>
<td>138.21</td>
</tr>
<tr>
<td>3</td>
<td>121.58</td>
<td>117.09</td>
</tr>
</tbody>
</table>

### TABLE 1
Pipeline eddy current $I_P$ for homogeneous earth (i.e. when resistivities of the three layers shown in Fig. 1a are $\rho_1 = \rho_2 = \rho_3 = \rho$) for various separation distances $d$.

<table>
<thead>
<tr>
<th>$\rho = 100 \Omega m$</th>
<th>$\rho = 1000 \Omega m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_P$ [A]</td>
<td>$I_P$ [A]</td>
</tr>
<tr>
<td>25</td>
<td>333.48</td>
</tr>
<tr>
<td>100</td>
<td>234.77</td>
</tr>
<tr>
<td>500</td>
<td>93.44</td>
</tr>
</tbody>
</table>

### TABLE 3
Eddy currents induced in OHGW, earth layers, buried pipeline and mitigation wires by the current $I_F = 1000$ A of the faulted conductor for a two earth layer model with resistivities $\rho_1 = 100 \Omega$ and $\rho_2 = 1000 \Omega$. The depth of the first earth layer $I_I$ is equal to 250 m, the separation distance $d$ is equal to 200 m, the bare mitigation wires have a radius equal to 5 mm and they are located at a distance equal to 1 m right to the pipeline center.

<table>
<thead>
<tr>
<th>$I_F$</th>
<th>$I_P$ [1000 A]</th>
<th>$I_P$ [0 A]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{g1}$ (1st OHGW)</td>
<td>60.53 A $&lt; -142^\circ$</td>
<td>60.10 A $&lt; -141^\circ$</td>
</tr>
<tr>
<td>$I_{g2}$ (2nd OHGW)</td>
<td>50.93 A $&lt; -144^\circ$</td>
<td>50.48 A $&lt; -143^\circ$</td>
</tr>
<tr>
<td>$I_{EEL}$ (1st Earth Layer)</td>
<td>501.03 A $&lt; -160^\circ$</td>
<td>467.98 A $&lt; -157^\circ$</td>
</tr>
<tr>
<td>$I_{SEL}$ (2nd Earth Layer)</td>
<td>274.09 A $&lt; 147^\circ$</td>
<td>256.18 A $&lt; 150^\circ$</td>
</tr>
<tr>
<td>$I_P$ (Pipeline)</td>
<td>216.02 A $&lt; 159^\circ$</td>
<td>121.58 A $&lt; 139^\circ$</td>
</tr>
<tr>
<td>$I_{m1}$ (1st mitigation wire)</td>
<td>55.21 A $&lt; 167^\circ$</td>
<td>55.21 A $&lt; 167^\circ$</td>
</tr>
<tr>
<td>$I_{m2}$ (2nd mitigation wire)</td>
<td>52.51 A $&lt; 160^\circ$</td>
<td>52.51 A $&lt; 160^\circ$</td>
</tr>
<tr>
<td>$I_{m3}$ (3rd mitigation wire)</td>
<td>58.75 A $&lt; 173^\circ$</td>
<td>58.75 A $&lt; 173^\circ$</td>
</tr>
<tr>
<td>$I_R$ (Total return current)</td>
<td>991.7 A $&lt; -179.3^\circ$</td>
<td>992.27 A $&lt; -179.4^\circ$</td>
</tr>
</tbody>
</table>
IV. CONCLUSIONS

The electromagnetic field problem of a power transmission line in the presence of nonhomogeneous earth and buried conductors has been numerically solved, using finite element method. Magnetic vector potential distribution in the cross-section of the parallel exposure and eddy currents induced in OHGW, earth layers, pipeline and mitigation wires may be easily computed with the proposed method, for any nonhomogeneous earth model. Many different earth models have been examined. Useful conclusions concerning the influence of the earth structure on the inductive interference levels are presented. Mitigation wires performance has also been investigated. It is concluded that a nonhomogeneous earth model is essential for the accurate computation of the inductive interference levels and for the effective design of mitigation systems.

V. REFERENCES


VI. BIOGRAPHIES

Kostas J. Satsios (S'94) was born in Serres, Greece, in May 1971. He received the Dipl. Eng. degree from the Department of Electrical Engineering at the Aristotle University of Thessaloniki in 1994. Since 1994 he is a Ph.D. student in the Department of Electrical and Computer Engineering at the Aristotle University of Thessaloniki. His research interests are in finite elements and artificial intelligence applications in power systems. Mr. Satsios is a member of the Society of Professional Engineers of Greece.

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