Archiv für Elektrotechnik 76 (1993) 285–292

Archiv
für Elektrotechnik

Coupled magneto-thermal field computation in three-phase gas insulated cables

Part 1: Finite element formulation

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Contents: A numerical procedure employing the finite element technique is developed for the computation of the coupled magneto-thermal field in three-phase gas insulated cables. The finite element formulation of both the electromagnetic and temperature field problems, the iterative procedure and the effective thermal conductivity of the insulation gas needed for the solution of the problem are presented here. Calculations made with the proposed method are presented in Part 2.

Berechnung des gekoppelten magnetisch-thermischen Feldes dreiphasiger gasisolierter Kabel

Teil 1: Lösungsansatz mit finiten Elementen

Übersicht: Es wird ein auf der Methode der finiten Elemente basierendes numerisches Verfahren zur Berechnung des gekoppelten magnetisch-thermischen Feldes dreiphasiger gasisolierter Kabel entwickelt. Hierzu werden der Lösungsansatz mit finiten Elementen für das elektromagnetische Feld und das Temperaturfeld, das Iterationsverfahren und die zur Lösung des Problems benötigte effektive thermische Leitfähigkeit des Isoliergases angegeben. Berechnungsergebnisse mit der vorgeschlagenen Methode werden in Teil 2 vorgestellt.

1 Introduction

The essential duty of a power cable is that it should transmit the maximum current (or power) for specified installation conditions. There are three main factors that determine the safe continuous current that a cable may carry:

a) The maximum permissible temperature at which its components may be operated with a reasonable factor of safety,
b) the heat-dissipating properties of the cable and
c) the installation and ambient conditions.

Gas insulated power cables are used for the transmission of high electrical power because they have advantages as compared to oil-paper insulated pipe-type cables. Some of these advantages are:

- Easy construction,
- good heat transfer through the dielectric SF_6,
- low ohmic losses,
- longer critical length and
- inexpensive terminations.

Therefore, the calculation of the coupled electromagnetic and thermal fields in gas insulated cables is of importance for the design and reasonable operation of the transmission system.

In an oil-paper insulated pipe-type cable the maximum permitted insulation temperature is a limitation to its ampacity. In a gas cable, due to the good heat transfer through the SF_6, the maximum sheath temperature is the limit. The enclosure temperature has to be limited so that it will not dry out the soil and increase its thermal resistivity. Fixing the maximum permissible sheath temperature is a matter of judgement. The literature on this subject mentions temperatures ranging from 40 °C to 60 °C.

By solving the steady-state heat conduction problem, the temperature distribution in the cable and in the surrounding soil is obtained. The inputs for the heat conduction problem are the power losses due to the imposed and induced currents in the conductors and the sheath. It is known that the magnitude of these losses depends upon the current density distribution inside the conductive parts of a multiconductor system. By solving the steady-state electromagnetic problem, the field distribution in the cable cross-section is obtained and the operating parameters of the cable (losses, forces and inductances) may be calculated. The input for the electromagnetic field calculation is the rms of the current flowing through each conductor. A complication originates from the temperature dependence of the electrical conductivities of the conductors and of the sheath. Therefore a solution of a coupled set of partial differential equations, representing thermal and magnetic diffusion problems, respectively, is required.

An additional complication of the problem is the heat transfer through the gas dielectric, in which both convection and radiation are important. These two mechanisms have a dependence on the conductor's and the sheath's temperatures, leading to an iterative solution for the thermal problem. In this paper an equivalent thermal conductivity of the SF_6 is used, based on the geometry, the temperatures and the other physical parameters of the cable. This does not overcome the necessity for an iterative solution, but allows the use of a single conduction-based thermal diffusion equation. The complication of the cable geometry and of the boundary condition of the two diffusion problems leads to a finite element computational solution.

The finite element method (FEM) has been used extensively in the solution of the steady-state and transient thermal problems in underground cable systems, using approximations for the input of the problem. In [1] the current distribution inside the conductors is assumed to be uniform, the sheath losses are set to 25% of the conductor losses and the electrical conductivity is considered to be temperature independent. In [2] and [3] the solution is given for cables that are approximated as line heat sources with an a priori known heat input rate. In [4] the heat input rate in the cables is taken from tables, while in the more general approach of [5] the conductors are approximated by a boundary having a known and constant heat flux.

On the other hand, the FEM has also been used for solving the steady-state multiconductor skin effect problem [6–8]. In [9] a finite element formulation has been presented for a three-phase gas cable, leading to the calculation of the losses in the conductors and the sheath. However, in these approaches, the electrical conductivity has been considered independent of the temperature.

The first attempt to couple the electromagnetic and heat transfer processes has been reported in [10], in which a temperature distribution around a three-phase cable system is computed with a finite element procedure. In that paper the surfaces of the cable sheaths have been assumed to be equitemperatural, the sheath eddy current losses are calculated from the Fredholm integral equation (approximating the sheaths with thin homogeneous cylinders), while the skin and the proximity effects in the conductors have been neglected. In [11] both diffusion equations are solved with the FEM, using the same mesh and taking the output of the magnetic problem (i.e. the power loss density) as the input of the thermal problem. Although the method has been applied to an eddy current analysis of an induction motor, assuming the electrical conductivity independent of temperature, the use of the FEM in that problem leads to an accurate description of the temperature distribution.

In this paper, a finite element formulation is used to model the three-phase gas cable electromagnetic and thermal steady-state diffusion problems, taking into account the real geometry and the real electromagnetic and thermal properties of the involved materials. The finite element mesh is the same for the two problems, applying different boundary conditions. An iterative procedure for the solution of the coupled electromagnetic and thermal diffusion is presented. The given quantity is the rms of the measurable current flowing through each conductor. As a result, the field distributions of the magnetic vector potential (MVP) and of the temperature can be calculated.

A general-purpose finite element program has been developed. From the MVP distribution, the current density distribution and finally the losses of the cable are easily obtained. From the temperature distribution, using a given maximum sheath temperature as a limitation, the ampacity of the cable is also easily determined. Calculations made with the method proposed in this paper are presented in Part 2.

2 The model

The cable consists of three tubular phase conductors with external radius r_a and wall thickness d_c, in equilateral configuration and in distance m between their centres as shown in Fig. 1. The conductors are located inside a tubular sheath with internal radius R_i and wall thickness d_s. The insulating gas between conductors and sheath is SF_6 at a pressure of 345 kPa (50 psi). The cable is directly buried in the ground in depth D, that is the distance between the soil surface and the outside sheath wall, as shown in Fig. 2.

The assumptions used in the calculation are the following:

- The cable is of infinite length, so that the coupled diffusion problem becomes a two-dimensional one.
- Charges and displacement currents are neglected.
- The conductors and the sheath have constant relative permeabilities μ_{rc} and μ_{rs}, respectively.

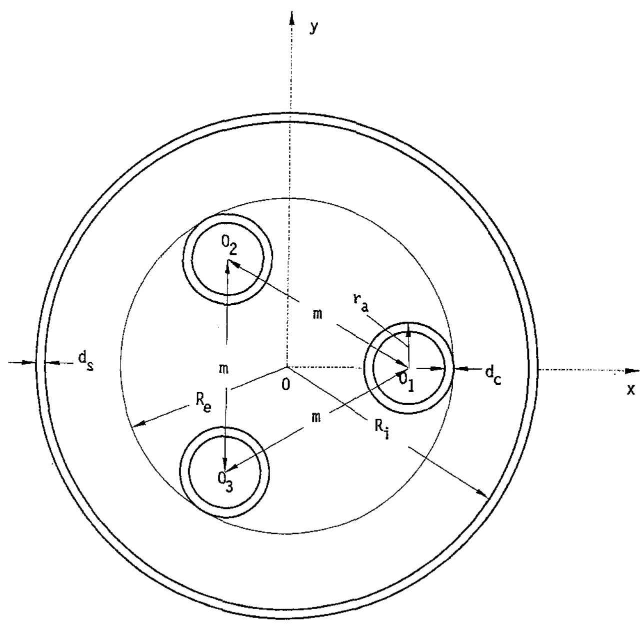

Fig. 1. Cross section of the three-phase gas insulated cable

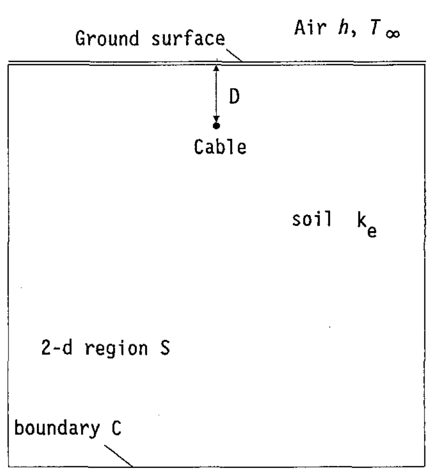

Fig. 2. Cable installation in depth D from soil surface

- The conductors' and the sheath's electrical conductivities σ_c and σ_s, respectively are functions of temperature.
- The thermal conductivities of the conductors, sheath and soil (k_c, k_s and k_e) are independent of temperature.
- The air inside the tubular conductors is still, therefore a thermal conductivity of air k_a is used in this region.
- An effective thermal conductivity k_{eff} of SF_6, including the effects of free convection and radiation between conductors and sheath, is calculated. Conduction becomes the dominant mode of heat transfer in the whole domain. This effective thermal conductivity k_{eff} is a function of sheath and conductors mean temperatures.
- The phase currents are sinusoidal and balanced.

On the basis of the last assumption, the following complex functions for the time variation of the three conductor currents are introduced:

$$\begin{aligned}\tilde{I}_1 &= \sqrt{2}\, I_{rms} e^{j\omega t}\\ \tilde{I}_2 &= \sqrt{2}\, I_{rms} e^{j(\omega t - 2\pi/3)}\\ \tilde{I}_3 &= \sqrt{2}\, I_{rms} e^{j(\omega t - 4\pi/3)}\end{aligned} \tag{1}$$

where ω is the angular frequency and I_{rms} is the rms of the current flowing through each conductor of the cable.

3 The electromagnetic field problem

3.1 Equations and boundary conditions

The assumptions made lead to a piecewise linear, steady-state, time harmonic electromagnetic field in a two-dimensional region S bounded by the curve C, as shown in Fig. 3. Following the analysis presented in [9] and supposing that the current density vector J and the magnetic potential vector A have z-direction, the two-dimensional electromagnetic diffusion problem is described by the system of equations

$$\begin{aligned}\frac{1}{\mu_0\mu_r}\left[\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2}\right] - j\omega\sigma A + J_s &= 0\\ - j\omega\sigma A + J_s &= J\end{aligned} \tag{2a}$$

where μ_0 is the permeability of vacuum and J_s is the uniformly distributed source current density.

Because it is assumed that J_z is the only component of current density and the problem is solved in terms of the vector potential component A_z, the required boundary conditions to ensure the uniqueness of the solution of the problem and of the magnetic field are [12]

$$A_z|_C = A_0(x, y) \tag{2b}$$

and

$$\iint_S J\, ds = I_{rms} \tag{2c}$$

So the unknowns in the system of equations (2a) are A and J_s, while the values of A at the limit C of region S are specified by the Dirichlet condition (2b) and the total current density J is specified in the integral form (2c).

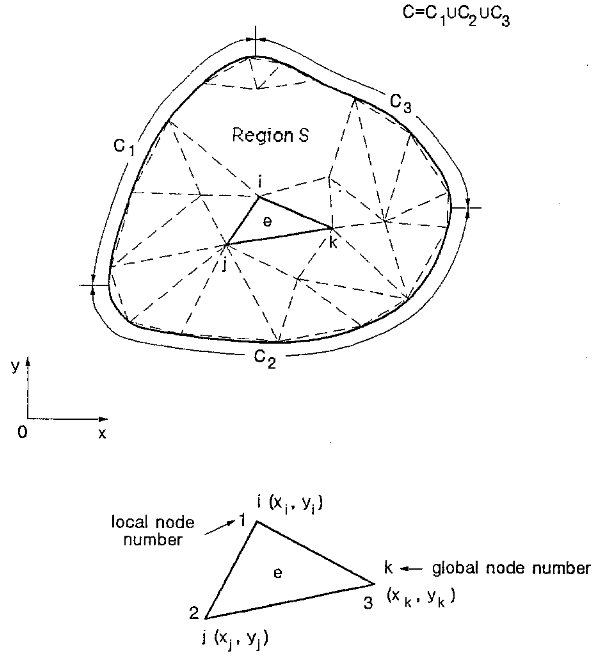

Fig. 3. Region under consideration S with limit the curve C and typical finite element e

The problem also consists of many interconnected regions and therefore is required to use the two continuity relations at the interfaces. The normal component of the flux density **B** and the tangential components of the magnetic field **H** have to be continuous across the boundary between two different media (provided that the boundary has no current sheet).

3.2 Finite element formulation

It has been shown [7] that for a straight conductor the source current density J_s is constant over its cross-sectional area. So the unknowns A and J_s may be approximated [9] in terms of linear interpolation polynomials $N(x, y)$ and $N_s(x, y)$ (see Appendix I) as

$$A(x, y) = \mathbf{N}^t\mathbf{A} \tag{3a}$$

and

$$J_s(x, y) = \mathbf{N}_s^t\mathbf{J}_s \tag{3b}$$

where the superscript **t** means the transposed vector and, in a multiconductor problem with m nodes and n conductors,

$$\mathbf{A}^t = [A_1\ A_2\ \ldots\ A_m], \quad \mathbf{J}_s^t = [J_{s1}\ J_{s2}\ \ldots\ J_{sn}] \tag{4a}$$

$$\mathbf{N}^t = [N_1\ N_2\ \ldots\ N_m], \quad \mathbf{N}_s^t = [1\ 1\ \ldots\ 1] \tag{4b}$$

According to the assumptions, the electrical conductivities σ_c and σ_s of the conductors and sheath, respectively are functions of temperature. Since conductivity values are usually quoted at 0 °C and if this conductivity is σ_0, then the conductivity $\sigma(T)$ of the material at temperature

T measured in °C is approximated by

$$\sigma(T) = \frac{\sigma_0}{1 + \alpha T} \quad (5)$$

where the temperature coefficient α is well known for the metals and alloys used in power cables.

Applying the Galerkin method to the system of equations (2) and assembling the element contributions in the usual way [6], leads to the following matrix equation

$$\begin{bmatrix} \frac{1}{\mu_0 \mu_r} \mathbf{S}_F - j\omega\sigma(T)\,\mathbf{T}_F & -j\omega\sigma(T)\,\mathbf{Q} \\ -j\omega\sigma(T)\,\mathbf{Q}^t & j\omega\sigma(T)\,\mathbf{W} \end{bmatrix} \begin{bmatrix} \mathbf{A} \\ \mathbf{G} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} \quad (6)$$

where $\mathbf{S}_F$ and $\mathbf{T}_F$ are the usual finite element matrices encountered in the solution of eddy current problems [13], while the vectors $\mathbf{Q}$, $\mathbf{I}$ and $\mathbf{G}$ and the diagonal matrix $\mathbf{W}$ are defined [9] for the multiconductor finite element formulation.

The boundary condition (2b) is an essential condition, therefore it must be imposed upon the interpolation polynomials $N(x, y)$ that are associated with the curve C of Fig. 3. In the multiconductor problem, the function $A_0(x, y)$ is usually set to zero at a great distance from the cable, so (2b) becomes a homogeneous Dirichlet condition of the form

$$A|_C = 0 \quad (7)$$

in which subscript z is omitted (since it is the only component of the magnetic vector potential A). The condition (7) is now easily imposed on the assembled matrix (6) by deleting the columns and rows that correspond to nodes belonging to curve C. On the other hand, boundary condition (2c) has been already imposed, during the Galerkin process, in the system (2a). Concerning finally the continuity relations of the electromagnetic field, it is well known, that the continuity of B_n is always satisfied exactly, since the MVP is chosen to be continous across the element boundaries, while the continuity of H_t is a natural condition. Therefore the continuity relations need not to be imposed explicitly on (6).

4 The temperature field problem

4.1 Equation and boundary conditions

As was stated earlier, an effective thermal conductivity k_{eff} of SF_6, that includes the effects of free convection and radiation between conductors and sheath, is introduced (see Appendix II). Also, according to the assumptions, conduction is the mechanism of heat transfer inside the tubular conductors and the thermal conductivity of air k_a is used in this region. So in order to determine the temperature distribution $T(x, y)$ in the two-dimensional region S bounded by the curve C (as shown in Fig. 3), a heat conduction problem is to be solved in the whole region of interest. When the problem is to be solved under steady-state conditions, the differential equation governing the heat conduction is

$$-k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] = \dot{q} \quad (8)$$

where $\dot{q}$ is the rate of heat generated per unit volume per unit time and k is the thermal conductivity of the material.

The solution of equation (8) depends on the physical conditions existing at the boundaries of the medium. With regard to the boundary conditions, there are several common possibilities which are simply expressed in mathematical form. Since the differential equation is of second order, two boundary conditions need to be specified. The possible cases are those in which:

1. The temperature T is specified (Dirichlet condition or boundary condition of the first kind)

$$T|_{C_1} = T_0(x, y) \quad (9)$$

2. The heat flux q_s per unit area at the boundary surface is specified (Neumann condition or boundary condition of the second kind)

$$k\left[\frac{\partial T}{\partial x} l_x + \frac{\partial T}{\partial y} l_y\right] + q_s = 0 \quad \text{on} \quad C_2 \quad (10a)$$

A special case of this condition corresponds to the perfectly insulated or adiabatic surface

$$k\left[\frac{\partial T}{\partial x} l_x + \frac{\partial T}{\partial y} l_y\right] = 0 \quad \text{on} \quad C_2 \quad (10b)$$

3. The convective heat loss $h(T - T_\infty)$, where h is the heat transfer coefficient and T_∞ is the ambient temperature, is specified (boundary condition of the third kind)

$$k\left[\frac{\partial T}{\partial x} l_x + \frac{\partial T}{\partial y} l_y\right] + h(T - T_\infty) = 0 \quad \text{on} \quad C_3 \quad (11)$$

where

l_x, l_y: direction cosines of the outward drawn normal to the surface,
C_1: boundary on which the value of temperature is specified,
C_2: boundary on which the heat flux q_s is specified and
C_3: boundary on which the convective heat loss $h(T - T_\infty)$ is specified.

The three boundaries of the thermal problem C_1, C_2 and C_3, as shown in Fig. 3, are a subset of the curve C, i.e. of the bound of the two-dimensional region S.

4.2 The finite element formulation

The problem of finding the temperature distribution inside the whole region of interest involves the solution of (8) using the boundary conditions (9) to (11). Using the FEM technique, the unknown T may be approximated in terms of linear interpolation polynomials $N(x, y)$ (see Appendix I) as

$$T(x, y) = \mathbf{N}^t\mathbf{T} \quad (12)$$

Applying the Galerkin method to equation (8) and assembling the element contributions in the usual way [16], leads to the following matrix equation

$$\mathbf{KT} = \mathbf{P} \tag{13 a}$$

where the matrix **K** and the vector **P** are given by

$$\mathbf{K} = \sum_e (\mathbf{K}_1{}^e + \mathbf{K}_2{}^e) \tag{13 b}$$

and

$$\mathbf{P} = \sum_e (\mathbf{P}_1{}^e - \mathbf{P}_2{}^e + \mathbf{P}_3{}^e) \tag{13 c}$$

The elementary matrices $\mathbf{K}_1{}^e$ and $\mathbf{K}_2{}^e$ and vectors $\mathbf{P}_1{}^e$, $\mathbf{P}_2{}^e$ and $\mathbf{P}_3{}^e$ that appear in (13 b) and (13 c) are defined for the temperature field problem in Appendix I.

Equations (13) are the desired equations. These have to be solved, after incorporating the boundary conditions, to obtain the values of nodal temperatures. The advantage of this formulation is that the boundary conditions (9) – (11) of the thermal problem have to be imposed on the same curve C, on which the homogeneous Dirichlet condition (7) of the electromagnetic problem has already been imposed. So it is possible to use the same finite element mesh for both diffusion problems and to define the appropriate boundaries for the problem that is currently solved.

5 The iterative procedure

The solution of the system of equations (2 a), that describe the electromagnetic diffusion problem, is equivalent to the matrix equation (6). In order to compute the nodal values of the MVP A and of the source current density J_s, we have to determine:

- The current I_{rms} flowing through each conductor of the three-phase cable, in order to define the vector **I** (see Appendix I) on the right-hand side of (6)
- The temperature distribution on the cross-section of the conductors and of the sheath, in order to calculate the value of the electrical conductivity σ at every point of these materials, as given in (5).

The solution of equation (8), that describes the thermal diffusion problem, is equivalent to the matrix equation (13 a). In order to compute the nodal values of the temperature T, we have to determine

- The average loss density $\dot{q}$ at every point on the cross section of the conductors and of the sheath, in order to define the vector $\mathbf{P}_1$ (see Appendix I) that appears on the right-hand side of (13 a)
- The mean temperatures of the conductors and of the sheath T_c and T_s respectively, in order to estimate the effective thermal conductivity k_{eff} of the insulating gas, as given in (A 12) (see Appendix II).

For the calculation of the loss density $\dot{q}$ we have to solve the electromagnetic diffusion problem. From the solution of the system in (6), this average loss density $\dot{q}^e$ for the typical element e may be calculated [9] from

$$\dot{q}^e = \frac{J^e J^{e*}}{\sigma^e} \quad [\mathrm{W/m^3}] \tag{14}$$

where J^e and σ^e are the total current density and the electrical conductivity of element e, respectively. The electrical conductivity σ^e is again a function of temperature, using the approximation (5). For all the above reasons, an iterative procedure is necessary in order to solve this strongly coupled electromagnetic and thermal diffusion problem. The iterative procedure contains 4 steps that will be explained in detail.

Step 1: The electromagnetic diffusion problem, described by the system of equations (2 a), is solved with the finite element formulation (6) for the computation of the unknown values of MVP A at every node and of the source current density J_s at every conductor. This solution is a function of the given current I_{rms} flowing through each conductor of the three-phase cable.

In the first iteration, the temperatures of the conductors and of the sheath are set equal to arbitrary and constant values (i.e. every point of the conductors material is assumed to have a constant temperature equal to T_{ca} and every point of the sheath material a constant temperature equal to T_{sa}). For all further iterations, the temperature values at every point will be obtained from step #4 and they will be different from point to point.

With the known temperature distribution at every point, the electrical conductivities σ^e at every finite element e that lies on the cross-section of the conductors and of the sheath is computed according to (5) as

$$\sigma^e = \frac{\sigma_0{}^e}{1 + \alpha^e T_m{}^e} \tag{15 a}$$

using the local values $\sigma_0{}^e$ and α^e and the mean temperature $T_m{}^e$ of element e given by

$$T_m{}^e = (T_1{}^e + T_2{}^e + T_3{}^e)/3 \tag{15 b}$$

Step 2: Using the values of A and J_s from the solution at step #1, the total element current density J^e is computed for every element e as [9]

$$J^e = -\frac{\mathrm{j}\omega\sigma^e}{3}(A_1{}^e + A_2{}^e + A_3{}^e) + J_{si} \tag{16 a}$$

if element e lies on the cross-section of conductor i with source current density J_{si}, or

$$J^e = -\frac{\mathrm{j}\omega\sigma^e}{3}(A_1{}^e + A_2{}^e + A_3{}^e) \tag{16 b}$$

if element e lies on the cross-section of the sheath. Using (16) and (14), the average loss density $\dot{q}^e$ of element e is calculated.

Step 3: Using the values of T from the solution at step #4 (or the arbitrary values T_{ca} and T_{sa} of the first iteration), the mean conductors temperature T_c and sheath temperature T_s are calculated. Using these values, the effective thermal conductivity k_{eff} of SF_6 is computed using (A 12).

Step 4: The thermal diffusion problem, described by equation (8), is solved with the finite element formulation (13 a) for the computation of the unknown values of temperature T at every node. The values of iteration i $T_{(i)}$ are compared with the values $T_{(i-1)}$ of the previous iteration $i-1$. If $|T_{(i)} - T_{(i-1)}| \leqq T_{err}$ at every node, where T_{err} is a small temperature, the iterative procedure is terminated.

As will be shown from the calculations presented in Part 2 of the paper, the convergence of the iterative procedure is very fast.

6 Conclusions

The computation of the coupled magneto-thermal field in a three-phase gas insulation cable based on a finite element formulation is the main purpose of this paper. Real geometry of the cable and real electromagnetic and thermal properties of the involved materials are taken into account. A single conduction based thermal diffusion equation is used for the thermal problem based on the calculation of an equivalent thermal conductivity of the SF_6. The coupled problem is solved using an iterative procedure based on temperature convergence at every nodal point. Given the current flowing through each conductor, both the electromagnetic and thermal field can be calculated.

Appendix I

Basic formulae for first-order triangular elements

Consider the typical first-order triangular element e shown in Fig. 3, with local MVP nodal values

$$\mathbf{A}^e = \begin{bmatrix} A_1{}^e \\ A_2{}^e \\ A_3{}^e \end{bmatrix} \tag{A 1}$$

and local temperature nodal values

$$\mathbf{T}^e = \begin{bmatrix} T_1{}^e \\ T_2{}^e \\ T_3{}^e \end{bmatrix} \tag{A 2}$$

using the first-order shape functions

$$\mathbf{N}^e(x, y) = \frac{1}{2S^e} \begin{bmatrix} a_1 + b_1 x + c_1 y \\ a_2 + b_2 x + c_2 y \\ a_3 + b_3 x + c_3 y \end{bmatrix} \tag{A 3}$$

where the coefficients a_i, b_i and c_i are well known from the literature [15] and S^e is the area of element e, we shall have the space approximation of the MVP as a phasor in the complex domain

$$A^e(x, y) = \mathbf{N}^{et}\mathbf{A}^e = A_{\text{real}}(x, y) + jA_{\text{imag}}(x, y) \tag{A 4 a}$$

and the space approximation of the temperature

$$T^e(x, y) = \mathbf{N}^{et}\mathbf{T}^e \tag{A 4 b}$$

where the superscript **t** means the transposed vector.

Suppose the element e lies in the conductor i, at which the source current density is J_{si}. Because J_{si} is a constant over the cross-section of the conductor, the phasor of the element e source current density will be

$$J_s{}^e = J_{si} \tag{A 5}$$

Assembling the element contributions over the whole domain, the space approximation for the vector potential will be obtained as in (3 a). Likewise, assembling the conductor contributions, the space approximation for the source current density will be obtained as in (3 b).

Assembling also the element contributions over the whole domain, the space approximation for the temperature will be obtained as in (12).

The elementary matrices $\mathbf{K}_1{}^e$ and $\mathbf{K}_2{}^e$ of (13 b) are given by

$$\mathbf{K}_1{}^e = \frac{k^e}{4S^e} \begin{bmatrix} (b_i{}^2 + c_i{}^2) & (b_i b_j + c_i c_j) & (b_i b_k + c_i c_k) \\ & (b_j{}^2 + c_j{}^2) & (b_j b_k + c_j c_k) \\ \text{symmetric} & & (b_k{}^2 + c_k{}^2) \end{bmatrix} \tag{A 6}$$

and

$$\mathbf{K}_2{}^e = h \int_{C_3} \begin{bmatrix} N_i{}^2 & N_i N_j & N_i N_k \\ & N_j{}^2 & N_j N_k \\ \text{symmetric} & & N_k{}^2 \end{bmatrix} \mathrm{d}C \tag{A 7}$$

where k^e is the thermal conductivity of element e and h is the heat transfer coefficient on curve C_3, on which we have supposed that the convection boundary relation (11) applies.

Finally, the elementary vectors of (13 c) are given by

$$\mathbf{P}_1{}^e = \iint_{S^e} \dot{q}\mathbf{N}^{et}\,\mathrm{d}S \tag{A 8}$$

$$\mathbf{P}_2{}^e = \int_{C_2} q_s \mathbf{N}^{et}\,\mathrm{d}C \tag{A 9}$$

and

$$\mathbf{P}_3{}^e = \int_{C_3} hT_\infty \mathbf{N}^{et}\,\mathrm{d}C \tag{A 10}$$

where (A 9) and (A 10) correspond to the boundary relation (10) and (11), respectively.

Appendix II

The effective thermal conductivity of the insulation gas

Explicit ampacity equations can only be derived when the radial temperature drops are proportional to heat flux, as in thermal conduction. Because the heat must be transferred across two surfaces in series, i.e. the outside of the conductor and the inside of the sheath, both free convection and radiation are important. So the ampacity must be determined by successive approximations. Conduction is the dominant mode of heat transfer from the sheath outward.

In order to calculate the thermal resistivity (based on both free convection and radiation) from conductor surfaces to sheath, the following formulae were used.

The temperature drop $(T_c - T_s)$°C across a concentric fluid gap, in the case where one conductor is located inside a sheath, may be expressed as

$$Q = \frac{2\pi k_{\text{eff}}(T_c - T_s)}{\ln(R_i/R_e)} = \frac{(T_c - T_s)}{R} \qquad \text{(A 11)}$$

where

Q:	heat flow per unit length	[W/m]
k_{eff}:	effective thermal conductivity	[W/m °C]
T_c:	mean conductor temperature	[°C]
T_s:	mean sheath temperature	[°C]
R_i:	inner sheath radius	[m]
R_e:	outer conductor radius	[m]
R:	thermal resistivity	[m °C/W]

The effective thermal conductivity k_{eff}, including the effects of free convection and radiation, may be calculated from (A 11) as

$$k_{\text{eff}} = \frac{\ln(R_i/R_e)}{2\pi R} \qquad \text{(A 12)}$$

The problem here is related to a three-phase gas cable. So a simplifying assumption has to be made about the heat transfer through the gas insulation. For this reason we consider an imaginary cylindrical heat dissipation surface of radius R_e (Fig. 1). This is an envelope of the three conductors and should be considered to consist of the same material as the conductors, from which the Joule loss of the three conductors emanates uniformly around the periphery.

The problem now is to find an effective thermal conductivity across this annulus gap. Fukuda [17] and Doepken [18] have discussed the heat transfer across a horizontal annulus gap. Based on their analysis and on the previous assumptions, a three-phase cable generalisation is accomplished. Let us first consider radiation. If ξ_s and ξ_c are the emissivities of the cylindrical surfaces of sheath and conductor, respectively, the Stephan-Boltzman law for a coaxial line is

$$Q_r = 5.67 \times 10^{-12} \times 2\pi R_e \xi'(T_c^4 - T_s^4) \quad \text{[W/m]} \qquad \text{(A 13)}$$

In this equation the temperatures are in [K] and R_e is in [m], while for diffuse reflection the effective emissivity ξ' is

$$\xi' = \frac{\xi_c \xi_s}{\xi_s + (R_e/R_i)(1 - \xi_s)\,\xi_c} \qquad \text{(A 14)}$$

The emissivity of a metal depends on its electrical conductivity. Assuming that the conductors are made of EC aluminium and the enclosure pipes of some high-strength aluminium alloy, it is recommended [19] to use the values $\xi_c = 0.3$ and $\xi_s = 0.8$.

On the other hand, the convective heat transfer depends on the gas density and therefore on its pressure and temperature. For an SF_6 insulated cable that has been filled to 50 psi at 20 °C, which is typical for isolated phase systems, Doepkens [18] recommends the following formula for convection

$$Q_c = \frac{3.75 \times 10^{-3}(T_c - T_s)^{1.2}\,(R_i - R_e)^{0.6}}{\ln(R_i/R_e)} \quad \text{[W/m]} \qquad \text{(A 15)}$$

Finally, the overall heat flow Q per unit length of cable is

$$Q = Q_r + Q_c \quad \text{[W/m]} \qquad \text{(A 16)}$$

After calculating this overall heat flow, the thermal resistivity R and finally the effective thermal conductivity k_{eff} of the gap may be calculated from equations (A 11) and (A 12), respectively.

In Fig. 4 the effective thermal conductivity k_{eff} of SF_6 is shown vs. different conductor and sheath temperatures. The calculation was based on the equations stated above, using the following typical values for a three-phase gas cable:

$$\begin{aligned} R_i &= 0.3429 \quad \text{[m]} \\ R_e &= 0.2365 \quad \text{[m]} \\ \xi_s &= 0.8 \\ \xi_c &= 0.3 \end{aligned} \qquad \text{(A 17)}$$

As shown in Fig. 4, the variations of the two temperatures lead to significant changes of the value of k_{eff}. Finally, in Figs. 5 and 6 the effective thermal conductivity k_{eff} of SF_6 is shown vs. mean sheath temperature T_s and for a constant mean conductor temperature $T_c = 80$ °C.

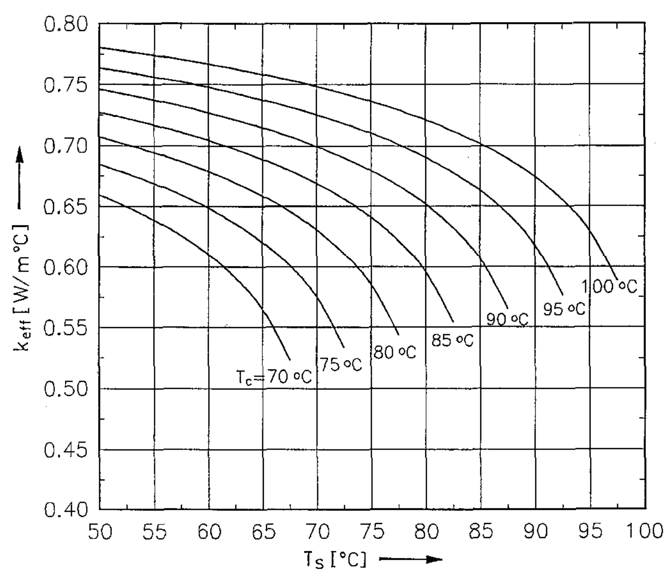

Fig. 4. Effective thermal conductivity k_{eff} of SF_6 as a function of mean sheath temperature T_s for seven different mean conductor temperatures T_c. The cable geometry and the emissivities of the materials are given by (A 17)

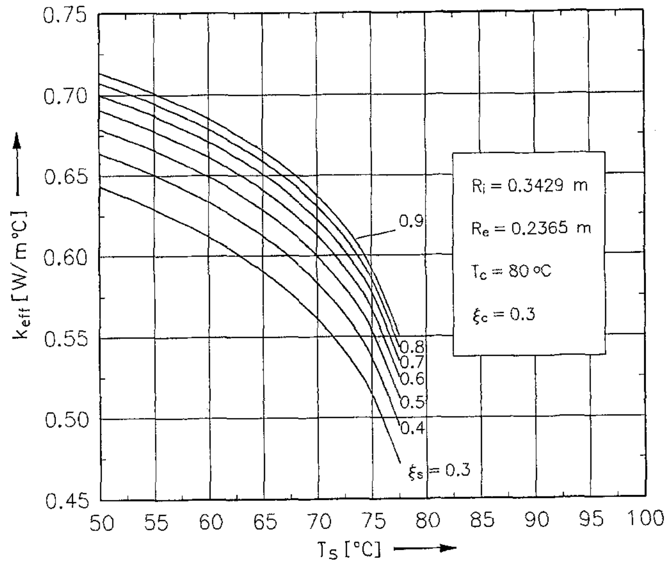

Fig. 5. Effective thermal conductivity k_{eff} of SF_6 as a function of mean sheath temperature T_s for nine different sheath emissivities ξ_s. The conductor emissivity ξ_c is 0.3 and the mean conductor temperature T_c is 80 °C

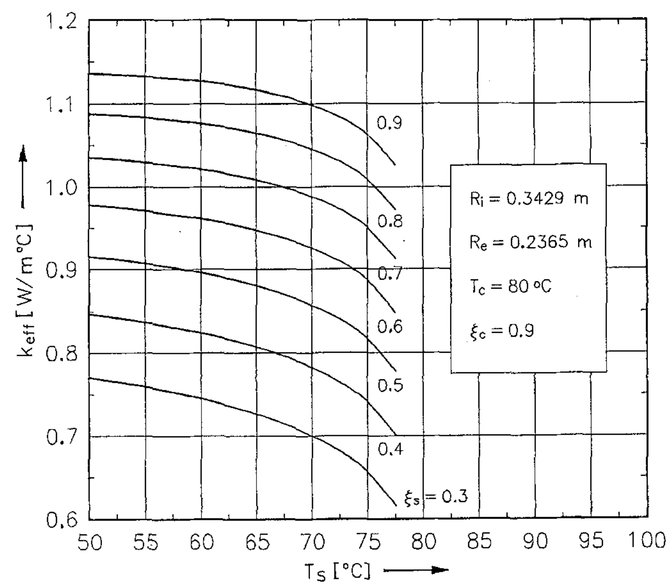

Fig. 6. Effective thermal conductivity k_{eff} of SF_6 as a function of mean sheath temperature T_s for nine different sheath emissivities ξ_s. The conductor emissivity ξ_c is 0.9 and the mean conductor temperature T_c is 80 °C

In Fig. 5 the conductor emissivity ξ_c is constant and equal to 0.3 while in Fig. 6 ξ_c is constant and equal to 0.9. The parameter in these two figures is the sheath emissivity ξ_s, which varies from 0.3 to 0.9. The range of values of k_{eff} for a typical mean sheath temperature $T_s = 65\,°C$ is between 0.59 W/m °C (Fig. 5 with $\xi_s = \xi_c = 0.3$) and 1.12 W/m °C (Fig. 6 with $\xi_s = \xi_c = 0.9$), i.e. an increase in the emissivity values from 0.3 to 0.9 leads to an almost 100% increase of the effective thermal conductivity of the insulation gas.

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Received July 28, 1992

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