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1. Introduction

Nonlinear processes in strongly coupled dusty plasmas (DP) have attracted theoretical interest recently, motivated by recent experiments. Dust (quasi)-lattices (DL) (2D or 3D) are typically formed in the sheath region above the negative electrode in discharge experiments, horizontally suspended at a elevated equilibrium position, at $z = z_0$, where gravity and electric (and/or magnetic) forces balance. Appropriate trapping potentials have also enabled the realization of 1D lattices, dominated by electrostatic interactions.

The linear regime of low-frequency oscillations in DP crystals, in the longitudinal (acoustic mode) and transverse (in-plane, shear acoustic mode and vertical, off-plane optical mode) direction(s) is now quite well understood. However, the nonlinear (NL) behaviour of DP crystals is little explored, and has lately attracted experimental [1-3] and theoretical [4-7] interest. We have recently considered the coupling among the horizontal (x) and vertical (off-plane, z) degrees of freedom in dust monolayers; a set of NL equations for coupled longitudinal and transverse dust lattice (LLDL, TDL) motion was thus derived [4]. Here, we review the nonlinear dust grain excitations which may occur in a DP crystal (assumed quasi-one-dimensional and infinite, composed from identical grains, of equilibrium charge q and mass $M$, located at $z_n = n \cdot \lambda$, $n \in \mathbb{N}$). Bunching is omitted here.

2. Transverse envelope structures (continuum)

Taking into account the intrinsic nonlinearity of the sheath electric and/or magnetic potential, the vertical (off-plane) n-th grain displacement $\delta z_n = z_n - z_0$ in a dust crystal (where $n = -1, 0, 1, 2, \ldots$), obeys the equation

$$\frac{d^2\delta z_n}{dt^2} + \omega_D^2 \delta z_n + i\phi(t) + \omega_k^2 (\delta z_n + 2\delta z_{n+1} + 2\delta z_{n-1} - 6\delta z_n) + \cdots = 0, \quad (1)$$

(where coupling anharmonicity and second plus nearest neighbors are omitted)

The characteristic frequency

$$\omega_k^2 = \frac{4q^2(\zeta_0/\lambda M)}{27/2}$$

is related to the (electrostatic) interaction potential, for a Debye-Hückel potential $U(h) = (q^2/\varepsilon) h^2$, one has

$$\omega_D = \omega_k^2 \exp(-1/\zeta_0) \approx \omega_k^2/\sqrt{2} \approx \omega_k^2$$

The on-site characteristic dust lattice frequency $\omega_D$ is the Debye length, $\kappa = \omega_D / \phi$ is the DP lattice parameter.

3. Transverse Intrinsic Localized Modes (ILMs) – Discrete Breathers (DBs)

ILMs.h i.e. highly localized Discrete Breather (DB) and multi-breather-type four-site vibrations have recently received increased interest among researchers in solid state physics, due to their omnipresence in periodic lattices and remarkable physical properties [6]. Dusty plasma DB excitations were shown to occur in transverse DL motion [1-11] from first principles (figure from [9]).

The existence of such DB structures at a frequency $\omega_{DB}$ generally requires the non-resonance condition

$$n\omega_{DB} \neq \omega_k \quad \forall n \in \mathbb{N}$$

which is remarkably satisfied in all known TDLB experiments [1]. The existence of DBs in 2D (hexagonal) dusty plasma structures is now under investigation [10].

4. Longitudinal envelope excitations

The NL longitudinal equation of motion ($\delta x_n = x_n - \bar{x}_n$) reads:

$$\frac{d^2\delta x_n}{dt^2} + \omega_L^2 \delta x_n + \hat{U}(\delta x_n) + \delta x_{n+1} - \delta x_{n-1} = 0, \quad \text{for} \quad n = -1, 0, 1, \ldots$$

One obtains (to lowest order et) $\delta x_n \approx \zeta(0) + \zeta(1) \phi(t) + \zeta(2) \phi(t)^2 + \cdots$

where $\zeta(0) = 0, \zeta(1) = 1, \zeta(2) = -1/2, \cdots$

One may employ the extended KdV (eKdV) (5), which accounts for both compressive and rarefactive lattice excitations (exact expressions in [12]). Alternatively, Eq. (7) can be reduced to a Generalized Boussinesq (GB) equation [13], for, again, $n = 0, \pm 1, \cdots$, one recovers a Boussinesq (B) equation, widely studied in solid chains. The GB (B) equation yields, like its eKdV (B) counterpart, both compressive and rarefactive (only compressive, respectively) solutions; however, the (supersonic) propagation speed v no longer have its close to $c_0$. The lengthy analysis (see in [12] for details) is not reproduced here.

5. Longitudinal Discrete Breathers?

Following existing studies on Discrete Breathers (ILMs) in FPU chains, one may expect the existence of such localized excitations associated with longitudinal dust grain motion. A detailed investigation, in terms of real experimental parameters, is on the way and will be reported soon.

References