TYPES OF CONTROL IN COLLABORATIVE PROBLEM SOLVING

Ioannis Papadopoulos(∗), Ioanna Sekeroglou(**)
ypapadop@eled.auth.gr(∗) ioanseke@eled.auth.gr(**)

Dep. of Primary Education, Aristotle University of Thessaloniki, Greece

In this paper, the types of social metacognitive control actions during collaborative problem solving are examined. Six pairs of future elementary school teachers work on two problems, an arithmetical and a geometrical one. The analysis of the collected data gave evidence of two types of control actions, Global and Context-based ones. At the same time, the factors that trigger control actions in a collaborative problem solving were also examined. It was found that some of these factors are a multiple solution task, an unreasonable result, the mismatching between the outcome of the problem-solving and verification processes, and finally the collaborative nature of the problem-solving.

INTRODUCTION

Performance on many tasks is positively correlated with the degree of one’s metaknowledge (Schoenfeld, 1985). Executive control and decision making is a way to talk about metacognition in problem-solving. Both are concerned with the solver’s continuous evaluation of her/his current working in conjunction with her/his aim. This means that the solvers monitor and assess both the state of their knowledge and the state of the solution and avoid „wild chases” that often guarantee failure in the evolution of the solution (Schoenfeld, 1985, 1988). The solver’s focus is on decisions about what to do in order to solve a problem. „Behaviors of interest include making plans, selecting goals and subgoals, monitoring and assessing solutions as they evolve, and revising or abandoning plans when the assessments indicate that such actions should be taken” (Schoenfeld, 1985, p. 27).

The vast majority of the relevant studies emphasize control actions taken by individuals. The study of Stillman and Galbraith (1998) is a nice example. In their study they found that the students exhibited knowledge of strategies (i) to aid understanding of the problem (such as re-read the problem and relate the problem to other problems within experience), (ii) to organize information (they were aware that organizing information into a table facilitates the process), (iii) for developing and executing plans, (iv) for monitoring process, and (v) for verification of the final results.

However, recent studies (Chiu & Kuo, 2009; Chiu, Jones & Jones, 2013) raise the importance of social metacognitive control during collaborative problem-solving. In this setting of collaborative problem-solving we examine the kinds of control actions
the solvers take in their effort to solve a problem as well as the factors that trigger these control actions. Therefore, our research questions are:

- What control actions can be identified during collaborative problem-solving?
- What factors trigger control actions during collaborative problem-solving?

**SOCIAL METACOGNITIVE CONTROL**

According to Chiu et al. (2013) social metacognitive control refers to „monitoring and control of the regulation and evaluation of others’ knowledge”, linking thus metacognitive judgments with communication, and applying one’s subjective metacognitive experiences to a group context (p. 74). In addition to the traditional components of individual metacognitive control, social metacognitive control includes group interaction and social influence. When mathematics problems are solved by groups rather than individuals, team members who can monitor and control other’s behaviors effectively, can increase their likelihood of solving difficult problems. This is a vital issue and it benefits both the problem-solving procedure and the solvers. On the one hand, this control can ensure that the answer is mathematically accurate and reasonable (Goos & Galbraith, 1996). On the other hand, it helps solvers to develop self-reflection, mental thinking and become capable solvers (Wilson, Fernandez & Hadaway, 1993). Chiu et al. (2013) following two students during their attempt to solve a problem found that the use of metacognitive control strategies made them able to understand, evaluate and build on each other’s thinking respectfully, creating thus new useful information that broadened their mathematics problem-solving. Through social metacognitive control, the cognitive and metacognitive demand that is necessary for solving a complex problem, is distributed among the group members, allowing the solvers to focus on smaller and simpler responsibilities according to their skills (Chiu & Pawlikowski, 2013). Nelson, Kruglanski and Jost (1998) found that as the group members model for one another, trying to understand and solve a problem, they actually help them interpret it and make effective metacognitive control decisions. Collaborative work however, might have also some limitations. To obtain effective social metacognitive control, sufficient communications skills are necessary (Kaiser, Cai, Hancock & Foster, 2001). Therefore, poor communication skills might influence performance and result to ineffective feedback (Efklides, 2009). Moreover, limited individual content knowledge might also result to inappropriate feedback that could mislead the group members (Holton & Thomas, 2001).

**DESIGN OF THE STUDY**

Twelve undergraduate students of the Department of Primary Education in the Aristotle University of Thessaloniki participated voluntarily in this study. The students’ mathematical background was weak despite they attended two optional courses (Elementary Mathematics and Problem-solving) plus the obligatory one on
Mathematics Education. They were asked to form pairs based on their personal preferences in order to ensure efficient co-operation.

Two problems were given to each pair, an arithmetical and a geometrical one. They had in total thirty minutes to solve each problem.

This is the arithmetical problem:

“*If we line up the school’s students in rows of three, two of them are left. If we place them in rows of four or five, two of them are also left. What is the number of students, given that this is a 3-digit number and the sum of its digits is 5?*”

This is not a difficult problem. But, it is interesting for two reasons. First, its solution combines a variety of mathematical concepts and/or mathematical ideas (use of Lowest Common Multiple and its multiples, use of the divisibility rules, all the possible combination of three digits that give a sum of five, etc.). Second, it has more than one possible solution. Initially there are multiple numbers that satisfy the first three conditions (2 students are left when in rows of 3, 4, or 5). But, the number of the possible solutions is reduced to 2 solutions due to the fourth condition (sum of digits is 5). The two solutions are 122 and 302.

This is the geometrical problem:

“*Find the area of the shape*” (Fig. 1)

![Figure 1. The shape of the geometrical problem](image)

This is also a typical geometrical problem related to the concept of area which is easy for an experienced solver. There is only one correct answer (245.5 square units) but this answer can be approached in various ways. For example, one could use Pick’s formula to calculate the area \( A(P) \) of any lattice polygon: \( A(P) = I + B/2 - 1 \) (‘I’ denotes the points in the interior of the polygon whereas ‘B’ the points on its boundary). Another approach is the division of the shape to sub-shapes. This means that the solver must find the area of each sub-shape and then, their sum makes the area of the
whole shape. An alternative could be to circumscribe a rectangle around the polygon, calculate its area, then find the area of the shapes that exist between the initial shape and the rectangle, add their areas, and subtract their sum from the area of the rectangle. The fact that both problems are open problems, amenable to multiple solutions/strategies (Silver, 1997) is a factor that triggers control actions. As Cifarelli and Cai (2005) claim, in open ended situations the solvers continually monitor and modify their problem-solving actions.

The students were also asked to vocalize their thoughts while performing the tasks. The think-aloud strategy refers to having students verbalize what they are thinking and doing throughout the problem-solving session. Students who verbalize seem to gain a better understanding of their own thinking (Albert, Bowen & Tansey, 2009) and therefore are prepared to ask appropriate questions to enhance their understanding of the problem-solving process. They also seem better able to trouble-shoot their own problems, and in many cases, solve those problems on their own (Pugalee, 2001). These make the „think aloud” another factor that triggers control actions during problem-solving.

The whole procedure was audio-recorded and then the students’ efforts were transcribed for the purpose of the paper. Their worksheets and the transcribed protocols constituted our data. The students’ protocols were parsed into episodes (according to Schoenfeld’s (1985) proposed Framework for Analysis of Problem-Solving Protocols), giving emphasis on the points showing aspects of control and on instances indicating that certain factors trigger control actions.

RESULTS AND DISCUSSION

The control actions identified in the analyzed protocols were finally organized in two collections according to whether these actions dependent on the task’s context or not. The first category is called „Global control actions” (not depended on the task). It is about control actions that can be applied to any problem no matter the content/mathematical topic of the problem (Fig. 2). This makes an analogy to the notion of heuristics. The second category is called „Context-based actions” (task-dependent). These actions are closely related to the context of the specific task. It is the nature of the task, its content, that provokes these specific control actions. These actions can be transferred only to similar tasks. In the next section, each reference to excerpts from the transcribed protocols is accompanied with an alphanumeric part on the left that needs to be explained in advance. This includes the number of lines of the protocol (e.g., 27-29 or 130). Then, a combination of letters and numbers is used to indicate the number of the pair, the number of the student, and the kind of the problem. For example, P3.1.A means Pair-3, Student-1, Arithmetical problem (and similarly G for the geometrical one).
Global control actions.

G1 – Understanding the problem

This refers to actions that aim to control the correct understanding of the problem. The collected data allowed us to identify several types that fit to this global control action. In some cases, the solvers tried to check their understanding by using the principle „Check the kind of the involved numbers to see whether you understand the problem” (G1). This means that they connected the correct understanding of the problem with the identification of the kind of numbers that might be (or not) a possible solution. For example, pair-3, in their early discussions while trying to correctly understand the arithmetical problem, made clear that any solution including decimal numbers must be considered as unacceptable. The problem asks for the number of students and therefore the answer must be a positive integer.

27-29 P3.1.A: We speak for students, so … We are not allowed to find decimal numbers.

Another principle in this global action was: „Restate the problem in your own words”. This refers to the well-known suggestion of Polya (1973) that „the verbal statement of the problem must be understood” (p.6). The solver “should be able to
state the problem fluently”. This was a quite frequent instance of control action in our study. Pair-4, in their effort to understand the arithmetical problem restated it in this way:

17-18 P4.1.A: The total number of students in a primary school is not a multiple of 3, 4, and 5 since each time we line up the students in rows of 3, 4, and 5, two of them are left.

The third way to ensure the correct understanding was to „Re-read the statement of the problem from time-to-time”. All the pairs took advantage of this action while solving both problems. When pair-3 got two answers for the arithmetical problem there was a doubt whether they understood correctly the problem:

130 P3.1.A: So, we made a mistake. We should find only one answer.

131 P3.2.A: Not necessarily. Let’s re-read the problem to check whether there is a clue for the existence of a unique answer.

Finally, another way to check the correct understanding was to „Draw a sketch or diagram to show connections and relationships to increase understanding”. There is an initial understanding and then a sketch is used to confirm or challenge this understanding. Trying to solve the arithmetical problem, one member of pair-5 drew a simple sketch to make sure that both solvers understand in the same way the problem (Fig. 3).

Figure 3. Draw a sketch to check your understanding

G2 – Errors’ detection

This type refers to the solvers’ efforts to check for the existence of errors in the intermediate results (thus not connected with the issue of verifying the final result) during the problem-solving process. Two types were identified in this global control action.

The first one is based on „Retracing the whole process step-by-step”. In this case, the solvers retrace the whole problem-solving process step by step and check each one of their steps to make sure they didn’t make any mistake. For instance, pair-3, decided to use Pick’s formula but they did not remember it. So, they decided to re-invent the formula. They worked on a series of lattice polygons and finally they came up with ‘the formula’. Then, they wanted to examine whether this formula is the correct one. So, they followed the same series of steps, using the same shapes, in the same order, which actually helped them to identify an error and correct the formula.
The second one is the „Re-execution of arithmetical operations”. The aim of the solvers was to ensure that the intermediate calculations were correct. For example, a member of pair-2 explained to his partner:

107  P2.2.G:  Are you sure?
108  P2.1.G:  Yes! It’s okay to find a decimal number
108  P2.1.G:  \[2 + 3 = 5, \ 5 \times 1 = 5, \ 5 / 2 = 2.5.\]
110  P2.1.G:  I just did it again to see whether we made a mistake.

**G3 – Satisfy the problem’s conditions**

The aim of this control action was to check whether the problems’ conditions are satisfied. The solvers decide their next step based on whether this decision is in alignment with the problem’s conditions. One of the conditions in the arithmetical task was that the answer must be a 3-digit number. Pair-1 examined whether certain numbers would be valid choices for the first digit. Based on the above-mentioned condition they decided that:

85  P1.1.A:  The first digit cannot be zero…. For the same problem, pair-4, based on the condition that if students are lined up in rows of five, two are left over, decided that:

153-154  P1.2.A:  Neither zero nor five are proper options for the last digit

**G4 – Check the reasonableness**

This action concerns the students’ engagement in checking the appropriateness and reasonableness or accuracy of their current work (argument, intermediate result, numerical values, measurements). The Common Core State Standards (National Governors Association, & Council of Chief State School Officers, 2010) give emphasis on the necessity for students being able to „Reflect on the reasonableness of their answers”, and „self-assess to see whether a strategy makes sense as they work, checking for reasonableness prior to getting the answer”. The relevant examples are drawn from the geometrical problem concerning the area of an irregular shape. Pair-3, trying to find the area of the shape got -199 square units as their final result. Checking the reasonableness of their result they soon realized that this was incorrect:

152-153  P3.2.G:  The area is minus 199 square units. What have we done? This is a negative area.

In the same spirit, pair-6, in order to calculate the area of the irregular shape, circumscribed a rectangle. Then, they found the area of each sub-shape that exists between the initial shape and the rectangle, added these areas, in order to subtract their sum from the area of the rectangle (Fig. 4). However, according to their calculations, the total area of the irregular shapes within the rectangle was bigger than
the area of the whole rectangle. Checking the reasonableness of their result they were able to realize that they made a mistake:

106-108 P6.2.G: The area of the shapes within the rectangle is bigger than the area of the rectangle. This is not reasonable.

Figure 4. The circumscription of a rectangle outside of the irregular shape

G5 – Verification

According to Eizenberg and Zaslavsky (2004), „verification is one aspect of metacognitive control skills” and is connected with Polya’s ‘Looking back’ final stage addressing the question „Can you check the result?”. So, to check the final result the participating pairs followed two different approaches.

The first one was to „solve again the problem using the same strategy”. The students repeated the same steps in the same order using the same problem-solving strategy. If this leads to the same answer, then the answer is correct. Otherwise, each step should be checked again separately. Pair-1 used the Pick’s formula to calculate the area of the irregular shape and then they wanted to verify the result. Their decision was:

234-235 P1.1.A: Let’s do the same again. Let’s count the dots one more time please.

The second approach was to „solve again the problem using an alternative strategy”. In this case the calculated result was verified through the use of an alternative strategy for solving the problem. If this alternative strategy leads to the same result then the answer is correct. So, the students apply strategically two problem-solving paths. The first one for solving the problem and the second for verification purposes. Pair-4 decided to use initially the Pick’s formula to calculate the area of the irregular shape. Then, in order to verify the result, they decided to divide the shape to sub-
shapes. They calculated the partial areas of these shapes and checked their initial result through the sum of these partial areas:

115-119  P4.2.G:  This second method was the verification. So, the area is 245.5 square units… We measured the area of the smaller shapes, added them and got the same number.

**Context-based control actions**

These actions are exclusively connected to the specific problem since it is the content of the task that provokes them. Consequently, such local actions were identified separately for each problem (Fig. 5). More specifically, two types of actions were identified for each problem.

**Arithmetical problem**

*CA1 – Systematically exhaust cases*  
The first control action for the arithmetical problem is to systematically exhaust all the possible cases. Based on one of the problem’s conditions the pairs wanted to form all the possible triads of digits that have the sum equal to 5. This could be done by forming these triads in a random way asking only for the sum of the digits. However, such an approach cannot guarantee that the solver will be able to find all the possible
triads. Therefore, the question for the pairs was to find a way of systematically working on triads so as to make sure they did not miss any triad. This made them move towards a systematic way of controlling the way the digits can be combined. So, they decided to start with the smallest digit (i.e., 1) in the first place (Fig.6, left). They kept it unaltered asking for the remaining two digits that would have the sum equal to 4 (so as to meet the demand for three digits that have the sum equal to 5). Then, the next choice for the first place was number 2, and so on. This resulted to a list of all the possible triads. Some pairs chose to work on the reverse order, starting with the biggest possible digit in the first place (i.e., 5) (Fig.6, right). This also ensured that all the possible triads were considered. The main aim of this control action was to check that all the possible triads have been examined.

Figure 6. Systematic working using a list

**CA2 – Divisibility rules**

The second context-based control action for the arithmetical problem was the use of the divisibility rules in order to check possible solutions. These rules determined the decision to keep or abandon a possible solution. This can be seen clearly in the discussion between the members of pair-4. They considered number 104 as one of the possible solutions for the arithmetical problem:

<table>
<thead>
<tr>
<th>48-49</th>
<th>P4.1.A:</th>
<th>It could be 104. The sum of its digits is 5.</th>
</tr>
</thead>
<tbody>
<tr>
<td>50-51</td>
<td>P4.2.A:</td>
<td>Yes. But, the last two digits form a 2-digit number that is divisible by 4, which means that the whole number is divided by 4. However, the number we ask for cannot be a multiple of 4</td>
</tr>
<tr>
<td>52</td>
<td>P4.1.A:</td>
<td>You are right</td>
</tr>
</tbody>
</table>
**Geometrical problem**

**CG1 – Ensure numbers/shapes**

For the geometrical problem, the first task-dependent control action was to ensure numbers or shapes. The use of the Pick’s formula needs both the number of the lattice points that are on the boundary of the shape and those that are in its interior. The solvers had to make sure that all these points had been considered. This refers to actions aiming to control that all the boundary or interior points have been considered without missing or duplicating any of them. Therefore, the participating pairs took actions such as the use of their own marks (circles, lines, different colors, and numbers) to indicate that a specific point had been considered (Fig. 7).

![Figure 7. Ensure numbers](image1)

In the same spirit, when the solution was based on the division of the initial shape to sub-shapes, the pairs had to be sure that they considered all the participating shapes for calculating the total area also without missing or duplicating any shape. In this case they used numbers to indicate that the specific sub-shape had been considered (Fig. 8). This use of numbers in their worksheets constitutes a control action.

![Figure 8](image2)
CG2 – Formula confirmation

This control action is connected with the solvers’ aim to re-discover the Pick’s formula because they wanted to use it, but they did not remember it. So, their first step was to find the formula. This was not an easy step. They examined a series of simple shapes to extract the formula (Fig. 9).

This was a process of problem-solving within problem-solving. They left aside the initial problem to work with this new one. Their effort resulted to a formula and then they wanted to check whether this formula was the correct one. They decided to use random shapes to check the strength of their formula (Fig. 10).

They were able to calculate the area of these shapes (for example using the grid). Then they applied their formula for the same shape. If the two results were identical, this meant that the formula was correct. This decision to use random shapes to check the validity of the formula was actually a control action at the local level of the geometrical problem. This is evident in the work of pair-3:

226  P3.2.G: If the formula is correct, it means that we can find the area of any shape.

227  P3.1.G: Yes

228  P3.2.G: Make a random shape quickly

What triggers control actions?

The analysis of the transcribed protocols gave evidence about the possibility for some factors that trigger control actions. We were able to identify four of them:
• The existence of multiple solutions.
• An unreasonable result
• Both problem-solving and verification processes lead to different answers.
• The collaborative nature of problem-solving.

The existence of multiple solutions
The successful solution of the arithmetical problem results in two solutions. The required number of students is 122 or 302. Both are correct answers since they satisfy all the conditions of the problem. However, it was not easy for most of the pairs to accept two solutions for the same problem. The extracts below are representative of this difficulty:

130 P3.1.A: We rather made a mistake since we had to find only one answer.
129 P5.1.A: Now we must find out which one (of the two) is the correct answer
93 P6.2.A: If we could have only one number our solution would be complete

This difficulty in accepting more than one solutions made them either looking back again to their problem-solving path to check for possible mistakes or using an alternative approach to see whether their multiple answers make sense.

An unreasonable result
Another factor that triggered control actions was the emergence of an unreasonable result. The negative area in the geometrical problem as already mentioned earlier was an example of an unreasonable result that triggered control actions. Another example was the result 100.25 for the population of the school in the arithmetical problem. The answer to this problem cannot be a decimal number. By realizing the unreasonableness of this result the students turned towards checking their argumentation to see what was the reason that misled them.

119-120 P1.1.A: We cannot have 100 point something… when talking about the number of students.

Problem-solving vs. verification
The third factor that appeared to trigger control actions was when the problem-solving and verification processes resulted in different answers. In this case, the solvers were led to check their results to resolve the discrepancy. This was noticed in two slightly different ways as it has already been mentioned in the subsection about verification. In the first, the solvers used the same method twice. Initially to solve the problem, and then they followed the very same process step-by-step to check the correctness of the result. In the second, the solvers chose two different approaches, one to solve the problem and one to check the correctness of the result. In both cases,
any disagreement between the two results was the reason for undertaking control actions to decide how to proceed.

The collaborative nature of problem-solving

Perhaps the most powerful factor that triggered control actions was the collaborative problem-solving in itself. During this collaborative work, "partners attend to both their own and the other’s knowledge, actions and emotions through invitational questions, evaluations, and responses" (Chiu, Jones & Jones, 2013). Usually, one of the members suggested the next step. Then, his/her partner was wondering why this was the best choice, or whether this was the right step. So, questions, such as "What do you think?", "Is that right?" arose. These questions aimed to monitor and control their individual effort to solve the problem as well as to make explicit their mutual understanding, to make each one participate and pay attention to the other’s actions and thoughts. In the 130.P3.1.A-131.P3.2.A extract it was the objection of the second student ("Not necessarily") and his prompt ("Let’s re-read the problem to check whether there is a clue") that led to this specific action. In this case a proposal ("We made a mistake. We should find only one answer") was aborted due to the doubt of the co-solver. Similarly, in the 234-235. P4.2.G excerpt, the pair repeated the same steps in the same order to ensure that the result is correct. This control action was the consequence of the prompt made by one of the students ("Let’s do the same again. Let’s count the dots one more time please"). So, in this case a proposal for a control action was initiated by one member of the pair and was approved by the other.

CONCLUSION

We know that metacognitive control has been mainly, but not exclusively, examined from the perspective of the individual, rather than the collective one. Examining control actions during collaboratively problem solving, gave evidence about the way pairs divided complicated tasks into sub-tasks, allocated these sub-tasks appropriately in order to distribute the demand in cognitive and metacognitive level. For example, in their effort to solve the given problem they posed a new one, i.e., the problem of finding the Pick’s formula which was a necessary tool for their solving approach. The social metacognitive control sometimes increased the transparency of the used argumentation and this is why we made an effort to distinguish two general types of such control actions. On the one hand, there are control actions that have a universal character and can be applied potentially to any problem. On the other hand, there are actions connected exclusively with the specific tasks and cannot be applied in other tasks, unless they are similar to the given one. In addition to these, we observed four certain factors that seem to trigger control actions. These are the existence of multiple solution for a problem, the emergence of an unreasonable result, the case that the problem-solving process and the verification result in different answers and the collaborative problem solving itself.
REFERENCES


