MEANINGS AROUND ANGLE WITH DIGITAL MEDIA DESIGNED TO SUPPORT CREATIVE MATHEMATICAL THINKING

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In this study two groups of Grade-8 students interact with a new expressive digital medium, experimenting with the concept of angles through a "tool-shaping" process. The medium, designed to foster students' Creative Mathematical Thinking (CMT), provides a novel set of affordances that are studied under the focus of a meaning-generation process. The study indicates that the students can arrive to mathematical meaning that enriches the more abstract understanding of angles, while at the same time improving upon certain aspects of CMT.

INTRODUCTION

Creative Mathematical Thinking (CMT) possesses a central role in the research in mathematics education. However, there is no consensus between the researchers as far as its definition is concerned. It can be seen as a product or process, general or domain-specific ability, situated within the 'genius' approach, or the problem solving and posing approach, or lately, in the so-called approach of 'techno-mathematical literacies' (Noss & Hoyles, 2013). Only the last one addresses the use and role of digital media for CMT. But, as Healy and Kynigos (2010) noticed, the development of CMT with the use of exploratory and expressive digital media has rarely been centrally addressed in providing users with an access to and a potential for creative engagements in meaning-generation activities. According to Ruthven (2008) the uses of these media is mainly instrumented towards contexts of traditional lecturing and demonstration of exercise solutions, which may not be characterized as learning environments that provoke exploration and dense construction of mathematical meanings by students Consequently students are not supported to develop CMT. Our paper studies the impact of such media in the development of students' CMT in conjunction with the generation of mathematical meaning by the students. We call this new kind of mediation 'c-book', ('c' for creativity) which is designed to afford CMT to the end users. The paper therefore focuses on the question: To what extend does this medium foster the meaning-making process? Are there indicators that aspects of CMT emerge during the meaning-making process?

THEORETICAL FRAMEWORK

Given the diverse approaches to CMT outlined above, and the relative lack of connection to math activity with expressive digital media, the concept remains fuzzy in the literature. In this study, which is part of a broader one (Papadopoulos et al, 2015),

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CMT is matched with: (i)‘construction’ of math ideas or objects which is in accordance to the constructionism that sees CMT being expressed through exploration, modification and creation of digital artefacts (Daskolia & Kynigos, 2012), (ii) Fluency (as many answers as possible) and Flexibility (different solutions/strategies for the same problem) (both seen as characteristics of a creative mathematical process in the literature, see for example Leikin & Lev, 2007), (iii) novelty/originality (Liljedahl & Sriraman, 2006) which is related with new/unusual/unexpected ways of applying mathematical knowledge in posing and solving problems, not easily met in students’ solutions (Vale et al., 2012), and (iv) usability/applicability (Stenberg & Lubart, 2000) through associations between different mathematical areas or between mathematics and other scientific fields and through elaboration which extends the (personal) body of knowledge via formulating new questions, making and checking conjectures, generalizing mathematical content, and reflecting on the mathematical work that takes place.

On the other hand, students’ engagement with expressive media provides rich opportunity for making appropriate mathematical meaning (Kynigos & Psycharis, 2003). Microworlds are such environments, allowing at the same time creativity –in our case CMT, customization and personal construction of tools (Healy & Kynigos, 2010). C-books exploit half-baked microworlds (Kynigos, 2007) which are incomplete by design, challenging students to explore the reason for the buggy behavior they show, and foster learning through tinkering. To understand the process of making meaning in this context, the instrumental approach (Verillon & Rabardel, 1995) as a ‘tool shaping’ procedure seems a useful theoretical tool which refers to how the affordances of an artefact, are adjusted by the student in order to be used as a tool for specific reasons.

THE DIGITAL MEDIUM

The C-book technology

C-book is a new expressive medium that affords the design and use of modules named c-book units. Each c-book-unit includes diverse “widgets” into the text and between the lines of the narrative, which a student can browse through, explore, experiment with, reconstruct and be actively involved in tasks and problems designed to promote CMT (Kynigos, 2015). The term ‘widget’ refers to objects, other than text, such as hyperlinks, videos and most importantly instances, or activities, from a broad range of digital tools in mathematics education such as Geogebra and MaLT2, a web-based Turtle Geometry environment which integrates Logo-mathematics symbolic notation with dynamic manipulation of 3D geometrical objects using sliders as variation tools. C-book also includes the “Workspace”, an asynchronous tool providing the interface for discussions organized in ‘trees’ (Fig.1).

The Don Quixote c-book unit

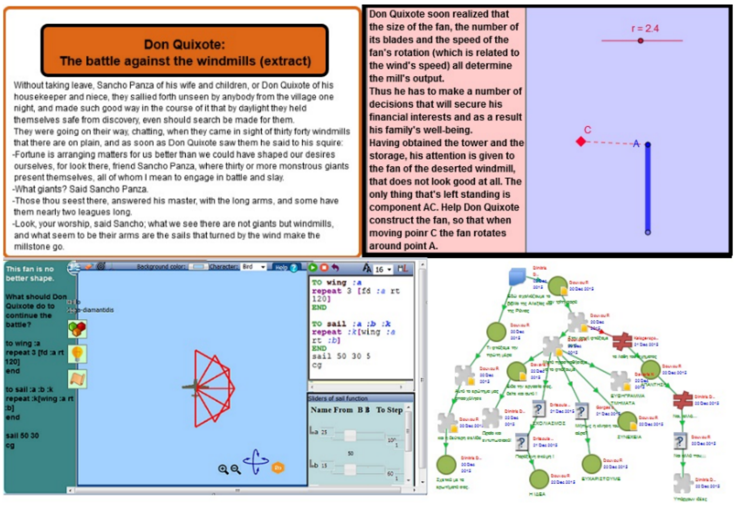

Figure 1: The C-book environment

The c-book unit used in this study presents a different twist of Don Quixote's story, agglomerated with a series of half-baked microworlds and other challenging tasks mostly in MaLT2 and Geogebra, in relation to the storyline. Its design aimed to provoke students to tinker and reconstruct windmills buggy by design, with many functionality issues. Even though the c-book technology affords a non-linear browsing of the c-book unit and engagement in any activity that the students find interesting it makes more sense to read and interact with the c-book unit starting from the beginning because of its narrative.

THE DESIGN OF THE STUDY

In the present study the methodological tool of "design experiments" (Collins et al., 2004) was used, designing and implementing an educational intervention in classroom and searching for relationships between the learning process and the use of digital media used by the students during the implementation phase.

Eighteen Grade-8 and six Grade-9 students of a public Experimental School in Athens, as well as two mathematics teachers and two researchers participated. The study took place in the school's pc lab during after-class Math Club activities (four sessions of two teaching hours each in approximately one month period). The students were divided into eleven groups of two and most of them were familiar with the usage of 2D E-slate Turtleworlds. Researchers took the role of 'participant observers' searching for students' interactions with the digital medium. The teachers' main role was to offer assistance in technical issues when required. Conversations between students or groups of students and their constructions on the screen constituted our data. This is why we used voice recorders and a screen-capture software (HyperCam2) to record students' interactions with the c-book unit tools. The data corpus was completed by the researchers' field notes.

The process of meaning-generation around angle of a group of students while engaged in the tinkering of two diverse widget instances of the c-book unit is presented. For the first one (Fig. 2) Logo code was developed in MaLT2 producing a quite abstract representation of an unfinished squared paper. Through the narrative, students were prompted to use it as a guide, in order to make a fan of a windmill like an origami made construction. The second one (Fig. 3) was about another half-baked logo code in MaLT2, were the stake was to shape up a windmill's fan by finding and fixing the bug.

Our hypothesis is that in a creative process (in terms of CMT) the students move from a static conceptualization to a more dynamic one, linking different angle aspects, through consecutively tinkering three diverse challenges of the same c-book unit.

RESULTS

A step towards conceptualization of angle, through elaboration of an artefact

Following the narrative linearly, the students initially had to address the first challenge, creating the fan of a windmill. It was easy for them to create the horizontal and vertical parts of the fan but they needed effort and systematic approach to overcome the difficulty of creating the oblique parts (in terms of angle and length) (Fig. 2).

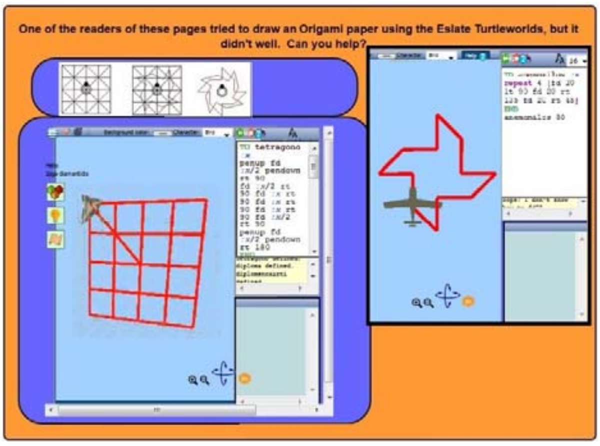

Figure 2: The 'squared paper' and the fan students constructed

To achieve their goal they constructed several angles in a more static context, where both arms of each angle were visible. Then they proceeded to the second task trying to shape up a windmill's fan by fixing the bug (Fig 3, left). This fan is much different not only as a geometrical figure, but as an abstract representation of a windmill as well. A line segment stands for the windmill's tower and the turn is represented by a variable in the code. By dragging a slider the user can make the fan rotating around a point. As students were observing the rotating figure, an original and unexpected idea came up. Instead of trying to fix the Logo code -as it was implicitly suggested in this task- they preferred to use the origami-made fan of the previous task.

Student1: What we want our fan to look like? How many blades should they be?

Student2: Let me go back… here look at the Origami code.

Student1: Let's use this figure as a fan.

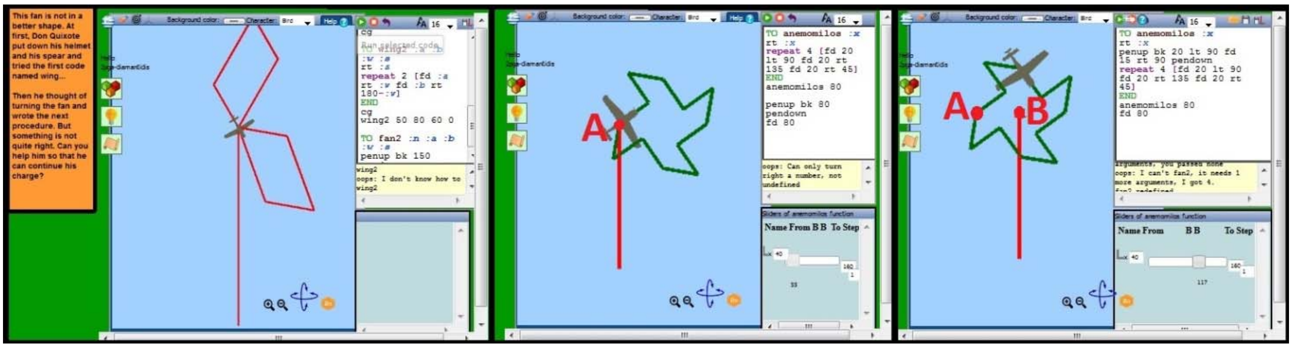

Figure 3: From the original (left) to their own elaborated artefact

This decision raised two extra issues. They had to not only reproduce the previous shape but at the same time to draw it exactly on the top of the 'tower', as well. However, by using their own code meant that the affordance of the slider for fan's rotation was no longer valid.

Student1: The 'name' of the slider that causes the rotation in the initial fan is x. So we have to use the variable x, to make our fan rotating.

So the students transferred the command line 'rt :x' (turning x degrees to the right), to the new code, just because of its usability. Thus, they changed the affordances of their original artefact, being able now to make it turn around a point, using a slider. However, their initial efforts resulted in a fan that turned around point A, instead of B (Fig.3, middle, right). This made them to focus more on the mathematical aspect of the fan:

Student2: It doesn't turn well. It should turn around this point! [*The point B*]

Student1: So we must identify the centre of the shape.

Student2: It is turning around this point [*A*], because A is the starting point [*She moves the slider x*]... Instead of going straight forward vertically, it turns right and then goes forward, drawing this line. As we move the slider it turns right by x degrees.

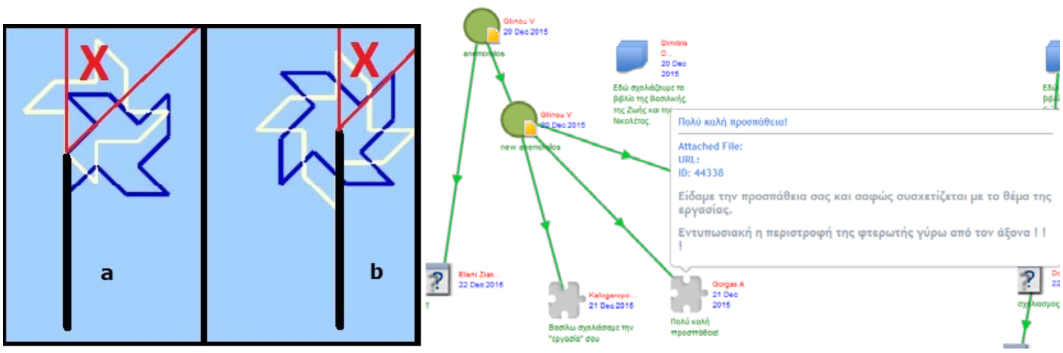

Figure 4: Constructing the new fan (left), comments on Workspace (right)

Actually, Student2 refers to an angle partly shaped. Only one arm of this angle is visible (Fig 4, left-a). The process of visualization is supported by dynamic manipulation–the dragging of the slider, while this angle is formed between the initial and the ending position of fan's same side. A development of angle's conceptual image takes place from a more physical representation (two arms visible) to a more abstract representation (a dynamic one, with only one arm visible) under the same context (the elaborated Logo-code of a fan used as a tool with more affordances). In the next phase this conceptualization moves another step forward.

The development of a more abstract concept

As they did not achieve their goal yet, students stressed their efforts in making the fan turn around point B (Fig. 3, right). Through tinkering with the Logo-code to change the position of the fan, they decided to add an extra movement of the aeroplane (character in MaLT2) right after executing the command 'right x' to ensure a new starting point A for the fan.

Student2: Can we move the plane without leaving its trace? [*Addressing to the teacher*]

Teacher1: Yes, use the command 'penup'.

Student1: Ok, so we can move the plane to start drawing from another point, in order to turn around the centre of the shape.

Their investigation resulted in a set of moving and turning commands to make the fan turn around point B. Both arms of angle x were now invisible (Fig. 4, left-b). However students refer to this angle to describe their construction.

Student1: Now the fan rotates around its centre.

Student2: Is it right? Is it really the centre?

Student1: Yes, it is. At least the angle's vertex is on the top of the windmill's tower.

This is indicative of a more abstract conceptualization of the angle, in the same context, reflecting on and elaborating the same artefact, to make it usable and appropriate to address these challenges of the c-book unit.

This rotating Origami-inspired fan was then posted in the 'Workspace' and commented by another group as '*An impressive rotation of the fan around its axis!*' (Fig. 4, right), since it was different from the work done by the other groups who preferred to work with the code suggested by the c-book unit.

DISCUSSION

The whole work done by the students can be seen through the two lenses of the meaning-generation process and aspects of CMT. In terms of meaning generation two levels in the students' actions can be identified. The first one is related to the instrumental aspect of their actions whereas the second highlights the evolution in the students' knowledge base about the concept of angle. The students seemed to 'carry' along with them this artefact, as a tool under consideration and development. As an artefact, 'windmills' constitute a central object of this c-book unit, something that motivated students to shift their viewpoint from a pure mathematical context to a more generic framework, using this artefacts again and again as a tool to address other tasks that refer to windmills. The progressive reflection on their constructions resulted to usable and appropriate tools that not only addressed the tasks in an effective way but also had a crucial impact on the process of meaning making. The students moved from a concrete static image about angle to a more abstract and dynamic one since they started talking about a specific angle given that they were not able to see its sides. In parallel with the meaning generation process, aspects of CMT were also apparent in the students' engagement with the c-book technology. The students' decision to abort the suggested fan and make their own is a construction inspired by the affordances offered by the c-book environment. Within this environment they also were supported by the available technology to exhibit an alternative solution (flexibility) to the problem of the design of the specific fan in a way that indicates originality. Originality can be judged on the basis of the low frequency since this was the less preferred approach (actually the only one) and the same time it was acknowledged as such by the class community (see the peer's comment in the Workspace).

So, on the one hand there is a new medium that provides a context and within this context the students work with activities that contain half-baked microworlds, change and/or fix them, connect the narrative with mathematics, make connections between the tasks. On the other hand, the combination of Constructionism, Flexibility and Originality, despite they come from different theoretical frameworks, constitutes a conceptual tool enabling researchers to better understand the students' CMT.

CONCLUSIONS

Given the lack of consensus about CMT in the Mathematics Education community it is important to look for conceptual tools that would enable us to understand CMT. In this paper it was evident that the affordances of the specific medium enabled processes of meaning generation and the presence of some aspects of CMT. In order to identify and understand the students' CMT a combination of three different theoretical constructs (constructionism, flexibility, and originality) was used. The decision to combine different theoretical frameworks seems to be a conceptual tool that contributes to our understanding of CMT and lessens the fuzziness around it. However, we need further research to develop more precise tools that will enable us to obtain a deeper understanding of CMT.

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