In this paper we follow three Grade 6 students trying to solve (at first individually, and then in a group) arithmetical and geometrical problems. The focus of the study is to identify and compare the various types of control actions taken during individual and collaborative problem-solving to show how the collective work enhances the range of the available control actions. At the same time the analysis of the findings give evidence about the impact of the collaborative problem-solving on the way the students can benefit in terms of aspects of social metacognition.

INTRODUCTION

Metacognition, use of metacognitive strategies and monitoring actions are core aspect of mathematical problem solving (Garofalo & Lester, 1985). Schoenfeld (1992) gives emphasis on the importance of metacognition in mathematical learning pointing out that the development of metacognitive actions helps solvers to have better performance and thus it is important in mathematical problem-solving. When students solve mathematical problems, they select to implement certain strategies, they pursue the right ones or recover from inappropriate choices, making thus decisions that determine the problem-solving success or failure (Schoenfeld, 1985). Executive control and decision making is a way to talk about metacognition in problem-solving. Both are concerned with the solver’s continuous evaluation of his current working in conjunction with his aim. Schoenfeld (1985) claims that task performance is positively correlated with the degree of one’s metaknowledge.

Given that most studies use to give emphasis on control actions taken by individuals, it is important that recent studies (Chiu, Jones, & Jones, 2013; Chiu & Kuo, 2009) make an effort to raise the importance of social metacognition during collaborative problem-solving.

In the spirit of this social metacognition the first purpose of the current study is to make a comparison between individual and collaborative problem-solving in order to make clear whether the control actions that emerge in the collaborative setting are due to the social nature of problem-solving or they are already possessed individually by the students who simply bring and use them in the collaborative settings. Our second aim is to examine the specific benefits that emerge in relation to social metacognition according to the model of Chiu and Kuo (2009).
TYPES OF CONTROL ACTIONS

During the last ProMath conference, Papadopoulos and Sekeroglou (2018) in their study examined the types of social metacognitive control actions during collaborative problem solving and organized them in two categories according to whether these actions are not depended on the task’s context (Global control actions) or they are closely related to it (Context-based actions). Global control actions (see Figure 1) can be applied to any problem no matter the context and/or the mathematical topic of the problem. Five such types of control actions were identified, and they can make an analogy to the notion of heuristics.

![Figure 1. Global control actions (Papadopoulos & Sekeroglou, 2018).](image)

The first type is related with the “understanding of the problem” and refers to actions aiming to control the correct understanding of the problem (students check the kind of numbers, restate the problem in their own words, re-read the statement, draw a sketch to show relationships and increase understanding). The second type refers to “errors’ detection” and can be of two kinds. The first one is based on ‘retracing the whole process step-by-step’ where students check each one of their steps to make sure they did not make any mistake. The second one is the ‘re-execution of arithmetical operations’ to ensure that the intermediate calculations
are correct. The third type is called “satisfy the problem’s conditions” and aims to check whether the problem’s conditions are satisfied. This will determine the solver’s next step. The next strategy refers to actions aiming to “check the reasonableness” (of an argument, a result, some numerical values), and the last one is the “verification of the final result” which can be of two kinds: ‘solve again the problem using the same strategy’ and ‘solve again the problem using an alternative strategy’. According to Papadopoulos and Sekeroglou (2018) this list of control actions is not exhaustive, and it is expected that future studies might add new types in this list.

Context-based actions are exclusively connected to the context of the specific task. It is the content of the tasks that triggers these actions, and this is why such actions are identified separately for each problem.

In this study our interest lies in the Global control actions.

**SOCIAL METACOGNITIVE CONTROL**

Metacognition has traditionally been understood as a person’s own knowledge about cognition and the regulation of cognitive processes (Brown, 1987; Flavell, 1976), giving emphasis on individual learning. However, learning is not merely an individual process. Many researchers highlight that social and cultural factors contribute to a more integrated type of learning (Lehtinen, 2003; Vauras, Salonen, & Kinnunen, 2008; Volet, Vauras, & Salonen, 2009). This explains why researchers involved in metacognition have gradually considered it as a process which is both individual and social in nature making obvious the need for broadening the traditional view of metacognition from individual processes to collaborative ones.

Social metacognition is an extension of metacognition into group interactions and consists of group members’ monitoring and control of one another's knowledge, emotions, and actions (Chiu & Kuo, 2009). According to Chiu et al. (2013) social metacognitive control refers to “monitoring and control the regulation and evaluation on others’ knowledge, linking thus metacognitive judgements with communication, and applying one’s subjective metacognitive experiences to a group context” (p. 74). Using social metacognitive control team members can understand, evaluate and build on one’s knowledge with respect and with purpose to create new and useful information that promote their problem-solving process. When mathematical problems are solved by groups rather than individuals, team members who can monitor and control other’s behaviors effectively, can increase their likelihood of solving difficult problems (Chiu et al., 2013). Additionally, it seems that social metacognition has both benefits and challenges (see Figure 2). It distributes metacognitive demands among group members, increases the visibility of one another’s metacognition, improves also individual cognition, promotes reciprocal scaffolding, and enhances motivation (Chiu et al., 2013; Chiu & Kuo, 2009).
<table>
<thead>
<tr>
<th>Dimension</th>
<th>Metacognition</th>
<th>Social metacognition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scaffolding</td>
<td>Self-scaffold</td>
<td>Reciprocal scaffold</td>
</tr>
<tr>
<td></td>
<td>● Set goals</td>
<td>● Recognize correct ideas</td>
</tr>
<tr>
<td></td>
<td>● Organize resources</td>
<td>● Detect flawed ideas</td>
</tr>
<tr>
<td></td>
<td>● Evaluate information</td>
<td>● Build shared knowledge</td>
</tr>
<tr>
<td></td>
<td>● Retrieve relevant information</td>
<td>● Expand understandings</td>
</tr>
<tr>
<td>Regulate emotions</td>
<td>Manage personal experiences</td>
<td>Motivate one another</td>
</tr>
<tr>
<td></td>
<td>● One’s beliefs affects views of one’s experiences by one’s belief systems</td>
<td>● Distributed risk of failure reduces individual risk</td>
</tr>
<tr>
<td></td>
<td>● Enhance motivation</td>
<td>● Aids emotional support</td>
</tr>
<tr>
<td>Resource demand</td>
<td>Distribute metacognitive demands</td>
<td></td>
</tr>
<tr>
<td></td>
<td>● More brain resources, more metacognitive resources</td>
<td></td>
</tr>
<tr>
<td></td>
<td>● Divide responsibilities</td>
<td>● Specialized roles according to each person’s strengths</td>
</tr>
<tr>
<td>Visibility</td>
<td>Attend to cognitive processes</td>
<td>Increase metacognition visibility</td>
</tr>
<tr>
<td></td>
<td>● Increase visibility of cognitive processes</td>
<td>● Social, public expression of metacognitive actions</td>
</tr>
<tr>
<td></td>
<td>● Increase visibility of their consequences</td>
<td>● Multiple sources of attention</td>
</tr>
<tr>
<td>Management</td>
<td>Manage one’s own cognition</td>
<td>Shared management improves individual cognition</td>
</tr>
<tr>
<td></td>
<td>● Coordinate own cognitive processes</td>
<td>● Others simultaneously monitor and evaluate</td>
</tr>
<tr>
<td></td>
<td></td>
<td>● Focus on a subset of the problem</td>
</tr>
<tr>
<td></td>
<td></td>
<td>● Reduce distractions</td>
</tr>
<tr>
<td></td>
<td></td>
<td>● Reduce mistakes</td>
</tr>
</tbody>
</table>

*Table 1.* Benefits of metacognition and social metacognition (Chiu & Kuo, 2009).

At collaborative settings each group member’s cognitive demand is reduced and the members directly interact with one another and share responsibilities.
Does collaborative problem-solving matter in primary school?

(Johnson, Johnson, & Smith, 2007). When collaborators understand one another’s skills, talents and capabilities, then they can apply social metacognitive control for decomposing a complex problem in sub-problems and distribute them to the group members according to their individual skills and strengths (Chiu & Pawlikowski, 2013). In more structured situations, each member can undertake a specific role such as the proposer of new ideas, critic, reporter etc. (Johnson et al., 2007). They also can focus on smaller, and more suitable for them activities and they will undertake less cognitive and metacognitive responsibilities. As a result, each solver has fewer possibilities to make mistakes and more to solve the sub-problem efficiently (Chiu & Kuo, 2009).

The second benefit is related to the greater visibility of cognitive and metacognitive processes of group members. When collaborators directly interact, they can capitalize on distributed processing. At minimum, multiple group members can observe, attend to, and evaluate one another’s work (Chiu et al., 2013; Chiu & Kuo, 2009). Moreover, distributed metacognition reduces the metacognitive demand on individuals, enabling them to focus on a subset of the problem, reducing distractions and mistakes, and specializing in their strengths. As a result, problem-solving efficiency is reinforced (Chiu & Kuo, 2009).

Furthermore, group members provide reciprocal scaffolding to the others. Reciprocal scaffolding refers to the type of questions, evaluations, repetitions, and elaborations that help students see their limitations, build a shared knowledge and expand their understandings.

The distribution of responsibilities shares also the risk of failure among the group members and provides emotional support to enhance motivation (Johnson et al., 2007). When collaborators position themselves together saying for example “we” instead of “I”, they highlight their shared situation and it is more likely to share responsibilities, risks, and rewards strengthening thus their sense of working together as a group (Chiu et al., 2013; Chiu & Kuo, 2009).

In these situations, the personal risk and the cost of failure are decreased. As a consequence, group members seem to feel less anxious, have a team spirit, and also a bigger motivation in order to work together for their group’s best (Chiu et al., 2013; Chiu & Kuo, 2009).

So, in this landscape, our research questions are:

(1) What (types of) control actions are identified in individual and collaborative mathematical problem-solving?

(2) What are the benefits of collaborative problem-solving in terms of social metacognition?
DESIGN OF THE STUDY

Sample of participants

Twelve Grade 6 students (7 boys and 5 girls) of two different public schools in a suburban area in Thessaloniki, Greece, participated in this study. Students were formed in four groups of three.

Table 2. The arithmetical problems for individual and collaborative problem solving.

<table>
<thead>
<tr>
<th>Individual</th>
<th>Collaborative</th>
</tr>
</thead>
<tbody>
<tr>
<td>If we line up the children of a camp in rows of four, one of them is left. If we place them in rows of five or six, one of them is also left. What is the number of children, given that this is: a 3-digit number less than 700, and the sum of its digits makes 10?&quot;</td>
<td>Solutions:</td>
</tr>
<tr>
<td>181</td>
<td></td>
</tr>
<tr>
<td>361</td>
<td></td>
</tr>
<tr>
<td>541</td>
<td></td>
</tr>
<tr>
<td>When George forms with his marbles groups of five, two of them are left. If he forms groups of seven or ten, two of them are also left. What is the number of marbles, given that this is: a 3-digit number greater than 200, and multiple of 3?&quot;</td>
<td>Solutions:</td>
</tr>
<tr>
<td>282</td>
<td></td>
</tr>
<tr>
<td>492</td>
<td></td>
</tr>
<tr>
<td>702</td>
<td></td>
</tr>
<tr>
<td>912</td>
<td></td>
</tr>
</tbody>
</table>

The criterion of their selection was their performance in mathematics which was – according to their teacher – above average and also their willingness to participate. Each triad was students from the same class and therefore this was the criterion for forming the specific groups since the students knew each other. The students worked initially individually. They were invited to solve individually an arithmetical and a geometrical problem. Then, on another day, in groups of three they were asked to solve again two problems that were of similar structure with the ones they solved individually. Some prefer to use the term “analogous problems” instead of problems with similar structure. In this paper we follow Pólya (1945/1973) who describes analogy as a sort of similarity implying that analogous problems have a similar structure to another problem. The specific characteristics and the relationships among the various problem features remain the same so much for both arithmetical problems as well as for the two geometrical ones. The students had as much time as they wanted for solving each problem.

For the purpose of this paper, the work of a group of 3 students will be presented here.
**Tasks**

In Table 2 the arithmetical problems for both the individual and collaborative problem-solving are presented.

It is not so easy for these problems to be solved but, at the same time, they cannot be considered as especially demanding ones. They are interesting for two reasons. On the one hand, their solutions combine a variety of mathematical concepts and/or approaches such as the use of Lowest Common Multiple and its multiples, the use the divisibility rules, all the possible combination of three digits that give a sum of 10, etc. On the other hand, they have more than one possible solution.

<table>
<thead>
<tr>
<th>Individual</th>
<th>Collaborative</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Individual Shape" /></td>
<td><img src="image2.png" alt="Collaborative Shape" /></td>
</tr>
</tbody>
</table>

Calculate the area of the shape (Answer: 71 sq. units)  
Calculate the area of the shape (Answer: 60 sq. units)

*Table 3*. The geometrical problems for individual and collaborative problem solving.

In Table 3 the geometrical problems for both the individual and collaborative problem solving are presented.

These problems are typical geometrical problems related to the concept of the area and they were chosen for the following two reasons. The first reason is that they...
are non-standard in the sense that the shapes are not-regular and therefore the students cannot rely on known formulas to calculate their area. Moreover, such problems are not included in students’ mathematical textbooks. Secondly, there is only one correct answer for each problem, but this answer can be approached in a variety of ways. For example, one could divide the initial shape to known sub-shapes. This means that the solver must find the area of the sub-shapes and then their sum makes the area of the whole shape. Another approach is to use the dot paper to count the complete square units and combine partial units to form complete ones. An alternative might be to circumscribe a rectangle around the polygon, calculate its area, find the area of the shapes that exist between the rectangle and the initial shape, then add their areas, and finally subtract this sum from the area of the rectangle.

Arithmetic and geometry are the two main areas of mathematics for primary school students. This is why we chose two problems one of each area as representative examples that could at the same time allow the exhibition of control actions that are global and not connected to a specific domain.

The fact that all of the above problems are open problems is a factor that triggers control actions and allows solvers to monitor and modify their problem-solving actions continually.

**Collecting and analysing data**

The students were also asked to vocalize their thoughts while performing the tasks in the spirit of the Thinking Aloud approach (Schoenfeld, 1985). The whole procedure was audio-recorded and then the students’ efforts were transcribed. Their worksheets and the transcribed protocols constituted the data of the study. The analysis of the data took place in two levels. Firstly, on identifying instances indicative of control actions for both individual and collaborative problem solving. These instances were organized according to the typology of Papadopoulos and Sekeroglou (2018). Secondly, on identifying instances indicating aspects of the benefits gained due to the social metacognition that were organized in the spirit of the Chiu and Kuo (2009) model.

**RESULTS**

Before presenting the types of control actions used by the students it would be helpful to present the problem-solving path followed by the groups while coping with the problems. Due to the limited number of pages it is not possible to present the individual problem-solving paths for each student.

For the solution of the arithmetical problem, the students made a list with 3-digit numbers that are multiple of 3 and checked which of them are divided by 5, 7 and 10. The solvers had in their mind that the last digit must be 2 because the number of the students must be multiple of 10 and 2 are left. After finding the first two possible answers (A1 and A2), students calculated their difference (A2 - A1 = d)
and calculated the next two answers using the formulae $A_3 = A_2 + d$ and $A_4 = A_3 + d$. They did not continue since they realised that the next answer ($A_5$) will be a 4-digit number. Then, and in order to verify their solutions, the students followed the same series of steps.

For the solution of the geometrical problem, solvers in the collaborative settings divided the shape to sub-shapes (known geometrical shapes). This means that the students had to find the area of the sub-shapes and then add these areas in order to find the area of the whole shape (see Figure 3 below). For the verification process, they used two alternative approaches to check the correctness of their solution. Initially, they used the dot paper in order to count square units (see Figure 2 below). Then, they applied the method of the circumscribed rectangle (see Figure 4 below).

**Types of global control actions: individual vs collaborative settings**

As it can be seen in Table 4, the range of the Global control actions (Papadopoulos & Sekeroglou, 2018) used by the students in the individual sessions is quite narrow. “Satisfy the problem’s condition” control action was the only one that was used by all the three of them. They wanted to check whether their possible solutions can be accepted as legitimate ones that meet the properties asked by the statement of the task.

<table>
<thead>
<tr>
<th>Control type</th>
<th>Students</th>
<th>Group</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S1</td>
<td>S2, S3</td>
</tr>
<tr>
<td>Understanding the problem</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Errors’ detection</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Satisfy the problem’s conditions</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Reasonableness</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Verification</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Effectiveness</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

*Table 4. Individual vs. collaborative global control strategies.*

For S1 this was actually and the only control action undertaken. Two of them (S2, S3) made an effort to check for errors in the intermediate steps (mainly calculations). Only one of them (S2) checked his understanding (for example, the last 2 digits cannot be 99 since their sum is greater than 10) and felt the need to verify the solution in the geometrical problem. No one thought to check the reasonableness of the result (for example the case of decimal numbers as potential solutions for the arithmetical problem). On the contrary, when the same students were engaged in collaborative problem-solving, they exhibited an impressively broad range of control activities. According to Table 4, in the collaborative
setting, instances of all the types of control actions have been identified. Interestingly, one of them (‘check for reasonableness’), took place only when students worked as a team (for example, the problem deals with children and therefore their number cannot be decimal). Finally, what is interesting is that indeed this list of types of control actions as presented by Papadopoulos and Sekeroglou (2018) is not exhaustive. During the collaborative problem-solving the students exhibited a new type of control action: “effectiveness” or ‘check whether the problem-solving path you followed is the most straightforward’.

**Collaborative problem-solving and Benefits of social metacognition**

Following the model of Chiu and Kuo (2009) for the benefits of social metacognition the collected data indicate that the social metacognition distributes metacognitive demands among group members, increases the visibility of one another’s metacognition, improves individual cognition, promotes reciprocal scaffolding, and enhances motivation.

**Distribute metacognitive demands**

The distribution of metacognitive demands refers to two partial aspects. On the one hand, during collaborative problem solving, group members reduce each peer’s cognitive demand interacting with one another and sharing responsibilities (Johnson et al., 2007). For example, during the arithmetical problem solving, S1 assigns certain parts of the problem-solving process to her peers.

S1: S2, check the 3-digit numbers that begin with 4, I will do those that begin with 5, and S3 those with 6.

On the other hand, the distribution of responsibilities is depended on each person’s skills and strengths. For instance, in her effort to calculate the area of the irregular shape on the basis of the square units on the dot paper, S1 realized that it was difficult for her to get the result through this strategy. So, appreciating the relevant skills of S2, she turned to him asking his help (see Figure 2).

S1: (Talking to S2) I cannot work with the complete and partial square units, but you can. Could you please do it?

*Figure 2. Working with the dot paper.*
So, each member of the group deals with fewer (meta)cognitive demands and thus the chance for making mistakes is reduced whereas the chance for solving correctly the sub-problems is bigger.

**Making metacognition visible**

Social metacognition increases, also, the visibility of one another’s metacognition (Chiu & Kuo, 2009). By distributing responsibility, the visibility of metacognitive processes is increased, and individual cognition is improved.

S1: The last digit must be 2 since the multiple of 10 should have 0 as its last digit and then 2 are left. Can it be 212?

S3: Why 212?

S1: Because if we divide it by 7, 5 and 10 the remainder is 2.

S3: Yes, but 212 is not divided by 3.

S2: (Talking to S3) You are right.

In this example which is drawn from the arithmetical problem-solving, S1 is wondering whether 212 could be a possible solution given that for each multiple of 10 the last digit is 0. Because 2 are left, this means that the last digit must be 2. The question of S3 “Why 212?” is an invitation for S1 to explain her thought “Because if we divide it by 7, 5 and 10 the remainder is 2”. The other two members attend her reasoning, express their metacognitive actions and reject the above number “Yes, but 212 is not divided by 3” and “You are right”.

When collaborators invite expression of one another’s ideas or share their own thoughts, they visibly express their cognitive and metacognitive processes. This greater visibility of cognitive and metacognitive processes seems to facilitate metacognitive evaluation recognizing thus correct ideas and detecting flaws.

**Improve individual cognition**

Improvement of individual cognition refers to allowing each group member taking responsibility in his strength and reducing distractions through the simultaneous monitoring and evaluation by the remaining members of the team. This is evident in Figure 3 below.

S2: 2, 4, 6, 8 (For sub-shape A).

S3: Yes, 8 (He also makes the calculation). And this is 6 (For sub-shape B).

S2: 2, 4, 6. Yes!

S3: Is this 1? (For sub-shape C).

S2: Let’s see.

S3: It’s 2 (he works visually).
S2: 4 divided by 2 equals 2. Yes! It is 2! *(use of the formula for the area of the triangle C, $4 \times 1 = 4$, $4 \div 2 = 2$)*.

*Figure 3. Working with splitting the shape to sub-shapes.*

The continuous mutual monitoring of the work between S2 and S3 results to an increase of their efficiency.

**Reciprocal scaffolding**

*Figure 4. Working with the circumscribed rectangle.*

It seems that during collaborative problem solving and through group member’s proposals, repetitions, and evaluations team members build a shared knowledge and also expand their understandings. The example below illustrates how such a shared knowledge was built in the specific group while solving the geometrical problem. More specifically, in order to verify the area of the irregular shape S1 proposes the method of the circumscribed rectangle (see Figure 4). This was the method he followed in the individual problem solving. S2 and S3 were not aware of this approach. Instead, in their individual attempts they preferred to divide the
irregular shape to sub-shapes and add their partial areas to find the total area. However, after S1’s proposal and explanation they accepted the new idea as an alternative correct approach and they proceeded to the verification of their answer using the method of the circumscribed rectangle.

The fact that S2 and S3 accepted S1’s proposal of circumscribing a rectangle as a correct one indicates a shared understanding (between the three students), which at the same time expands the understanding of S2 and S3 by adding new content in their tool bag of the available methods for calculating the area of irregular shapes.

**Greater motivation**

Finally, it seems that social metacognition has an impact on students’ motivation. As they share responsibilities they also share the risks of failure and its consequences which results in less anxiety about solving a problem (Johnson et al., 2007).

When the group decided to apply the specific strategy of the list of 3-digit numbers that end to 2 and use the difference between the first 2 answers (A2-A1) to find the remaining ones, S2 expressed his doubt.

S2: I am not sure this works.
S1: Let’s try it! We have nothing to lose!
S3: It’s not a bad idea!

This example is indicative of the willingness to take such risks and try things because of the feeling that the risk is shared. S1 gives emphasis on the fact that it is a common endeavour saying ‘WE’ have nothing to lose. This contributed positively and S2 continued to this direction.

**DISCUSSION AND CONCLUSIONS**

As the coda of this paper it would be interesting to give some feedback to the following question: “Do the students see any value in this collaborative problem-solving effort?” So, after each collaborative session, the students were asked to describe their feeling about the whole experience. The following statements reinforce the view that collaborative settings and social metacognition benefit students in their problem-solving activity.

S2: When one of us makes a mistake someone else detects the flawed idea. Actually, there is always a triple control.
S2: We found ourselves with an increased number of thoughts.
S1: More suggestions.
S3: One says something but three are thinking on this.

As far as our first research question is concerned it seems that collaborative problem solving facilitates the appearance of more global control actions than the
ones that take place during individual-problem solving. Moreover, it is important that the students exhibit new types of control actions (for example the control action of efficiency) making obvious that this list of control actions is not exhaustive. We hope that more control actions will be added gradually to this list.

Moreover, it seems that Social Metacognition facilitates a series of benefits for the members of the groups such as the distribution of metacognitive demands among group members, the increase of the visibility of one another’s metacognition, the improvement of individual cognition. These benefits result in reciprocal scaffolding and enhanced motivation.

What still remains open is the impact of this experience when the students come back to individual problem-solving. Will they be able to transfer the experience and knowledge gained in the social level to the individual one? This a challenge for a future research study.

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