GEOMETRY PROBLEM-SOLVING IN A COMPUTATIONAL ENVIRONMENT. ADVANTAGES AND RESERVATIONS

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Abstract

In Maths. Education tasks designed originally to target enrichment of a concept sometimes may be more appropriately regarded as problem-solving activities associated with the concept. This issue is particularly significant for tasks solving with computer environments and tools. In this paper I will try to illustrate these points in some tasks done dealing with area for primary school students.

Introduction.

A lot of research has dealt with the positive effects of the ICT (Information and Communication Technology) in the process of teaching and learning. They show that computers provide environment to test ideas and feedback, provide mirrors to mathematical thinking, encourage autonomy, link the general and the specific, link the symbolic to the visual, facilitate problem solving, serve as effective manipulatives (Clements, 2000). Computers also are the ideal gadget to make students independent from the teacher's direction (Bender, 1999), offer a new access to direct manipulation of geometrical drawings, offer open worlds in which they freely explore problem situations (Balacheff & Kaput, 1997). Finally the pupils focus on the relevant aspects of a problem and function at higher levels of geometric understanding (Glass, 2001).

But despite the positive effect concerning the contribution of technology to Geometry Problem-Solving, a more careful study of the results of relevant researches makes obvious a reservation that appears to exist in the research community with
regard to the suitable usage of technology in the Problem-Solving process. It seems also that tasks designed originally to target enrichment of a concept sometimes be more appropriately regarded as Problem-Solving activities associated with the concept. In such framework a study was conducted aiming at considering questions such as:

- If a task intended for conceptual enhancement is not carefully designed or it is a complex one, is it more likely to bring out associated Problem-Solving activities?
- To which extent should Problem-Solving activities associated to the concept be considered as integral or separate with the concept, and how much they should be studied in conjunction with the concept?
- How is this issue affected by a computational environment vs. a traditional one? That is: a) the way students confront such activities in both environments, b) the positive contribution of technology and c) the important skills and initiatives that the students may lose in a computational environment.

Generally, our data showed that when students solve problems, which are designed from the teacher-researcher to facilitate the acquisition of a certain concept, they are mostly engaged in the solving activities without necessarily drawing on the conceptual target. The accumulation of the experience of solving such problems may lead the student to the concept enhancement.

**Description of the Study.**

The tasks were presented to a primary school, computer aided population compared to one without computer. It was about pupils of the 5th grade. They already had been taught the concept of area and the formulas for the calculation of the area of
known shapes (triangles, squares etc.) The study lasted 2 weeks. The topic was the calculation and the comparison of areas of complex and not regular shapes. When we say “complex shapes” we mean the ones that can be divided in other shapes (triangles, squares etc) that are simpler and familiar to the children, while with the term “not regular” we mean the shapes that are complicated and cannot be split in other simpler (e.g. the outline of a lake or an island etc). The tasks we selected were non-standard.

The reason was that we wanted to avoid the simple retraction of known mechanisms for the estimation of area. Such tasks also do not require a lot of formal knowledge of geometry and they are liable to dynamic approach in a computer environment while simultaneously we have the opportunity to see how easily the students work in such environment (Kuchermann and Hoyles, 2003; Dugdale, 1998).

The problems required comparison of areas (Activities 1A and 1B), estimation of areas (Activities 3A, 3B and 4) or estimation of the whole area but also parts of it (Activity 2) (Figures 1, 2, 3). More specifically activity 2 shows the form of the courtyard of the school and the children are called to separate it so the ½ of it
becomes football region, 1/4 volley region, the 1/8 to play “apples” (a traditional Greek game) and 2 regions equal with to the 1/16 of courtyard for “skipping rope”.

We used three different applications (Cabri, GeoComputer, MSPaint). Depending on the task the students had to cope with, either they used only one of the softwares, or a combination of them. The choice of the software was ours and not the students’. Additionally we used the Camtasia Studio applications in order to record the activities of students in the computer screen (as *.avi files) so as to see later the effort of each student and to locate and analyze errors and strategies. Finally we used a video recorder to record the two populations working in their environment and a tape-recorder for the interviews taken after the completion of children’s work.

Positive Effect of Computing from the Problem-Solving Perspective.

The following results were obtained as we analyzed the positive effect of computing in our study from the Problem-Solving perspective:

- Students used the computer as a tool of verification, of confirmation, to control their conjectures and conclusions (Federica, 2002). Most formulations of a Problem-Solving framework are based on Polya’s stages of Problem-Solving (Polya, 1962). He called his final stage “looking back” and maybe it is the most important part of the Problem-Solving process. Students used the tool of automatic measurement in order to verify that the “white” triangle has the same area with the square unit. So they controlled the validity of their initial conjecture concerning this triangle. (Figure 4).

![Figure 4: Verification of estimation](image)
They did the same in activity 2. Some of them estimated the area of the whole courtyard and of its parts and then used the “Auto Measure” tool to realize whether their solution was “correct”. If it was such they would stop. Other wise they would keep on. Some of them kept on trying in order to discover and clarify their mistakes so as to find the final “correct” result. Others kept on trying in order to arrive in such result so as to be identical with the answer of computer’s authority. It is obvious here that these students’ activity didn’t function positively to the “conceptual” attainment. It appeared that for them it was important just to be sure that their solution was the correct one without this being necessarily connected to the concept.

An important technique in Problem-Solving process in area determination is the division of a complex shape to simpler pieces that are recombined to create a new shape of equal area but of different form through the actions of “cut”, “paste”, “rotate”, etc (Baturo, 2002; Battista, 1998). Such tasks designed for concept enhancement, in practice become exercises in Problem-Solving. The “cut and paste” method is a common activity that is used for young children concerning area. The conceptual content of this activity is summarized by “area is preserved under an action concerning a cut and paste”. The Problem-Solving aspect is exactly what cutting and pasting must be done in order to obtain a given shape from another...
(implying the areas of the two are the same). This process of analyzing and recombining can be realized in paper-pencil environment too. But when the task is a complex one, then students confront it with a sense of insecurity. For example, in activity 3, students had to estimate the area of a complex shape and then to transform it to a familiar one with the same area. The possibility to make the transformation mentally in the paper-pencil environment without the support of the computer appeared prohibitory because during their effort the children lost the control (which piece is transferred and where). The task brings out Problem-Solving activities associated with the concept. But students lost the initial focus. They focused on these activities missing thus the concept. The use of computer gave the following advantage: The students could visualize each choice of removing or even undo wrong choices through the possibilities of copy, paste, rotate, different colouring (Figure 5).

In this phase it is important that we gain all the “intermediary situations” of the transformations until the student reaches the final shape starting from the initial. Cutting, pasting, rotating, superimposing are natural procedures to attain “intermediary situations” (Owens & Clements, 1998). We collect thus a lot of information about every student’s strategy, which could be lost in the traditional environment of paper-pencil.

Finally in this phase, students produce copies of a given shape or of a part of it, which are either congruent with the original shape or not and which can be situated at any position and orientation wanted. So, the students had the possibility to compare areas or to realize whether
what appears to the eye equal or unequal, is really such. In activity 1, students placed the pieces they had to compare in the remaining part of the computer screen. They even put them with the same orientation in order to realize whether the shapes have the same area (Figure 6). Thus, one could say that this problem solving activity might function clearly in conjunction with the concept itself. It might contribute to the concept perception from the student. But what we saw was that, the student was simply trying to solve the problem (compare areas) and he was not thinking to conceive the concept.

- It is possible for the external circumstances to influence the student. Thus, students under the influence of technology give much more attention to the didactic objectives. Access to important mathematical ideas can be broadened to include a wider sample of the student population (Balacheff & Kaput, 1997). In our study, we observed that even the weakest students who did not usually participate during the traditional teaching, now spend the whole time of the session dealing with the tasks.

In addition, we had the opportunity to make the problem accessible to every student in the laboratory in the sense of having multiple correct solutions (Pandiscio, 2001). This appeared intensely in the activity 1 B. All the students who worked to the paper-pencil environment presented the same solution. That was the method of transpositions of partial square units to create a complete one. In the laboratory we recorded a variety of different approaches to the task and the choice of the method was often highly idiosyncratic. Some of them used the superimposition to compare the areas. Others separated every shape to subregions and then they transferred these subregions to the remaining part of the computer screen and thus ascertaining whether they had the same area.
Students had the possibility to make direct comparisons and to find relations between what they thought or what they expected as a result and what they saw in their screen. Thus visualizing the results of their activities on the screen, they could immediately react to them. In activity 3 A, a student initially coloured the incomplete squares, in both sides of the shape. Counting the whole square units and adding the remaining partial square units that fit one to each other, she proceeded to an initial estimation of the area. The result of her activity (Figure 7, left picture) leaded her to the thought that the right vertical part fits to the left one. So she transferred the right one and pasted it to the left. Now the new result leaded her to a new reaction. The new shape is a familiar one and it’s very easy to estimate its area. The later result was different from the former one. Thus the student corrected the answer of the problem accordingly to the result of the above procedure (Figure 7). On the contrary, students in the paper-pencil environment did not follow this sequence of steps. Students estimated initially the area. They observed the partial square units that fit one to each
other. They could not realize mentally the new shape that is the result of the transformation. Thus, they lost the opportunity to correct their wrong initial estimation (e.g. 27 square units instead of 25) (Figure 8). Thus, visualization became convenient only for students’ work in the lab through repetitive reactions to the results of their own activities.

After finishing their work, students had the possibility to save their work to allow further studying later. This is important from the teacher’s perspective too. We gained essential information concerning the strategies students used and the mistakes they made. The whole class had the chance to watch, locate and comment on these strategies and errors.

**Computing, Problem-Solving and Reservations.**

Our analyzing of the effect of computing in the study from the Problem-Solving perspective did not show only positive effects as the above mentioned ones. We came across a series of problems suggesting that the students might lose important skills or initiatives during the Problem-Solving process that they might have attained in a traditional (i.e. non-technological) environment.
Do they miss the problem because of erroneous computational operations? It is generally accepted that a way to improve students’ skills in Problem-Solving, is to record the types of mistakes they often do during the Problem-Solving process. But in our case students missed the problem despite the fact that they had in mind the correct way of solving the problem. This concerns activity 3A. One student in the laboratory configured in her mind the correct way for the solution. It was the observation that the right part fits to the left one. She tried to select the right one and then by copy and paste to transfer it to the right side of the shape. Because of her inability to execute correctly the required computational operations, finally she abandoned and turned to a different approach of the problem. Thus, this task led the student to a series of Problem-Solving activities associated with the concept of area. The student insisted to these activities and the danger to miss the problem was apparent. She overrode her initial target (that of the estimation of area) and focused on a series of activities.

Fig. 9: Computational mistakes

In another case (activity 4) a student was led to an incorrect result because of an erroneous measurement of the height of a triangle. Despite the fact that he knew the formula to estimate the area of the triangle, he used the tool of automatic distance
measurement in a wrong way so he substituted these wrong numbers on the formula (Figure 9).

- Has the computer been treated only as a confirmatory and recording tool and not as an exploratory one? The point is whether the software is used in a systematic way aiming at concept acquisition. In many cases we saw that students used the computer initially as a means of verification. In activity 2 some of the students solved the problem mentally (sometimes correctly, sometimes not). Then, in order to divide the initial shape to the specific parts, they used the tool of automatic area measurement and by the “trial and error” method they were expecting the computer to show them the number they had had in mind. In this case these students needed simply to confirm practically their experiential conjecture. It makes clear (as it is mentioned previously) that the students are missing their orientation and despite the fact that “split” is a typical Problem-Solving activity concerning area measurement they misses the problem in favor of these activities.

Some others used the computer as a recording device, much like paper and pencil, though perhaps neater and more easily edited (Dugdale, 1999). In activity 4, we observed that some students used the tool of distance or length measuring, the calculate tool to make some arithmetic operations and the text tool to keyboard comments. They did not use any tool of the software of substantial relevance to area calculation. However we should comment here that in the Greek educational reality, the integration of technology in education is at a very early stage. So these students had a limited experience and this could explain the behavior.

- Do they miss some good opportunities? Is the authority of the computer an obstacle for the students to construct their own explanation? Even though students had learned some techniques suitable for these computer-based tasks, they...
did not recognize their applicability when they confronted relevant tasks. For example, in activity 4, many students “saw” the triangle $\triangle AB\Gamma$ (Figure 9) as a right one and they handled it accordingly as such. Only one of them used the software, as a tool to make sure that this triangle is a right one. So, some good opportunities like this went unnoticed.

Additionally sometimes students relied on computer’s authority and thus they avoided constructing their own explanation. In activities 1A and 1B some students counted easily the whole square units. But they had trouble in counting the parts of the shape including the incomplete squares. Instead of trying to find a way to confront the situation they resorted to the automatic measurement via the software, without examining whether the computer’s answer is indeed the right one or without trying to understand why this answer is the right one. They lost the process of measurement and the opportunity to deal with units of measure in a meaningful way.

- **Do we set a limitation for students’ choices?** We already mentioned the fact that there exists the possibility for a student to lose the problem because of his/her inability to apply correctly a strategy in a computational environment despite the fact that this strategy is a correct one. Conversely there exists the possibility for a student to miss the problem because of the restrictions of the software’s capabilities. There is a limitation on student’s activities. He/she finally is not so free to select and to apply a strategy.

On the one hand, a student’s actions are affected by the well-known technical, psychological and social shortcomings of the software used. On the other hand behind each software exists the person who designed it and he did not have in mind the particular student or the particular situation in which the teacher decides to use it. For GeoComputer and in activity 1B, the software permits the handling only of shapes
that are defined from the dots of the grid. So it was impossible for the students to copy, paste or rotate incomplete square units of which vertexes did not coincide with the dots of the grid. It was an obvious method to obtain the solution but there was the above-mentioned limitation because of software’s capabilities. Cabri also does not permit to handle parts of a shape (i.e. cut a part of a shape and then copy or paste or color only this). This was the reason we used a combination of these softwares.

- **Do they lose valuable time?** There is a question whether in some cases activities in a technological environment could lead to a weakening of the learning. This question is based on the fact that the computer screen functions to “impel” the student in front of it. The screen constantly demands for action either with the mouse or the keyboard. We had some cases where students were forced to make illogical actions. No strategy was hidden behind these actions. They simply were playing with the software or they were drawing segments or grids without a logical structure. These activities could typically be characterized as Problem-Solving activities. But in this case they are not associated with the concept of area measurement.

Assuming that some concepts demand mental effort we conclude that these students wasted valuable time, which could be better dedicated to mental effort in a non-technological environment. But this is not the only way in which time may be wasted. The presupposition that using Dynamic Geometry solely facilitates the solution process is a trap (Gawlick, 2002). The use of technology itself as well as the students’ specific reactions to it causes unforeseen difficulties. One such difficulty is time inflation. In the case of the courtyard activity (Activity 2) many students did not finish their work on the task. The effort to materialize their strategies led them to consume a considerable amount of time so they had not the possibility to finish their
work during the specific session. When they were asked why they could not solve the problem, their answer was the same: “I didn’t have enough time to do it”.

Do they lose skills in using geometric tools? Finally it is worthwhile to state a reservation concerning the issue whether working in a computational environment leads to the loss of skill in using geometric tools reducing the breath of the understanding of the concept. For example, in the first courses in the beginning of the school year, the children dealt with the concept of height (e.g. what is height, how we draw it in the case of a triangle or a parallelogram, how we measure it, etc). Later in the topic of area, height was not anymore the aim but became the means in order to calculate the area of certain shapes. The suitable usage of geometric tools is connected with a series of concepts that are closely related to the concept of area (i.e. the concept of the right angle, perpendicularity, vertex, opposite side). The complete absence of the real geometric tools and their replacement by “virtual” ones supplied by the computer environment has as a consequences a disconnection from the above mentioned subconcepts thus reducing the understanding of the main concept. Students in the laboratory drew the height in activity 4 creating a simple segment line that connected a vertex of the triangle with the opposite side. This segment seemed perpendicular to the side. They did not use the Cabri’s tool “Perpendicular line” to draw it. They did not even check whether the angle between their “perpendicular” line and the side of the triangle was a right one (90°).

Conclusions.

Generally we may contend that tasks designed originally to target enrichment of a concept sometimes may be more appropriately regarded as Problem-Solving activities associated with the concept.
As it concerns the Problem-Solving aspect of activities related to the concept of area, our data strengthen our conjecture that if a task intended for conceptual enhancement is not carefully designed or it is a complex one, it is more likely to bring out associated Problem-Solving activities. When students solve problems, which are designed from the teacher-researcher to facilitate the acquisition of a certain concept, they are mostly engaged in the solving activities without necessarily drawing on the conceptual target. But the accumulation of the experience of solving such problems may lead the students to the concept enhancement.

All the above-mentioned are based in the preceded Pilot Study, which has become the base for the design of the main research, which already is in progress. Only a group of students participated in the Pilot Study while many more groups of students are participating in the main research. Recently, we finished the training phase of these groups concerning the usage of Cabri. It was here where our data showed that the accumulation of the experience of solving such problems might lead the students to the concept enhancement. In this phase we worked with a shape that the students had been taught at the beginning of the school year. It was about trapezoid.

We asked the students to construct a trapezoid. The constructions of the groups without experience on Problem-Solving activities had been limited to the drawing of an accidental quadrilateral so as to look like a trapezoid. But when they tried to handle it in a dynamic way (dragging), it stopped being alike a trapezoid. Reversely, the Pilot Study’s group made a choice that strengthens our conjecture concerning the contribution of the Problem-Solving activities in a long-term period. Some students of this group constructed 2 parallel lines using Cabri’s possibilities in a suitable manner. They chose 2 points on each line, they “hid” the lines and then drew the quadrilateral
that is the trapezoid. They handled it dynamically. So they ascertained that it remains always a trapezoid because of the maintenance of the “parallel” of the two bases.

Finally as the question of how this issue is affected by a computational environment vs. a traditional one is concerned, we saw the positive contribution of the technology. The computer provided representations enhancing concept formation. It enhanced:

- The capability of “looking back” after the task is finished.
- The visualization of the results of students’ strategies on the computer screen so they can react to their decisions.
- The possibility of multiple correct solutions for the same problem.

Additionally we saw that the analysis of the results of the computing from the Problem-Solving perspective set a series of reservations suggesting that the students might lose important skills or initiatives during the Problem-Solving process that they might have attained in a traditional environment. This analysis brought to light indications strengthening reservations like:

- The possibility of failing to answer the problems because of weaknesses in applying a correct strategy.
- The usage of the computer only as a confirmatory and not as an exploratory one.
- The situation where the computer screen impels the students to make illogical actions, thus wasting time, which could be better dedicated to mental effort.
- The fact that students rely on the computer’s authority and they don’t construct their own interpretations.
- Finally, the issue whether the work in a computational environment leads to the loss of skills in using geometric tools.
References


Bender Peter (1999). Some Paradigms of Mathematics Education - And how the work with computers is related to them (with particular consideration of geometry teaching). *GDM, 7*-19.


