FRACTURE

• Fracture Modes

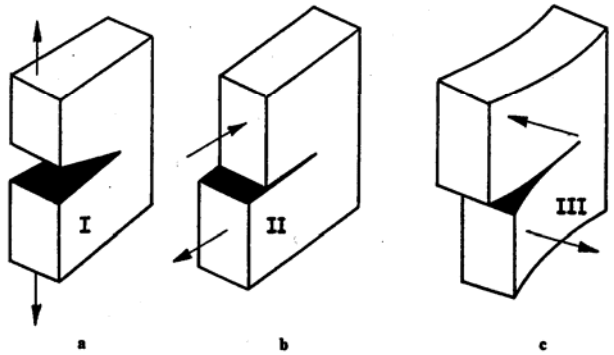


Figure 3.26 The three modes of fracture: (a) Mode I: opening mode; (b) Mode II: sliding mode; (c) Mode III: tearing mode.

• Mode I

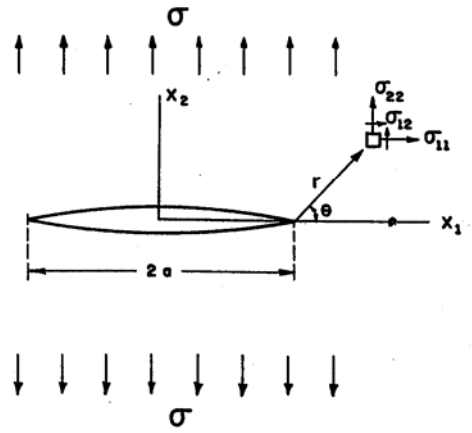


Figure 3.27 Infinite, homogeneous, and elastic plate containing a through-the-thickness central crack of length $2a$, subjected to a tensile stress σ.

Mode I:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \begin{bmatrix} 1 - \sin\frac{\theta}{2} & \sin\frac{3\theta}{2} \\ 1 + \sin\frac{\theta}{2} & \sin\frac{3\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{3\theta}{2} \end{bmatrix}$$

$$\sigma_{13} = \sigma_{23} = 0$$

$$\sigma_{33} = 0 \qquad \text{plane stress}$$

$$\sigma_{33} = \nu(\sigma_{11} + \sigma_{22}) \qquad \text{plane strain}$$

Mode II:

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{bmatrix} = \frac{K_{II}}{\sqrt{2\pi r}} \begin{bmatrix} -\sin\frac{\theta}{2} & \left(2\cos\frac{\theta}{2} & \cos\frac{3\theta}{2}\right) \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} & \cos\frac{3\theta}{2} \\ \cos\frac{\theta}{2} & \left(1-\sin\frac{\theta}{2} & \sin\frac{3\theta}{2}\right) \end{bmatrix}$$

$$\sigma_{13} = \sigma_{23} = 0$$

$$\sigma_{33} = 0 \qquad \text{plane stress}$$

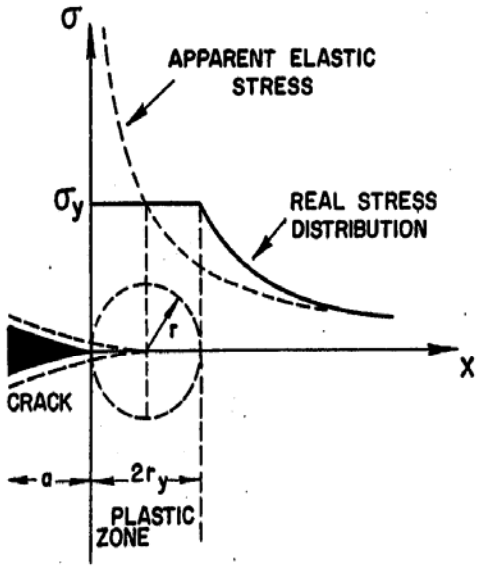
$$\sigma_{33} = \nu(\sigma_{11} + \sigma_{22}) \qquad \text{plane strain}$$

Mode III:

$$\begin{bmatrix}\sigma_{13}\\ \sigma_{23}\end{bmatrix} = \frac{K_{III}}{\sqrt{2\pi r}} \begin{bmatrix} -\sin\frac{\theta}{2} \\ \cos\frac{\theta}{2} \end{bmatrix}$$

$$\sigma_{11} = \sigma_{22} = \sigma_{33} = \sigma_{12} = 0$$

• Plastic Zone



$$(2a)_{\text{eff}} = 2(a + r_y)$$

$$\sigma_y = \frac{K}{\sqrt{2\pi r_y}}$$

$$r_y \approx \frac{1}{2\pi}\left(\frac{K}{\sigma_y}\right)^2 \qquad \text{plane stress}$$

$$r_y \approx \frac{1}{6\pi}\left(\frac{K}{\sigma_y}\right)^2 \qquad \text{plane strain}$$

Figure 3.29 Plastic zone correction (after Irwin). The effective crack length is $(a + r_y)$.