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CHROMATIC THIRD RELATIONS, SYMMETRICAL OCTAVE DIVISION AND PATHS IN PITCH SPACE: ANALYTICAL STUDY OF FRANZ LISZT'S IL PENSEROSO

INTRODUCTION

During the 19th century, a number of composers explored symmetrical organizations of tonal pitch space by its division to equal successive parts. The most frequent divisions were made with minor or major thirds and resulted in 4 or 3 equal parts, while divisions to 2 or 6 equal parts were very rare.

The reason was that only division by thirds offered the possibility of incorporation of the consequent chromaticism into the tonal system through chromatic mixture. In any case, the harmonic structure in these cases was at the limits of the diatonic tonal system, which is

1 A preliminary version of this article was presented (in Greek) at the Symposium “Music Theory and Analysis: Methodology and Practice” (School of Music Studies, Aristotle University of Thessaloniki, September 2006). The present complete version was presented at the 7th European Music Analysis Conference (VII EUROMAC), Rome, September 28 - October 02, 2011.

2 According to Aldwell - Schachter - Cadwallader, there are three levels of chromatic mixture: the first involves chord borrowing from the parallel major or minor mode (simple mixture), the second change of chord quality with unaltered chordal root (secondary mixture) and the third involves secondary mixture of a harmonic element yielded from basic mixture (double mixture) [ALDWELL - SCHACHTER - CADWALLADER 2011, 435-47, 590-596].
based on the asymmetrical division of the octave to perfect fifths and fourths.\(^3\)

Franz Liszt is considered one of the key composers of the 19\(^{th}\) century. His music not only represents most nineteenth century tendencies and norms, but also – especially in his mature songs and late works for piano – foresees the future evolution of music, through the use of symmetrical harmonic structures, loosening of tonal centres and questioning of the exclusiveness of the dominant to tonic fifth relation. The significance of symmetrical harmonic structures in Liszt’s mature works has been explored in theoretical/analytical work by Todd (1988), Skoumal (1994), Cinnamon (1986), Forte (1987), et al. These papers examine the works’ deeper harmonic structures and identify the importance of chromatic thirds relations and the weakening of the dominant-to-tonic diatonic fifth relation, while they project their conclusions mainly through schenkerian reductional diagrams or chord charts and references to dualistic harmonic theories of the 19\(^{th}\) century.

*Il Penseroso*, the piece analysed in the present paper, belongs to the second part (*Deuxième Année: Italie*) of the collection *Années de Pèlerinage* (Years of Pilgrimage, 1838-61, 2 versions) and conveys musically and emotionally the composer’s journey to Italy. The piece was composed in 1839, while Liszt was in Rome and San Rossore with Marie d'Agoult, and its programmatic title (The Thinker) refers to Michelangelo’s statue of Lorenzo de Medici in Florence. The piece is austere and contemplative, and contrary to Liszt’s typical style, it has no pianistic virtuoso effects. The piece was very special and intimate for the composer, since he used it again twenty years later as the basis for his piano piece *La Notte*, inspired by a Michelangelo quatrains (he even had expressed the wish that it should be played at his own funeral).\(^4\)

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3 As pointed out by music theorists, the asymmetrical division of the octave gives distinct structural personality and special functional role to each harmonic scale degree [e.g. Clough 1979, 45-61; Lerdahl 2001, 50-51].

4 *La Notte* is part of the *Trois Odes Funèbres* (1860-69). The *Trois Odes Funèbres* were composed after the death of his son Daniel, born while in Rome with Marie d’Agoult, 20 years earlier. The following quatrains by Michelangelo - entitled *La Notte* - is quoted in the score of the piece: «Grato m’è il sonno, e più l’esser di
Pensoeoso and La Notte have been studied comparatively and in relation to their symbolic meaning and the visual arts by Le Diagon-Jacquin's (2011). Il Penseroso has also been studied by DAMSCHRODER (1990) in relation to Beethoven's influence on Liszt and by EIGELDINGER (1996) regarding its relationship with Michelangelo's statue (see also SAFFLE 2004, 298, 356-357).

The present paper attempts — through the combined application of older and recent theories and analytical methodologies — a holistic (morphological, functional, structural and prolongational) analysis of Il Penseroso focusing on the harmonic relationships of chromatic thirds and the possible cognitive paths in diatonic or chromatic Tonal Pitch Space.

1. Theoretical issues

The theoretical background of the analysis involves four axes: Riemannian functional harmony, Neo-Riemannian transformations, Tonal Pitch Space and Reductive/Prolongational theory.

Theoretical research on chromatic third relations and symmetrical octave division has been active since the end of the 19th century, mainly in the Riemannian functional harmonic theory tradition [RIEMANN 1893; DE LA MOTTE 1976; KLUMPENHOUWER 2002], but it has recently been invigorated by newer theories of harmonic structure and/or formal function [IMIG 1970; LEWIN 1987; COHN 1996, 1998; HYER 1995; LERDAHL 1994, 2001; KOPP 2002].

Chromatic third relations were successfully described in Hugo Riemann's Functional Harmony (Vereinfachte Harmonielehre, 1893),

sasso./ Mentre che il danno e la vergogna dura,/ Non veder, non sentir m'è gran ventura./ Pero non mi destar, deb' — parla basso!» (I am thankful to sleep, and more thankful to be made of stone./ So long as injustice and shame remain on earth,/ I count it a blessing not to see or feel./ So do not wake me – speak softly!). For more information on the history of the composition of Il Penseroso and La Notte [HARWELL-CELENZA 2006, 18-20].
the most well-known dualistic theory of the 19th century. In this theory, all subsidiary chords (Nebenklänge, diatonic or chromatic) have a minor or major 3rd relation to the three basic chords (Hauptklänge: T, S, D), and are considered transformations of them. In W. Maler’s (Beitrag zur durmolltonalen Harmonielehre, 1957) and de la Motte’s (Harmonielehre, 1976) simplified functional harmony, for each basic chord there are Parallel Chords (Parallelklänge - minor 3rd root relationship), and Opposite Chords (Gegenklänge - major 3rd root relationship). Fig. 1 depicts all parallel and opposite chords of the chord pair C major-C minor. Third relations may be diatonic (1st grade, two common tones) or chromatic (2nd grade, one common tone or 3rd grade, no common tones) and may concern non-modulatory chord relations or modulatory region relations within an expanded chromatic tonality, capable of incorporating every major or minor chord. Functional harmony emerged as a powerful descriptonal and analytical tool.

1st grade relation (2 common tones): T-Tp, T-Tg, t-TP, t-tG;
2nd grade relation (1 common tone): T-TP, T-tG, T-TG, T-TP, t-kp, t-tg, t-Tg, t-tp;
3rd grade relation (no common tones): T-tg, t-TG.

The transformations are categorized into transpositions (Schritt) and inversions-exchanges (Wechsel).

Uppercase and lowercase letters denote major or minor chords respectively. In case of two-letter chords, the first one denotes the initial chord quality (major or minor) and the second one the resulting chord quality, e.g. tG is the major opposite chord of the minor tonic.
The transformational basis of Riemann’s theory has been recently re-discovered and exploited for the analysis of late 19th century music in the *neo-Riemannian Theory* [Proctor 1978; Krebs 1980; Lewin 1982, 1987; Hyer 1995; Cohn 1997, 1998; Kopp 2002]. The theory studies the transformational processes between chords independently from their initial functional dualistic framework. These processes are P, R, L, D (Parallel, Relative, Leading-tone exchange, Dominant)\(^7\) and their transformational result is described schematically in Fig. 2a, where each triangle represents a major or minor chord [Cohn 1998, 172]. The overall structure is symmetrical, constant and cyclic for each one of its dimensions (toroidal structure). The schematic structure is assembled from minor and major thirds (diagonally) and perfect fifths (horizontally). In this framework, all types of thirds or fifths relations can be described as simple combinations of transformations, regardless of harmonic functions; this process corresponds to the way many 19th century composers organized their chordal musical surface of regional harmonic background and justifies the chosen symmetrical structures. Systematic exploration of symmetrical tonal spaces and equal octave divisions by 3\(^{rd}\)s was made by Richard Cohn and one of his most important symmetrical structures is the hyper-hexatonic system (see diagram in Fig. 2b [Cohn 1996, 24]).

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\(^7\) Brief explanation of the four basic transformational processes (see Fig. 2): P (Parallel) transforms a major or minor chord to its parallel (e.g. C-E-G ↔ C-E♭-G), R (Relative) transforms a major or minor chord to its relative (minor 3\(^{rd}\) root relation, e.g. C-E♭-G ↔ Eb-G-B♭), L (Leading-tone exchange) transforms a major chord by moving its root one semitone down and a minor chord by moving its 5th one semitone up (major 3rd root relation, e.g. C-E♭-G ↔ A♭-C-E♭), D (Dominant) transposes a major or minor chord one perfect fifth down (e.g. C-E♭-G ↔ F-Ab-C).
Fig. 2a.

Neo-Riemannian transformations [COHN 1998, 172].

Fig. 2b.

Cohn’s hyper-hexatonic system [COHN 1996, 24].
An important step towards a possible cognitive explanation of the relations between chords and regions was made by Fred Lerdahl's *Tonal Pitch Space* (2001). The proposed model, represented mathematically by simple integer number formulas and geometrically by bi-dimensional tables, three-dimensional cones and four-dimensional toruses, can calculate the cognitive distances between the chords and tonal regions of a diatonic or chromatic tonal space. The application of the corresponding formulas can yield the distances between chords with third relation of all three grades in the diatonic space [Lerdahl 2001, 55, 60, 70].

For example:

\[ \delta(\text{C major} \rightarrow \text{A minor}) = \delta(I \rightarrow vi) = 7, \delta(\text{C major} \rightarrow \text{E minor}) = \delta(I \rightarrow iii) = 7 \]  

(1\text{st} grade third relation, 2 common tones, non-modulatory progression, R neo-Riemannian transformation),

\[ \delta(\text{C major} \rightarrow \text{A major}) = \delta(I/\text{I} \rightarrow V/\text{ii}) = 11 \]  

(2\text{nd} grade third relation, 1 common tone, non-modulatory progression, R+P neo-Riemannian transformation),

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8 The *Tonal Pitch Space* theory [Lerdahl 2001] evolved as an expansion of the *Generative Theory of Tonal Music* [Lerdahl - Jackendoff 1983]; its purpose was to provide explicit stability conditions and optimally clarified preference rules for the construction of GTTM's time-span reduction and prolongational reduction. The diatonic pitch space model is based largely on experimental research in music psychology, mainly by Krumhansl 1979, 1983, 1990 and Deutsch & Feroe 1981; it is also consistent with music theory models such as Weber's regional chart (1821-24), Riemann's *Tonnetz* (1893), Schoenberg's chart of regions (1954) and Lewin's neo-Riemannian transformations (1987). *Tonal Pitch Space* theory provides a model of tonal hierarchy, i.e. an atemporal and permanent knowledge schema expressed in simple algebraic formulae, which correlates spatial distance with intuitive musical distance and describes the cognitive distances between pitches, chords and regions in tonal music. Since the model provides quantification of the cognitive distances between the musical events of the analysis' reductional levels, it contributes to the formation of important preference rules regarding time-span stability and prolongational connection.
\[ \delta(\text{C major} \rightarrow \text{A}^b \text{ minor}) = \delta(\text{I/III} \rightarrow \text{i/vi/b}) = 23 \text{ (3rd grade third relation, no common tones, modulatory progression, P+L+P neo-Riemannian transformation).} \]

Particularly interesting for the present paper is TPS theory in regard to symmetrical Pitch Spaces [\textit{ibid.}, 249-297]. The distances that the model yields for the chords and regions of octatonic and hexatonic spaces is unexpectedly small (5 for octatonic and 4 for hexatonic regions), a result that indicates the structural affinity between regions that are one minor or major third apart, when they are perceived and studied outside the strictly tonal harmonic framework (see Fig. 3 [\textit{ibid.}, 257, 262]). Tonal Pitch Space Theory is essentially in congruence with neo-Riemannian Theory, with the former explicating the structures and transformations of the latter, while at the same time taking into account chordal roots, regional relations and diatonic vs chromatic hierarchy, aspects largely ignored in neo-Riemannian analyses.

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9 \textit{Chord Distance} (\( \delta \)) is defined by the TPS Theory as the cognitive distance between two chords \( x, y \) in tonal space. Its algebraic formula is \( \delta(x \rightarrow y) = i + j + k \), where \( i = \text{the regional key signature difference expressed in steps through the circle of 5ths} \), \( j = \text{the number of 5ths steps needed to shift} \ x \ \text{into} \ y \), and \( k = \text{the number of distinctive pcs in the basic space of} \ y \ \text{compared to those in the basic space of} \ x \) [\text{Lerdahl} 2001, 60]. As a reference for the following calculations, we cite the distance between the tonic and the dominant in C major: \( \delta(\text{C major} \rightarrow \text{G major}) = \delta(\text{I} \rightarrow \text{V}) = i + j + k = 0 + 1 + 4 = 5 \). Note that normal type letters denote chords and bold type letters denote tonal regions. Also, capital or low-case latin numerals denote major or minor chords/regions respectively.
Fig. 3.

Octatonic and Hexatonic Space with the cognitive distances between regions according to the TPS model.
Examining the issue through a prolongational perspective (according to CINNAMON [1986, 3]), equal octave division by thirds represents a type of prolongation in which the foreground harmonic relationships that generate it are not referential to the middleground harmony being prolonged, resulting in a fusion of contrapuntal and harmonic procedures: contrapuntal in terms of the harmony being prolonged, harmonic in terms of the more foreground chordal successions within. This division refers to tonal regions (modulatory) or to chromatic chords (non-modulatory). A feature of the modulatory operation is the discontinuity of voice-leading between the elements of the procedure, something that does not occur during the non-modulatory operation, where voice-leading creates a new tonal hierarchy between structural and intermediate chords (see Fig. 4).

![Fig. 4. Modulatory and non-modulatory equal division of pitch space by major 3\textsuperscript{rd}s [CINNAMON 1986, 4].](image)

2. Analysis

2.1. Form and motivic structure

The piece consists of the following parts: A (b. 1-22), A' (b. 23-39) and Coda (b. 40-48). The overall form is extended binary (parts A and A' can be considered as antecedent-consequent of a macro-binary form), with parallel parts A and A' differing at the cadence type (half cadence at the dominant for A – full cadence at the tonic for A') and at the rhythmical content of the left-hand accompaniment (homophonic...
chords in A, continuous 8-note step movement in A'). Parts A and A' contain phrases of different size, that end at full or half cadence: A1 (b. 1-4, full cadence in C# minor), A2 (b. 5-8, full cadence in E minor), A3 (b. 9-13, full cadence in G minor), A4 (b. 14-22, half cadence in C# minor),\(^{10}\) A'1 (b. 23-26, full cadence in C# minor) and A'2 (b. 27-39, full authentic cadence in C# minor). An important distinguishing aspect between the two parts is the large 13-bar consequent phrase (A'2) in part A', in the place of which there are three smaller phrases (4-bar A2, 5-bar A3 and 8-bar A4) in part A. In both parts the initial antecedent phrase (A1 or A'1) remains essentially the same (differing only at the accompaniment). Schematically:

\[
\begin{align*}
A & : A1 \text{ (b. 1-4, full cadence in C# minor)} \\
 & : A2 \text{ (b. 5-8, full cadence in E minor)} \\
 & : A3 \text{ (b. 9-13, full cadence in G minor)} \\
 & : A4 \text{ (b. 14-22, half cadence in C# minor)} \\
A' & : A'1 \text{ (b. 23-26, full cadence in C# minor)} \\
 & : A'2 \text{ (b. 27-39, full authentic cadence in C# minor)} \\
Coda & : \text{(b. 40-48)}
\end{align*}
\]

Parallelism between parts A and A' is enhanced by the exclusive use of one basic motive, from which all melodic phrases originate (the only exception being the last bars of phrase A'2). This basic motive features dotted 8\textsuperscript{th}-notes and double-dotted quarter notes, something that gives a funeral march character to the piece.\(^{11}\) The motive is encountered in two main forms (in various minor extensions and transformations, according

\(^{10}\) The half cadence occurs at the first beat of b. 21. The rest of b. 21 up to the end of b. 22 is a prolongation of the dominant that prepares the second part of the extended binary form.

\(^{11}\) An obsession with death as a symbol or programmatic theme can been observed in many of Liszt's mature works of his last creative period [WALKER 2001, 782-783].
to the length of the phrases): the antecedent type (in the onset of phrases)

\[\text{\includegraphics{antecedent_type}}\]

and the consequent type (at the end of phrases):

\[\text{\includegraphics{consequent_type}}\]

2.2. Rhythmic reduction of phrases: Harmonic analysis\(^{12}\)

Phrase A1 (b. 1-4, Fig. 5) starts with the chord progression C\(\#\) minor (tonic) to A minor (minor Gegenklang of tonic, tg) – minor chords with major 3\(\text{rd}\) root distance – and ends with a cadence to 3\(\text{rd}\)-less tonic (open 5\(\text{th}\) chord). The two initial minor chords have a 2\(\text{nd}\) grade third relation, and the use of a minor in the context of C\(\#\) minor tonality denotes a 2\(\text{nd}\)-level chromatic mixture (according to Aldwell - Schachter - Cadwallader 2011, see footnote 2).\(^{13}\)

\(^{12}\) In the present analysis, functional harmonic symbols were preferred over Roman numeral symbols for the harmonic analysis, because they can describe chromatic chord relationships more successfully. All letters referring to pitches are in low-case italics (e.g. d-f\(\#\)-a). All letters referring to chords are in capital and in normal type (e.g. G major, A minor chords), while when referring to tonalities they are in capital and bold type (e.g. G major, E minor tonalities). When referring to chords without specifying major or minor quality (i.e. in figures, formulas and diagrams), capital letters denote major chords and low-case letters minor ones. Moreover, capital or low-case letters in bold type, when used in figures, formulas and diagrams, denote major or minor tonalities respectively, e.g. C\(\#\) denotes C\(\#\) major tonality and e denotes E minor tonality.

\(^{13}\) The secondary mixture is justified by the fact that the root of the A minor chord belongs to C\(\#\) minor tonality as its VI degree, however the chord itself does not belong to either C\(\#\) minor or C\(\#\) major tonality.
Phrase A2 (b. 5-8, Fig. 6) starts as before, with the chord progression C$\flat$ minor to A minor, but now A minor is in first inversion and is used as subdominant for the modulation to E minor tonal region, in which the phrase cadences. The final chord lacks its third again.$^{14}$

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$^{14}$ Two more aspects of the harmonic analysis can be highlighted: a) the parallel octaves between 2$^{\text{nd}}$ and 4$^{\text{th}}$ voice at b. 5, which are a result of voice doubling in the context of a three-voice harmony and b) the second inversion of the C major chord at b. 7, which is used as a consonant chord, something fairly common in the harmonic idiom of the 2$^{\text{nd}}$ half of the 19$^{\text{th}}$ century.
In phrase A3 (b. 9-13, Fig. 7), the symmetrical properties of the augmented chord are exploited. Since it divides the octave into three equal parts (three major thirds), and engaging enharmonic equivalence of its pitches, each member of this chord can be considered root, 3rd or 5th. Its transformations are particularly useful: raising any chord member by a semitone yields minor chords and lowering by a semitone yields major chords. All three emerging chords of each category have a major 3rd root relation. So, in phrase A3 the augmented chord $d\#-g-b$ yields initially $e-g-b$ (E minor) and then (enharmonically as $e_b-g-b$) $e_b-g-c$ (C minor). These two minor chords have a root distance of descending major 3rd and a 2nd grade third relation, while the second chord (C minor) is used as subdominant for the modulation to the G minor tonal region. The modulation is declared with a full cadence to a 3rd-less tonic chord.

![Fig. 7. Reduction of phrase A3 (b. 9-13).](image)

In phrase A4 (b. 14-22, Fig. 8) the augmented chord is used again, but in a different way. Initially, the $f\#-b_b-d$ chord yields $g-b_b-d$ (G minor) through an ascending semitone transformation, but then its enharmonic form $g_b-b_b-d$ yields $g_b-b_b-d_b$ (Gb major) through a descending semitone transformation.

---

15 A full study on the special role of the augmented chord in music theory and in Liszt’s music can be found in Larry Todd’s paper [Todd 1988]. The paper also investigates the chord’s function as a structural, prolonged event, despite its ambiguous/dissonant intervallic nature (e.g. in the piano work Nuages Gris).
transformation. The produced chord leads through a linear – non-functional – chord progression (Gᵇ-F-E-Eᵇ-D, all major) to the D major chord, the dominant of the current tonal region. These two indirectly connected chords (Gᵇ/F♯ major – D major) have a descending major 3ʳᵈ root distance and a 2ⁿᵈ grade third relation. The D major chord is considered a neapolitan chord in root position in C♯ minor tonality, acquiring a subdominant function, and the phrase concludes with a half cadence to the dominant in the initial C♯ minor tonality. This phrase closes the first part of the binary form.

![Fig. 8.]

Reduction of phrase A₄ (b. 14-22).

The second part A' contains only two phrases, of which the latter is considerably longer. Phrase A'₁ (b. 23-26, Fig. 9), is essentially a repetition of the corresponding phrase A₁ of the first part, with the difference that the accompaniment in the left hand is step-wise linear instead of chordal-vertical. The phrase ends at a 3ʳᵈ-less tonic.
Fig. 9.

Reduction of phrase A’1 (b. 23-26).

Phrase A’2 (b. 27-39, Fig. 10) starts as the previous phrase A’1, with the chord progression C# minor to A minor. The next chord (F major) continues the descending major 3rd progression and although it can be described as the major Gegenklang of the major tonic (TG), it also implies (as a dominant) the tonal region of Bb minor, even though the region is not confirmed through cadential progression (the implied tonic appears only over a pedal of the dominant at b. 29). Afterwards, the descending major 3rds progression (C# - A - F) is interrupted by a chromatic circle of 5ths – which is repeated with minor alterations – consisting of dominant minor 9th chords. The descending circle of 5ths progression – lasting 6 bars – is the structural opposite of the interrupted descending circle of major 3rds. The common element of the two cycles is their structural goal, which is the main tonal center of the piece, the C# minor chord. The arrival to this goal at b. 39 completes for the first time both the interval cycle of major 3rds (C# - A - F - C#) and the 5ths cycle of dominants and comprises the structural end of the piece. Interestingly, this is the only full cadence of the piece that features a full final chord (containing its minor 3rd). The structural antithesis of the descending 5ths and descending major 3rds progressions is also denoted motivically, since b. 33-38 (the 5ths cycle) are the only ones not containing the funeral march motive of the dotted 8th-notes.
The following Coda (b. 40-48, Fig. 11) functions as a prolongation of the structural cadence and contains two main harmonic elements: an idiomatic plagal cadence (type sN-t) and a summary statement of phrase B2, with the full major 3rd interval cycle and the full cadence, but without the descending 5th progression, while re-establishing the main rhythmic motive.

2.3. Prolongational analysis

The rhythmical reductions presented above will be the basis for the prolongational reduction, expressed in hierarchical (foreground-middleground-background) schenkerian graphs. There are three graphs for the foreground level (parts A, A', Coda), and one for the middleground and background levels. The foreground graph of part A (Fig. 12) reveals the relation of the first phrase with the remaining three:
phrase A1 leads to a structural close at a level close to the musical surface (b. 4), while the other phrases to a structural opening through its “interruption” (half cadence to V7, b. 22). Worth noticing is the melodic arpeggiation by minor 3\textsuperscript{rd}s at a intermediate level: c\# (b. 4) - e (b. 8) - g (b. 13) - b\textsubscript{b} (b. 17). Also, the prolongational progression of G\textsubscript{b} major to D major yields whole-tone fragments from the pitches of the outer voices (b\textsubscript{b}-g\#-f\# and g\textsubscript{b}-e-d), while at the musical surface there is parallel chromatic voice leading at the upper three voices.

\textit{Fig. 12.}

Foreground prolongational graph of part A (b. 1-22).

The prolongational analysis of part A' (Fig. 13) reveals both the parallelism to part A and its different structural role and goal. While part A builds \textit{structural tension} through the ascending minor 3\textsuperscript{rd}s arpeggiation and the interruption of the structure at the dominant (with ^\textsuperscript{4}), part A' creates \textit{structural relaxation} through the arrival to the dominant (with ^\textsuperscript{2}), its prolongation and its resolution to the tonic (with ^\textsuperscript{1}). The cadential dominant chord (b. 35) is the progressional goal of both the F major chord (chromatic minor 3\textsuperscript{rd} relation) and the harmonic sequence of applied dominants (5\textsuperscript{th} relation), something that reinforces its structural role and also denotes the opposing harmonic forces beneath the musical surface (3\textsuperscript{rd}s and 5\textsuperscript{th}s harmonic relations). Also, worth noticing is the par-

\textsuperscript{16} According to schenkerian theory, the structural interruption occurs only at the ^\textsuperscript{2} melodic degree [SCHENKER 1979, 36]. In the present prolongational analysis, we consider the interruption to occur at the ^\textsuperscript{4} melodic degree, due to the prominence of f\# in b. 19-21 and the omission of d\# from the musical surface at the half cadence that occurs in b. 21 (d\# appears only in the prolongating downward arpeggiation of the dominant chord in b. 22).
allelism of the linear and chordal progressions at the end of parts A and A': in part A major 3rd relations are more prominent, while in part A' perfect 5th relations prevail; however, both parts of the binary form end with parallel descending chromatic motion of the three upper voices.

![Fig. 13. Foreground prolongational graph of part A' (b. 23-39).]

The Coda (Fig. 14) can be considered a prolongational summary of part A', with the difference that the F major chord progresses directly to the dominant, something that confirms the linear/contrapuntal role of the dominants' cycle at the end of part A'. Pitches a and c# of the musical surface are considered cover tones and they are removed from the prolongational graph.

![Fig. 14. Foreground prolongational graph of the Coda (b. 40-48).]

The middleground graph (Fig. 15) summarizes the tonal background of the piece. Part A builds structural-aesthetic tension through the
departure from the tonal center towards tonal regions located minor
third upward distance (transition from C# minor to E minor and then
G minor) and the structural interruption. Interestingly, the composer
avoids the B♭ minor tonal region, which would complete the symmetri-
cal tonal background, albeit with a foreseeable anticipated way for the
experienced listener, something that made this choice a compositionally
less favorable option. So, Liszt does not emphatically confirm this
region, but refutes the listener’s expectations twice: a) he uses B♭ melod-
ically (see Fig. 12), but harmonizes it with the G♭ major chord, and b)
implies the B♭ minor tonality within the suspended harmonic evolution
of phrase B2 by employing its dominant chord (F major in b. 29-31).
Part A' creates structural-aesthetic relaxation through the preservation
of the same tonality and the structural cadence to its tonic. Also, this
part contains the completion of the non-modulatory chordal sequence
of descending major 3rds (C#-A-F-C#).

Fig. 15.

Middleground prolongational graph of the piece.

The background graph (Fig. 16) does not include either the interrupt-
tion of the structure (the f# can be perceived as a neighbour note at this
level) or the chromatic 3rds relations and projects a normative Ursatz-
type fundamental structure. This level connects the piece with the tonal
tradition of the 19th century.
2.4. *Neo-Riemannian transformations and paths in pitch space*

Three main intervalic relations create the harmonic structure of the work (expressed as neo-Riemannian transformations): 

The *ascending minor third* (R+P), which is used for the structural connection of minor key tonal regions: C♯ minor - E minor - G minor - (B♭ minor)

The *descending major third* (types L+P, P+L, L, P+L+P), which is used for the main chord progressions in the same region or as a modulation tool in all the phrases of the piece. The connected chords are either minor or major and form pairs: C♯ minor - A minor (L+P), E minor - C minor (L+P), G♭ major - D major (P+L), A minor - F major (L), F major - C♯ minor (P+L+P) or full cycles: C♯ minor - A minor - F major - C♯ minor.

The *descending perfect fifth* (D), which defines the overall tonal idiom of the work, since it is the structural core of all phrasal cadences (dominant to tonic progressions). It is also used in foreground linear progressions (circle of fifths in phrase A'2).

These intervallic relations are key elements in the paths in tonal pitch space followed by the phrases and parts of the piece. Pitch space paths can explicate neo-Riemannian transformations and prolongation graphs
by adding psycho-acoustical chordal and regional distances and tonal center relationships.

The regional pitch space path of part A is presented in Fig. 17, as a segment of the overall diatonic chart of regions (the full chart is a symmetrical four-dimensional toroidal structure). Only minor tonal regions with minor third distance are used (solid rectangles or circles) or implied (dashed circles), and the final tonality, although the same as the initial, looks as if it lies at a different place. This happens due to the fact that the structure actually folds around itself so that the final C# minor region is the same as the initial, however, the diagram successfully describes the tonal journey.

![Diagram of pitch space path](image)

*Fig. 17.*

Regional pitch space path of part A.

Descending to the next prolongational level, the chordal/regional path is revealed in more detail (Fig. 18, with first row showing phrase A1 and the second row phrases A2-A4). All third relations have been specially indicated, and described by their neo-Riemannian transformation type and TPS chordal distance. Interestingly, the non-modulatory [L+P] transformation of phrase A1 has lower chordal distance from the modulatory [L+P] transformation of the next phrases.
Chromatic third relations, symmetrical octave division and paths in pitch space

Fig. 18.
Chordal/regional pitch space path of part A.

The regional pitch space path of part A' is presented in Fig. 19. An important difference with the regional path of part A is that there is only one solid tonal region, and the others are only implied (dashed circles). Also, an ambiguity regarding the transition from the implied E minor region to the implied B♭ minor region is expressed as two possible paths.

Fig. 19.
Regional pitch space path of part A'.
Descending to the chordal/regional pitch space path (Fig. 20), the two possible paths are presented in more detail, as well as the roles of the implied tonal regions that are penetrated by the third relations (described by their neo-Riemannian transformations). The first option connects the A minor chord (iv in the implied E minor region) directly to the F major chord (V of the implied Bb minor region), while the second option uses two pivot relations and also implies the A minor region. Both options are plausible and describable, and the choice is a matter of mental interpretation by the listener or analyst.

![Diagram of chordal/regional pitch space path]

*Fig. 20.*

Chordal/regional pitch space path of part A'.

Moreover, the analysis discloses the presence of a symmetrical background harmonic structure, in which the overall tonal pitch space is symmetrically divided into four tonal regions through the minor third interval and each region is symmetrically divided into three chordal regions through the major third interval. This description resembles
and outlines the hyper-hexatonic system (or triadic hexatonic space), which may be also considered as the implied background of the work. Fig. 21 depicts the pitch space paths followed by the directly or indirectly connected chords and tonal regions of the hyper-hexatonic system in the piece.

*Fig. 21.*

The work's hyper-hexatonic system (triadic hexatonic space) and the related tonal regions/chords.
Of course this piece is not post-tonal, however, it can be seen as a link between Liszt’s mainstream middle-period pieces and his experimental late-period pieces, which relied much more to symmetrical chords and uncompleted tonal gestures. If we accept this structural background, there is only one compact structure that includes all main chordal relationships, the pitch space distances are smaller, and the projected structural coherence and uniformity of the piece is more evident because of the “Principle of the shortest path” [LERDAHL 2001, 73-77]. It even unifies the harmonic elements implying the forming of a harmonic progression style. Conceivably, the pitch space path of the piece could be a hybrid structure combining the diatonic and chromatic (hexatonic) paths. This corresponds to the work’s harmonic nature, since it lies at the borders between diatonic and chromatic tonality.

3. Conclusion

As pointed out by Alan Walker, one of the tasks that contemporary Liszt scholarship has to undertake is to demonstrate how the strange, remote language of his old age flows from his earlier style (the composer’s shift seems extreme, and for many years these pieces lacked an audience) [2001, 782-783]. The present paper attempts a contribution towards this direction through the examination of a piece that connects the middle creative period with the late innovative works.  

The present multi-faceted analysis reveals that chromatic thirds and symmetrical harmony are core elements in Il Penseroso’s harmonic language, influencing both the surface linear chord progressions and the background tonal region relationships, as well as the work’s overall structural and morphological design.  

17 This is additionally helped by the fact that Liszt re-used Il Penseroso 20 years later for the composition of La Notte.

18 It is also worth commenting that the structural properties disclosed in the present paper could possibly form the base for a new hermeneutical approach to the piece, through the presumable symbolic connection of tonal space distances. and chromatic thirds relationships to the non-musical poetic or emotional aspects of the piece. However, such an approach will not be pursued in this paper.
the disclosed features through the use of mathematical and geometrical representations of the work’s harmonic structure. Therefore, in *Il Penseroso*, as in most of Liszt’s mature works, a loosening of traditional tonal structures through the modulatory or non-modulatory symmetrical division of pitch space takes place. However, this does not occur in an extreme, revolutionary way, so the chromatic symmetrical space coexists with the traditional diatonic space in a fragile, well-thought balance. This ambiguity is reflected in the deeper harmonic structure, which denotes the use of both the diatonic and the hyper-hexatonic pitch spaces.

Moreover, the tendency towards the exploration of symmetrical tonal structures, as encountered in the majority of Liszt’s mature works, makes the composer a forerunner and prophet of the harmonic revolution that took place in the first half of the 20th century, with the systematic exploitation of hexatonic, octatonic and other symmetrical harmonic spaces by important and influential composers such as Igor Stravinsky, Béla Bartók, Alexander Scriabin, Olivier Messiaen, and many others [Lendvai 1993].

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ABSTRACT

Chromatic third relations, symmetrical octave division and paths in pitch space: Analytical study of Franz Liszt's Il Penseroso

Chromatic third relations and symmetrical octave division into three or four equal parts consisting of major or minor thirds, as well as symmetrically constructed chords, were important structural elements of the music written in the second half of the 19th century. Theoretical research on these issues has been active since the end of the 19th century (mainly in the Riemannian functional harmonic theory tradition), but it has recently been invigorated by newer theories of harmonic structure and/or formal function (e.g. Imig, Lewin, Cohn, Hyer, Lerdahl, Kopp). The significance of symmetrical harmonic structures in Liszt's mature works has also been explored in theoretical/analytical research by Todd, Skoumal, Cinnamon, Forte, et al. These papers examine the works' deeper harmonic structures and identify the importance of chromatic thirds relations and the weakening of the dominant-to-tonic diatonic fifth relations, while they project their conclusions mainly through schenkerian reductional diagrams or chord charts and references to dualistic harmonic theories of the 19th century. The present paper attempts the application of a combination of older and recent theories and analytical methodologies (focusing on Riemannian functional harmony, schenkerian/reductional theory, neo-Riemannian transformational theory and Tonal Pitch Space theory) to the analysis of Franz Liszt's piano piece Il Penseroso from the collection Années de Pèlerinage - Deuxième Année: Italie. The analysis reveals that chromatic thirds and symmetrical harmony are core elements in the piece's harmonic language, influencing both the surface linear chord progressions and the background tonal region relationships, as well as the work's overall structural and morphological design. The piece's deeper harmonic structure denotes the use of both the diatonic and the hyper-hexatonic systems, in reference to which the paths in tonal space are drawn schematically and calculated mathematically according to the Tonal Pitch Space theory. The clarification of the piece's harmonic nature renders a holistic functional, structural and prolongational analysis of the work.
KEY WORDS: F. Liszt, Il Penseroso, neo-Riemannian analysis.

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