Modal Pitch Space —
A theoretical and analytical study

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**Abstract**
This paper presents an approach to the pitch space of the seven diatonic modes. The proposed theory is an expansion of Fred Lerdahl's tonal pitch space model; its purpose is a more accurate description of the situations involved in the analysis of diatonic modal music. The original methodology and set of algebraic calculating formulae is retained, but it is applied on modal instead of tonal space, on the basis that the latter is a subset of the former. In connection to Fred Lerdahl's theory, melodic motion, chord attraction and various cadence types are described within the modal context. Apart from the calculation of the pitch and the chordal and regional space algebraic representations, geometrical representations of all three levels of the modal pitch space are also included. Finally, the stability conditions arising from the new model are used as criteria to build the time span reduction and prolongational reduction parts of the Generative Theory of Tonal Music (GTTM) analysis of modal music. Two short GTTM analyses of 20th century modal music are being presented to illustrate the new model's analytical use.

1 INTRODUCTION — BACKGROUND

The present article comprises an approach to the pitch space of the seven diatonic modes. The proposed theory is an expansion and adaptation of Fred Lerdahl's *Tonal Pitch Space*, aiming to a more accurate description of the situations involved in the analysis of diatonic modal music. The original methodology and set of algebraic calculating formulae is retained, but it is applied to modal instead of tonal diatonic space, on the basis that the latter is a subset of the former. Following the mathematical and geometrical representations pertaining to the new model, two short GTTM (Generative Theory of Tonal Music) analyses of 20th century modal music are presented, in order to illustrate the model's analytical use.

The *Tonal Pitch Space* theory (Lerdahl, 1988 - 2001) evolved as an expansion of the *Generative Theory of Tonal Music* (Lerdahl and Jackendoff, 1983); its purpose was to provide explicit stability conditions and optimally clarified preference rules for
the construction of GTTM's time-span reduction and prolongational reduction. The diatonic pitch space model is based largely on experimental research in music psychology, mainly by Krumhansl (1979, 1983, 1990) and Deutsch and Fere (1981); it is also consistent with music theory models such as Weber's regional chart (1821-24), Riemann's Tonnetz (1893), Schoenberg's chart of regions (1954) and Lewin's neo-Riemannian transformations (1987). The algebraic representation of the diatonic pitch space model consists of integer number formulas; its geometric representation consists of matrix, toroidal and cone structures (Lerdahl, 1988; Krumhansl, 1983). Since its first formulation in 1988 (Lerdahl, 1988), the theory has been further complemented by the concepts of the Underlying Musical Schemata (Lerdahl, 1991), the regional Pitch-Space Paths (Lerdahl, 1992), the Octatonic and Hexatonic Pitch Spaces (Lerdahl, 1994) and the Tonal Tension and Attraction Theory (Lerdahl, 1996); the theory was summarized and published in its complete form in 2001 (Tonal Pitch Space, Lerdahl, 2001). As stated by Lerdahl (2001, p. 80), the tonal pitch space theory was initially conceived in order to explain the regularities observed in the experimental empirical results, contrary to the typical procedure whereby a preexisting theory is tested by subsequent experimentation. Up to now, the theory's empirical status has not yet been thoroughly investigated; therefore, there are aspects of the theory still waiting to be experimentally supported.

An Event Hierarchy (Bharucha, 1984) represents hierarchical relationships inferred from a sequence of musical events, normally cast as pitch reductions. The prolongational structures of GTTM, which are the most salient output of the theory's reductional analytic methodology, form event hierarchies. A Tonal Hierarchy is the sum of the hierarchical relations of the pitches, chords and regions within a tonal system modeled by a multidimensional space in which spatial distance equals cognitive distance. Having been established as a schema in the experienced listener's mind after long exposure to the musical idiom it represents, the tonal hierarchy is essential in the formation of event hierarchies from a sequence of perceived musical events. Pitch Space Theory provides such a model of Tonal Hierarchy, i.e. an atemporal and permanent knowledge schema expressed in simple algebraic formulae, which correlates spatial distance with intuitive musical distance and describes the cognitive distances between pitches, chords and regions in tonal music. Since the model provides quantification of the cognitive distances between the musical events of the analysis' reductional levels, it contributes to the formation of important preference rules regarding time-span stability and prolongational connection.

(1) Investigation of listeners' response to an induced music for the cognitive proximity of pitch classes, chords and regions and arrival to a geometric solution representing the multidimensional scaling of the patterns implicit in the empirical data.

(2) Theory of pitch sequence encoding through the use of hierarchically organized alphabets ranging from the octave to the chromatic scale.
2 Modal Pitch Space Calculations

2.1. The Diatonic Modes
Prior to the calculation of chordal and regional distances, it would be useful to discuss some properties of the diatonic modal system. The seven diatonic modes (the term *mode* is used with the meaning of *scale*) may be produced by the consideration of each pitch of a diatonic collection as a tonic. Thus, each diatonic collection has seven modes, all under the same key signature; therefore, there are twelve diatonic collections, all connected to each other by the circle of perfect fifths through the addition or the subtraction of sharps or flats. Instead of classifying the modes that have the same key signature in successive diatonic steps, the diatonic circle of fifths is used (Figure 1). Arranging the modes in such a way as to avoid the diminished fifth until the circle's completion, the following classification emerges (for the "white-key" diatonic collection): Lydian F - Ionian C - Mixolydian G - Dorian D - Aeolian A - Phrygian E - Locrian B.

![Figure 1. First classification of the modes.](image)

If the same tonic is kept but the key signature (*i.e.* the diatonic collection) is altered, another type of classification of the seven modes arises. The circle of perfect fifths is used again in this process, but in this case the circle represents the addition of flats (or the subtraction of sharps) through the progression from Lydian to Locrian (Figure 2). If C is taken as the constant tonic, the following classification emerges: Lydian C (F♯) - Ionian C (−) - Mixolydian C (B♭) - Dorian C (B♭, E♭) - Aeolian C (B♭, E♭, A♭) - Phrygian C (B♭, E♭, A♭, D♭) - Locrian C (B♭, E♭, A♭, D♭, G♭).

![Figure 2. Second classification of the modes.](image)
Thus, with the use of the circle of perfect fifths, both types of modal relationships are classified in the same order: Lydian through Locrian. At the diatonic level, the circle creates a relation between all the modes of a specific diatonic collection before becoming a diminished fifth and at the chromatic level it creates a relation between all the modes of a specific tonic before progressing to another. The center mode in both cyclic orders is the Dorian mode.

2.2. Distance from the Modal Tonic

The basic space is a hierarchical representation of a pitch space element organized in five successive levels (a to e: octave space to chromatic space), which derives (Lerdahl, 1988, p. 320; 2001, p. 47) from Deutsch and Feroe’s hierarchical model (1981) with the addition of the fifth level (level b). As an example, the basic space of the C Dorian mode is presented.

```
level a  0
level b  0  7
level c  3  7
level d  2  3  5  7  9  10
level e  1  2  3  4  5  6  7  8  9  10  11
          C  C#  D  D#  E  F  F#  G  G#  A  A#  B
```

*Figure 3. Basic space of the C Dorian mode.*

The distance from the tonic for every other pc (pitch class) of the mode can be calculated (Lerdahl, 2001, pp. 48-49) as the sum of this pc’s vertical distance (number of levels down in the basic space counted for the pc’s first appearance) and horizontal distance (shorter number of horizontal steps at all levels of the space for a pc to reach p0) from the modal center. As an example, the distances for the Dorian mode are calculated (Figure 4); the calculation of the distances for the other modes produces similar results.

- **Vertical distance:**
  - C=0
  - G=1
  - E_b=2
  - D,#; A; B=3
  - D_b; E; F,#; A#=4

- **Horizontal distance:**
  - C=0
  - G; E_b; D; B; D_b; B=1
  - A=2
  - F=3
  - E; A#=4
  - F#=6

- **Combined distance:**
  - 0  4  3  6  2  5  4

*Figure 4. Pcs distances from modal center for the C Dorian mode.*

2.3. Chordal Space within a Mode

The chordal space within a mode, meaning the distance of every chord from the tonic chord of the scale, can be calculated using the following formula (Lerdahl, 2001, p. 55): \( \delta(X/A \rightarrow I/A) = j + k \), where \( \delta \) = chordal distance, \( X \) = the chord in
Latin numeral notation (I to VII), A = the mode, j = the shortest number of steps in the circles of fifths in either direction that the root of this chord needs to make in order to reach the tonic and k = the sum of the non-common tones at all levels of the basic space (except the chromatic level, which is common for all the chords). The non-common tones are underlined in the following charts. The chordal space distances for C Dorian mode are calculated (Figure 5).

![Figure 5. Chordal space calculations for C Dorian mode.](image)

Regardless of the type of diatonic mode, the results produced by this process are the same. The model is constructed so that the diatonic chordal distance is calculated in relation to the interval distance of the chords' roots regardless of the chord quality (major, minor or diminished triad). Therefore, the overall diatonic chordal distances, expressed in relation to interval class, are:

\[ \delta(\text{ic 5,7}) = 5, \delta(\text{ic 3,4,8,9}) = 7, \delta(\text{ic 1,2,10,11}) = 8. \]

Also, the chordal distances expressed in relation to diatonic interval distance from the tonic are (the scale degree symbols used remain the same, regardless of chord quality):

\[ \delta(V\to I) = 5, \delta(IV\to I) = 5, \delta(VI\to I) = 7, \delta(III\to I) = 7, \delta(II\to I) = 8, \delta(VII\to I) = 8. \]

The result that arises from the above is that all diatonic modes are identical in the calculation of chordal distance. This generalization, however, is not entirely satisfactory at all circumstances: for example, \( \delta(F\to c) \) in Dorian mode "feels" greater than \( \delta(f\to c) \) in Aeolian mode, although they are arithmetically equal. This perceived unevenness is explained through the _chord attraction theory_ (Lerdahl 1996, 2001), which is incorporated in the theory of tonal space and will be discussed later.
The difference is that, when progressing from F major to C minor with C as the common tone, the other voices move by two whole steps; on the other hand, when progressing from f minor to c minor, they move by one whole and one half step. This creates different attraction values ($\alpha$) in each case, resulting in the perception of difference between their cognitive distances (see also section 3A).

At this point some facts about the Locrian mode must be discussed. This mode is problematic in relation to its inclusion in the modal pitch space as equally stable with the other modes. The reason is that the Locrian mode does not conform to Preference Rule 1 regarding the construction of scales and basic spaces (Lerdahl, 2001, p. 272) because levels b and c (fifth and chord level) of its space contain events with sensory dissonance (the diminished fifth interval and the diminished triad). So, although the Locrian mode is used in jazz, as well as in some music specimens in the borderline of the tonal idiom, it may not be considered equal to the other six diatonic modes in terms of its internal structural integrity.

2.4. Regional Modal Space with Constant Tonic Pitch

The next stage is the calculation of the regional distance between modes that have the same tonic but different key signatures; the regional distance is calculated as the modes' tonic chords' distance. The following formula will be used: $\delta(I(A_x,I(A_y)) = i + j + k$, where $\delta$ = the chordal distance between modal regions, $A_x$, $A_y$ = the modal regions (in bold face), $i$ = the integer expressing the difference in key signature between the two modes and $j$, $k$ as in the previous calculations; note that $j$ always equals 0 in these calculations because all the tonic chords have the same root. With C Lydian as reference mode, since it is first in the progression discussed in 2.1, the results presented in Figure 6 are obtained.

Each mode's distance from its neighbors is 2, except in the case of the two following transitions: from Mixolydian to Dorian, because of the change in chord quality from major to minor and from Phrygian to Locrian, because of the loss of the perfect fifth in the mode's tonic chord. These results yield the formation of two groups of modes, each group having three members equidistant from each other (the Locrian mode is not included in any of these groups):

- the major tonic group modes: Lydian - Ionian - Mixolydian
- the minor tonic group modes: Dorian - Aeolian - Phrygian.

Both of these groups have central modes from which the other two members of the group have equal distances of 2. The center for the major tonic group is the Ionian mode, while the center for the minor tonic group is the Aeolian mode. An interesting highlight is the coincidence of the two central modes with the major and minor tonal scales.

(3) A basic space preferably correlates height of level in the space with the degree of sensory consonance of adjacent intervals within a level.
2.5. MODAL TRANSPOSITIONS, MODULATIONS AND CHROMATIC INFLECTIONS

A transition - modulation between two modal regions can occur in four different ways:
- modulation to another mode in the same key
- modulation to the same mode in another key (transposition)
- modulation to another mode keeping the same tonic but altering key signature (modal interchange).
- modulation to another mode in another key.

The distance between two modulating modal regions can be calculated with the same algebraic formula, regardless of which of the four types is involved. The regional space distance formula is: $\Delta(A_x \rightarrow B_y) = i + j + k$, where $\Delta$ is the regional distance, $A_x, B_y$ are the modal regions ($A, B$ = tonic, $x, y$ = mode type) and $i, j, k$ as before. For the calculation of the chordal distance between regions, the formula is: $\delta(X/A_x \rightarrow Y/B_y) = i + j + k$, where $\delta$ is chordal distance, $X, Y$ = chord in Roman numeral notation (I to VII) and $i, j, k$ as before. Two examples are presented: the calculation of the regional distance between $C_{dor}$ and $A_{mix}$ (Figure 7)

$$\Delta(C_{dor} \rightarrow A_{mix}) = \delta(I/C_{dor} \rightarrow I/A_{mix}) = i + j + k = 4 + 3 + 10 = 17$$

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Figure 6. Regional space calculations with $C$ as a constant tonic.

Figure 7. Regional distance between $C$ Dorian and $A$ Mixolydian.
and the calculation of the chordal distance between $VI^7/A_{aeol}$ and $IV^7/A_{dom}$ (Figure 8).

$$
\delta(VI^7/A_{aeol} \rightarrow IV^7/A_{dom}) = i + j + k = 1 + 3 + 6 = 10
$$

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$VI^7/A_{aeol}$  $IV^7/A_{dom}$

Figure 8. Chordal distance between $VI^7/A_{aeol}$ and $IV^7/A_{dom}$.

The use of chromatic inflections and leading tones generates an interesting question: whether, and to what extent, they are to be regarded as inflections and borrowed chords or as members of new modal regions; this issue is also present in relation to the minor scale, which is not considered to include the chromatic leading tone. One proposed, non-universal solution is the following: when chromatic tones appear as temporary deviations from the scale, i.e. as leading tones, chromatic non-harmonic tones or members of chromatic chords, they may be considered to be chromatic inflections; if, on the other hand, their occurrence is more persistent — as, for example, when they are embedded in a melodic theme, or when they keep reappearing for a longer period of time — then they may be considered to establish a new modal region. However, the loose distinction between chordal and regional space allows for a high degree of ambiguity.

According to the pitch space theory, borrowed chords and chromatic inflections are calculated within the home region by the registration of the chromatically inflected pitch classes. Hence, $i = 0$, $j$ behaves as before and $k$ includes all chromatic pitch classes. To illustrate, the chordal distance from the tonic for a borrowed C minor chord in the context of $A_{dom}$ is calculated in Figure 9.

$$
\delta(c \rightarrow a/A_{dom}) = i + j + k = 0 + 3 + 6 = 9 \text{ (instead of 7 if it was C major)}
$$

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$A_{dom}$  $C/A_{dom}$

Figure 9. Distance of C minor chord from the tonic in Dorian.

3

The attraction theory and modal cadences

The Attraction Theory plays a crucial role in modal pitch space. Since large parts of modal compositions seem to lack strong indications of a tonal center (except for non-ambiguous melodic statements and cadence points), the strongest
“gravitational” forces that unify the flow of music are the horizontally related melodic and chordal attractions. The attraction principle and the voice-leading and harmonic attraction calculations play an important role in the full appreciation of modal music more so than in tonal music; they shed light in a domain where little analytical information is disclosed by the highly ambiguous chordal and regional distance considerations.

Considerable research on this topic has been carried out by Meyer (1973), Narmour (1990) and Bharucha (1996); furthermore, a Tonal Tension and Attraction theory has been developed as part of the Tonal Pitch Space theory (Lerdahl, 1996, 2001), aiming to define — in mathematical terms — the exact amount of tension or relaxation within or across the three categories of strong prolongation, weak prolongation and progression. The three main calculation rules introduced by Lerdahl (1996, 2001) for his attraction theory model are:

**melodic attraction rule:** \( \alpha(p_1 \rightarrow p_2) = s_2/s_1 \times 1/n^2 \), where \( \alpha \) = the melodic attraction of pitch \( p_1 \) towards pitch \( p_2 \), \( s_1, s_2 \) = the level position of the pitches in the basic space (the fifth level is omitted in these calculations) and \( n \) = the distance between them in semitones,

**voice-leading attraction rule:** \( \alpha_{vrl}(C_1 \rightarrow C_2) = \alpha_{v1} + \ldots + \alpha_{vn} \), where \( \alpha_{vrl} \) = the voice-leading-attraction of chord \( C_1 \) towards chord \( C_2 \) and \( \alpha_{rn} \) = the melodic attractions within each voice from the first to the second chord,

**harmonic attraction rule:** \( \alpha_{rh}(C_1 \rightarrow C_2) = k[\alpha_{vrl}(C_1 \rightarrow C_2)]/\delta(C_1 \rightarrow C_2) \), where \( \alpha_{rh} \) = the harmonic attraction of chord \( C_1 \) towards chord \( C_2 \), \( k \) = a constant equaling 10, \( \alpha_{vrl} \) as above and \( \delta(C_1 \rightarrow C_2) \) = the chord distance between \( C_1 \) and \( C_2 \).

In compliance with these rules, a number of modal cadence types will be examined, which will be grouped in two categories:

a) *The modal cadences that have common elements with tonal ones*, like the perfect V-I and plagal cadence IV-I (Figure 10).

![Figure 10. Modal cadences with tonal characteristics.](image-url)

The Lydian and Ionian modes share the same type of V-I cadence, which is identical to the major scale perfect cadence:

**Lydian - Ionian V-I cadence (a of Figure 10):**

\[
\alpha_{vrl}(V \rightarrow I) = \alpha(B \rightarrow C) + \alpha(G \rightarrow G) + \alpha(D \rightarrow E) + \alpha(G \rightarrow C) = 4/2 \times 1^{1/2} + 0 + 3/2 \times 1/2^2 + 4/3 \times 1/5^2 = 2 + 0.375 + 0.053 = 2.43
\]

\[
\alpha_{rh}(V \rightarrow I) = 10 \times 2.43/5 = 4.86
\]
The Aeolian and Dorian modes may have two possible dominant-tonic type cadences: one with minor dominant \( v \) and one with major dominant chord \( V \) (the major dominant is produced with the use of chromatic leading tones, not present in the modes).

**Dorian - Aeolian \( v-i \) cadence (b):**

\[
\alpha_{rvl}(v\rightarrow i) = \alpha(B\rightarrow C) + \alpha(G\rightarrow G) + \alpha(D\rightarrow E_b) + \alpha(G\rightarrow C) =
\]

\[
4/2 \times 1/2^2 + 0 + 3/2 \times 1/2^2 + 4/3 \times 1/5^2 = 0.5 + 1.5 + 0.053 = 2.05
\]

\[
\alpha_{rh}(v\rightarrow i) = 10 \times 2.05/5 = 4.1
\]

**Dorian - Aeolian \( V-i \) cadence (c):**

\[
\alpha_{rvl}(V\rightarrow i) = \alpha(B\rightarrow C) + \alpha(G\rightarrow G) + \alpha(D\rightarrow E_b) + \alpha(G\rightarrow C) =
\]

\[
4/2 \times 1/2^2 + 0 + 3/2 \times 1/2^2 + 4/3 \times 1/5^2 = 2 + 1.5 + 0.053 = 3.55
\]

\[
\alpha_{rh}(V\rightarrow i) = 10 \times 3.55/5 = 7.1
\]

The Mixolydian mode may also have two alternative dominant-tonic type cadences: one with minor \( v \) and one with major (chromatic leading tone) \( V \) (which is identical to the Ionian \( V-I \), already examined).

**Mixolydian \( v-I \) cadence (d):**

\[
\alpha_{rvl}(v\rightarrow I) = \alpha(B_b\rightarrow C) + \alpha(G\rightarrow G) + \alpha(D\rightarrow E) + \alpha(G\rightarrow C) =
\]

\[
4/2 \times 1/2^2 + 0 + 3/2 \times 1/2^2 + 4/3 \times 1/5^2 = 0.5 + 0.375 + 0.053 = 0.928
\]

\[
\alpha_{rh}(v\rightarrow I) = 10 \times 0.928/5 = 1.856
\]

The plagal type subdominant-tonic cadences of the Dorian and Aeolian modes are also worth considering.

**Dorian \( IV-i \) cadence (e):**

\[
\alpha_{rvl}(IV\rightarrow i) = \alpha(C\rightarrow C) + \alpha(A\rightarrow G) + \alpha(F\rightarrow E_b) + \alpha(F\rightarrow C) =
\]

\[
0 + 3/2 \times 1/2^2 + 3/2 \times 1/2^2 + 4/2 \times 1/5^2 = 0.375 + 0.375 + 0.08 = 0.83
\]

\[
\alpha_{rh}(IV\rightarrow i) = 10 \times 0.83/5 = 1.66
\]

**Aeolian \( iv-i \) cadence (f):**

\[
\alpha_{rvl}(iv\rightarrow i) = \alpha(C\rightarrow C) + \alpha(A_b\rightarrow G) + \alpha(F\rightarrow E_b) + \alpha(F\rightarrow C) =
\]

\[
0 + 3/2 \times 1/2^2 + 3/2 \times 1/2^2 + 4/2 \times 1/5^2 = 1.5 + 0.375 + 0.08 = 1.955
\]

\[
\alpha_{rh}(iv\rightarrow i) = 10 \times 1.955/5 = 3.91
\]

Having compared the values for these types of cadences, one can find justification for both the preferred use of leading tones in perfect cadences and the rare use of the Mixolydian \( v-I \) or the Dorian plagal cadences compared to their alternatives: the \( \alpha_{rh} \) values for the perfect cadences with leading tones are 7.1 and 4.86 in the Dorian and Mixolydian modes respectively, while the \( \alpha_{rh} \) values without leading tones are 3.55 and 1.856; also, the \( \alpha_{rh} \) value for the plagal cadence in the Dorian mode is 1.66, while the same cadence in the Aeolian mode has a \( \alpha_{rh} \) value of 3.91 (see also discussion about the IV-I progression in section 2.3).

b) **The modal cadences of purely contrapuntal origin**, like the Phrygian cadence and others.

In voice-leading (contrapuntal) type cadences, the calculation of harmonic attraction \( \alpha_{rh} \) will be omitted; only the voice-leading attraction \( \alpha_{rvl} \), will be
calculated, because the origin and the actual use of these cadences is based on melodic continuation and linear voice-leading, and not on harmonic progression in the perfect fifths circle.

The Phrygian mode has a characteristic cadence (Figure 11), which can be described as vii\textsubscript{6}-i (or vii\textsubscript{6}-I with the chromatic Picardy third).

**Figure 11. Phrygian cadence.**

**Phrygian vii\textsubscript{6}-i cadence (a of Figure 11):**
\[
\alpha_{\text{rvl}}(b\textsubscript{6} \rightarrow c) = \alpha(B \rightarrow C) + \alpha(F \rightarrow E\textsubscript{i}) + \alpha(F \rightarrow G) + \alpha(D\textsubscript{b} \rightarrow C) = \\
4/2 \times 1/2^2 + 3/2 \times 1/2^2 + 3/2 \times 1/2^2 + 4/2 \times 1/2^2 = 0.5 + 0.375 + 0.375 + 2 = 3.25
\]

**Phrygian vii\textsubscript{6}-I cadence (b):**
\[
\alpha_{\text{rvl}}(b\textsubscript{6} \rightarrow C) = \alpha(B \rightarrow C) + \alpha(F \rightarrow E) + \alpha(F \rightarrow G) + \alpha(D\textsubscript{b} \rightarrow C) = \\
4/2 \times 1/2^2 + 3/2 \times 1/2^2 + 3/2 \times 1/2^2 + 4/2 \times 1/2^2 = 0.5 + 1.5 + 0.375 + 2 = 4.375
\]

Also of interest are the contrapuntal cadences of the three-voice type vii\textsubscript{6}-I\textsubscript{omit 3} (Figure 12a), encountered mainly in 14th and 15th century music, with variants like the I with omitted fifth (Figure 12b) and the musica ficta vii\textsubscript{6}, known as the “Burgundian cadence” (Figure 12c).

**Figure 12. Contrapuntal type modal cadences.**

3-voice cadence vii\textsubscript{6}-I\textsubscript{omit 3} (a of Figure 12):
\[
\alpha_{\text{rvl}}(b\textsubscript{6} \rightarrow C\textsubscript{omit 3}) = \alpha(B \rightarrow C) + \alpha(F \rightarrow G) + \alpha(D \rightarrow C) = \\
4/2 \times 1/2^2 + 3/2 \times 1/2^2 + 4/2 \times 1/2^2 = \\
2 + 0.375 + 0.5 = 2.875
\]

3-voice cadence vii\textsubscript{6}-I\textsubscript{omit 5} (b):
\[
\alpha_{\text{rvl}}(b\textsubscript{6} \rightarrow C\textsubscript{omit 5}) = \alpha(B \rightarrow C) + \alpha(F \rightarrow E) + \alpha(D \rightarrow C) = \\
4/2 \times 1/2^2 + 3/2 \times 1/2^2 + 4/2 \times 1/2^2 = \\
2 + 1.5 + 0.5 = 4
\]

3-voice cadence vii\textsubscript{6}-I\textsubscript{omit 3} (c):
\[
\alpha_{\text{rvl}}(b\textsubscript{6} \rightarrow C\textsubscript{omit 3}) = \alpha(B \rightarrow C) + \alpha(F \rightarrow G) + \alpha(D \rightarrow C) = \\
4/2 \times 1/2^2 + 3/2 \times 1/2^2 + 4/2 \times 1/2^2 = \\
2 + 1.5 + 0.5 = 4
\]

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The comparison of these results provides justification for the extensive use of musica ficta in cadences and the preference for the final major chord in later periods: the "musica ficta" phrygian cadence has a $\alpha_{\text{phy}}$ value of 4.375 while the diatonic version has a $\alpha_{\text{r}}$ value of 3.25; also the older type three-voice cadence (with omitted third) has a smaller $\alpha_{\text{r}}$ value (2.875) than both the omitted fifth version (4) and the "musica ficta" version (4).

Possible expansions of the modal cadence attraction model would be its application to the quasi-organum style "coloring" parallel fifths (or any other interval) or the highly chromatic modal cadences used in the 19th and 20th century; but further elaboration on the subject will not be made at this point.

4
Modal space representations

The graphic representation of the basic (pitch class) space for any mode is similar to the cone structure introduced by Krumhansl (1983) but not identical, since it includes a level for the interval of fifth as well; in this structure, the tonic pitch is the tip of the cone and the other pitches are found in virtual zones on the surface of this cone corresponding to levels a-e of the space; their distance from the top is in proportion to their position in the basic space. As an example, the basic space of the C Dorian mode is represented in Figure 13.

![Figure 13. Representation of basic space for C Dorian mode.](image)

Projected geometrically, the chordal space for every diatonic mode is the structure of Figure 14, with the vertical axis representing the circle of fifths and the horizontal axis representing the circle of thirds. The horizontal and vertical distances are different (7 and 5 accordingly) and the area in dashes is the chordal core of the
mode. This chart can also be converted into a toroidal structure (Lerdahl, 1988, p. 334), the two representations (chart and torus) being interchangeable in principle. It should be mentioned that the model is not totally correct geometrically, because the diagonal distance should be $8.6$ (= the square root of $(5^2 + 7^2)$) instead of $8$, but the approximation is considered tolerable for the purposes for which the model is constructed.

![Figure 14. Representation of chordal space for all diatonic modes.](image)

The first level of description of the regional space, which includes the modes in the same diatonic collection (same key signature), stems directly from the chordal space description; it is constructed by the substitution of the chords in the chordal space around the major tonic chord with the modal regions that correspond to them. For instance, taking $C_{Ionian}$ as the central mode, the structure of Figure 15 arises.

![Figure 15. Regional space representation (constant key signature).](image)

An interesting aspect of the representation of Figure 15 is that all six modes (excluding the Locrian) are contained in the first two vertical rows only, the first one containing the minor tonic modes and the second row the major tonic ones; each member of the row connects to the next through the circle of perfect fifths and the bottom member of the first row connects to the top member of the second row in a similar manner. Each row has its center mode, which is the Aeolian for the minor
modes and the Ionian for the major modes; the distance values in this grid are 7 for the horizontal and 5 for the vertical axis. A similar structure was discussed in 2.4, describing the relationship between the modes with the same tonic but different key signature; there, the distance value operators were 7 and 2. In both cases the modes progress according to the circle of perfect fifths; in the first case, though, the circle defines the relation between the tonics of the modes, while in the second case the relation between the modes is defined by their key signatures. These two representations conform to the discussion about the modes in section 2.1.

The combination of these two structures (Figure 16a and Figure 16b) yields a new structure with two hyper-regions (Figure 16c), each containing all the major or minor tonic modes around C. The distances value operators are 7, 5 and 2.

![Diagram](image)

**Figure 16.**

Representation of modal hyper-regions with major and minor central chord.

It is possible to construct the next level of regional representation of the modal pitch space, which includes all the modes in all the key signatures, by combining the hyper-region structure and the tonal pitch space regional chart; the regional chart includes all the major and minor tonal regions and the distance value for both horizontal and vertical axes is 7. If one starts from the part of the regional chart that has C as its center (Figure 17a) and makes a draft version of the part of the regional modal space chart that includes the neighbors of the C related modes and indicates these modes' key signature origin, the result depicted in Figure 17b will be obtained. In this chart, the distance value which acts as constructing operator is 7 for both horizontal and vertical dimensions. Thus, the central Ionian mode is equidistant from its relative and its homonymous Aeolian modes, as occurs in the corresponding major and minor regions of the tonal regional chart.
However, if all six modes of each diatonic collection and their distances from the central modes (Ionian and Aeolian) are incorporated in the regional chart, the distance values make the key signature regions overlap and the overall shape of the structure slants (Figure 18, page 72). The three distance values that construct the grid are 7 (for the semi-diagonal and vertical axis between central regions), 5 (for the vertical axis between modes with the same key signature and their tonic pitches one fifth apart) and 2 (for the horizontal axis between modes with the same tonic chord and key distance 1). The solid rectangles are the pivot hyper-regions, the dashed rectangle is the modal interchange hyper-region and the modes in small rectangles are the ones that belong to the same (0 or 3½ key signature in this case) diatonic collection. The vertical distance between horizontal rows is 5 instead of 7 in the original region chart; the original vertical distance of 7 is maintained, albeit in a semi-diagonal direction. Orthogonal rectangles can be constructed from the modes that have the same key signature. It should be pointed out that the distance value of 8 instead of 7 for the e\textsubscript{phr} mode exists because in this geometrical chart e\textsubscript{phr} is associated with a\textsubscript{acol} and not directly with C\textsubscript{ion}, therefore j = 4 instead of 3; the same applies for the distance between c\textsubscript{acol} and B\textsubscript{mix}.

This representation has a geometrical imperfection beyond the approximation level; the imperfection concerns the semi-diagonal distance 7. According to Euclidean geometry, it should be the square root of \( (5^2 + 2^2) \), which equals 5.4; this may be corrected by projecting this structure in four-dimensional space (toroidal representation), so that the chart items are not on the same plane. However, this could be an indication of the limitations of this geometrical model and the need for a more precise mathematical and geometrical description. Furthermore, the model is not applied entirely correctly in the modulation between non-adjacent diagonally positioned modes, but it describes the relations between the modal neighbors of each mode; a similar situation occurs in the tonal regional chart. Depending on the way a modulation is carried out (for example, the pivot regions used), the space “bends” or “expands” to describe the distance of the two modes through the modulating procedure.
Figure 18. Regional space representation.
5

**ANALYTICAL APPLICATIONS**

The principle of the shortest path (Lerdahl, 2001, p. 73)\(^4\) links the tonal or modal pitch space with the *Generative Theory of Tonal Music* by providing two important stability conditions in the form of preference rules for the time-span reduction (TSR) and prolongational reduction (PR) (Lerdahl, 2001, p. 74):

**Time-span stability:** Of the possible choices for the head of a time span, prefer the event that yields the smallest \(\delta \) (or \(\Delta \)) value in relation to the superordinate context for this time-span.

**Prolongational connection:** Of the possible connections within a prolongational region, prefer the attachment that yields the smallest \(\delta \) (or \(\Delta \)) value in relation to the superordinate endpoints of this region.

Two short excerpt analyses will be presented to illustrate the use of GTTM in modal context, combined with the stability conditions derived from the modal pitch space theory. The first is from Béla Bartók’s “Romanian Folk Dances, VI” (bars 1-16) and the second from Eric Satie’s “Trois Gymnopédies, no. 1” (bars 32-39 and 71-78).

A) **ROMANIAN FOLK DANCES, VI (FIGURE 19, PP. 74-75)**

The first example is a harmonization of an original Romanian folk tune (Figure 19; note that level h is the musical surface and TSR level g is the first reductional level). The main melodic theme in this excerpt occurs twice (bars 1-8 and 9-16) and the harmonization of each occurrence is different. The theme itself consists of the transposed repetition of a four-bar phrase that spans a downward perfect fourth interval (A - E and D - A). According to the key signature and the starting and concluding chords, the piece is in D major (= D Ionian). However, according to the chromatic elements (the alternation of G sharp and G natural) and the bass line, the piece is explicitly divided into four modal regions (Figure 19, TSR level h): D Lydian (bars 1-4), D Ionian (bars 5-8), F\(\sharp\) Aeolian (bars 9-12) and A Mixolydian (bars 13-16)\(^5\).

The first harmonization (bars 1-8) is based on a D bass pedal note and has minimal chordal variation. The main structural melodic element, apart from the downward leap of perfect fourth in the upper voice, is a chromatic downward movement (visible at TSR level d: A - G\(\sharp\) - G). The ending of the phrase is a prolongation of the initial structure; superficially, it seems to be a right strong prolongation (both melodic and chordal content coincide after reducing out the G

---

(4) Among the possible paths in Pitch Space, the listener chooses the one that incorporates the least cognitive effort.

(5) The fourth phrase (bars 13-16) could also be interpreted as a G Lydian modal region, in parallel to the first phrase (bars 1-4), because of the G in the bass; however, the cadence to the A major chord (bar 16) makes the A Mixolydian interpretation more preferable.
Figure 19. GTTM analysis of bars 1-16 from Bartók's "Romanian Folk Dances, No. 1".
in the left hand at level c), but actually it is not, because of the modal region shift. The MPS (modal pitch space) chordal distance between the events of bars 1 and 8 (see PR level c at Figure 19) is: $\delta(l/D_{lyd} \rightarrow l/D_{ion}) = i + j + k = 1 + 0 + 1 = 2$, which equals the distance between the modal regions of D Lydian and D Ionian. Because of the small distance, the connection is considered an approximate strong prolongation (see tree structure of Figure 19); however, this subtle difference is important for the understanding of the work's nature.

The second harmonization (bars 9-16) starts with a F♯ minor chord that establishes a F♯ modal region for bars 9-12 and ends with an A major chord, center of the A Mixolydian region. The whole excerpt is considered a right progression from D major chord (bar 1) to A major chord (bar 16), and functions as a type of half cadence. The MPS chordal distance between these events also represents the regional distance: $\delta(l/D_{lyd} \rightarrow l/A_{mix}) = i + j + k = 1 + 1 + 5 = 7$, which is larger than the distance between a tonic and a dominant: $\delta(l/D \rightarrow V/D) = 0 + 1 + 4 = 5$. This part also contains two temporary chromatic inflections, which do not define new modal regions: the C natural of bar 12 and the F natural of bar 15. Both inflections create chromatic chords that are considered weak prolongations of their non-chromatic forms. In particular, the chord F♯-A-C-E (bar 12) is a weak right prolongation of the F♯ minor chord (bar 9) and the chord G-B-D-F (bar 15) is a weak prolongation of the G major chord with an added major seventh (bar 13). The MPS chordal distance in the first case is: $\delta(F\#-A-C-E \rightarrow i/F\#_{aeol}) = i + j + k = 0 + 0 + 4 = 4$ and in the second case: $\delta(G-B-D-F \rightarrow VII/7/A_{mix}) = 0 + 0 + 2 = 2$. The first distance is larger not only because C natural affects the fifth level but because of the addition of the seventh at the chord level as well; however, both distances are relatively small and justify the preference for weak prolongation. It is worth pointing out the uncovering of two structural melodic elements with downward chromatic motion: C# - C - B (bars 9-13, TSR level d) and F# - F - E (bars 13-16, TSR level e); these chromatic motions parallel the chromatic element A - G# - G (bars 1-5, TSR level d) of the first harmonization. Also, in the context of these downward motions, the left-hand chords on the second beats of bars 4 and 12 function as pick-ups to the following phrases, providing an internal structural link between the four-bar phrases of the piece.

At prolongational level c a choice has to be made about the connection of the F♯ chord (F♯ Aeolian region) of bar 9; the chord may either be connected to the D major chord (D Lydian region) of bar 1 as a weak right prolongation according to the interaction principle, or to the A major chord (A Mixolydian region) of bar 16 as left progression. The calculation of the MPS chordal distances yields the following results: $\delta(i/F\#_{aeol} \rightarrow l/D_{lyd}) = i + j + k = 0 + 3 + 4 = 7$ and $\delta(i/F\#_{aeol} \rightarrow l/A_{mix}) = i + j + k = 1 + 3 + 5 = 9$; according to the aforementioned preference rule, the first choice is more stable, therefore the event of bar 9 is attached to the structural beginning as a right weak prolongation at level b'.
b) TROIS GYMNOPEDIES, NO 1 (FIGURE 20, PP. 78-79)

The excerpts chosen for the analysis are two eight-bar closing phrases; the first phrase (bars 32-39) closes the first part and the second phrase closes the piece (bars 71-78). The structural beginnings are the same for both parts; they are only discussed at TSR level g (Figure 20, bars 6 and 45), as they do not play any significant role in the analysis until TSR level b and PR levels b-c. The choice of these specific phrases was made because of the interesting issues that arise from the comparative study of their modal characteristics, and especially the modal cadences they incorporate.

According to the key signature and the structural beginning (D major chord with added major seventh), the piece is in D major (= D Ionian). However, two important modal modulations (modal interchanges, since the modal center is retained) occur during the two ending phrases (Figure 20, TSR level g): the first phrase (bars 32-39) starts in D Ionian but ends in D Mixolydian (C sharp is replaced by C natural during the cadence) and the entire second phrase (bars 71-78) is in D Dorian mode (both Fs and Cs are always natural).

Both phrases contain perfect cadences of the type II - b - I, which are prolongationally described as double left (relaxating) progressions (Figure 20, PR level d-e). In both cases the II chord is prolonged for six bars, with the difference that in the first phrase the bass deviates temporarily from E (bars 33-34), while in the second E is being held as a pedal note. Also, the reappearance of the theme (bar 45) is considered a right strong prolongation of the structural beginning (bar 6) through the interaction principle. These structural elements define a complete normative prolongational structure at the deepest PR levels (Figure 20, PR level a-c). However, this structure does not describe the most characteristic and original aspect of the piece, which is the deviation from tonal to modal harmony. If bar 38 contained an A major chord instead of A minor, then a normal, but comparatively uninteresting for a composition of the late 19th century, tonal cadence would occur in the context of D major instead of D Mixolydian. The chordal distance δ would be the same in both cases: \( \delta(V/D \rightarrow I/D) = \delta(v/D_{\text{mix}} \rightarrow I/D_{\text{mix}}) = 5 \); however, as already mentioned in section 3a, the harmonic attraction of V towards I would be different in each case: \( \alpha_{\text{rh}}(V/D_{\text{ion}} \rightarrow I/D_{\text{ion}}) = 10 \times 2.43/5 = 4.86 \) and

---

(6) Interaction Principle (Lerdahl and Jackendoff, 1983, p. 228): The head \( e_i \) of a prolongational region \( (e_i-e_j) \) must be chosen from the events in the two most important levels of time-span reduction represented in \( (e_i-e_j) \).

(7) Normative Prolongational Structure (Lerdahl and Jackendoff, 1983, p. 234): A cadenced group preferably contains four (five) elements in its prolongational structure: a. a prolongational beginning, b. a prolongational ending, (c. a right-branching prolongation as the most important direct elaboration of the prolongational beginning), d. a right-branching progression as the (next) most important direct elaboration of the prolongational beginning, e. a left-branching "subdominant" progression as the most important elaboration of the first element of the cadence.
Figure 20. GTTM analysis of bars 32-39 and 71-78 from Satie's "Trois Gymnopédies, No. 1".
\( \alpha_{rh}(v/D_{mix} \rightarrow i/D_{mix}) = 10 \times 0.928/5 = 1.856. \) The more than double chordal attraction value of the A major chord makes it a more predictable choice; that is why Satie preferred the A minor and made a “perfect” cadence in the Mixolydian mode. Alternatively, if bar 77 contained an A major chord, the result would be a normative V - i cadence in D minor instead of a v - i “perfect” cadence in D Dorian. The chordal distances in both cases would be equal again but the harmonic attraction values would be different: \( \delta(V/d \rightarrow i/d) = \delta(v/D_{dor} \rightarrow i/D_{dor}) = 5, \alpha_{rh}(v/d \rightarrow i/d) = 10 \times 3.55/5 = 7.1 \) and \( \alpha_{rh}(v/D_{dor} \rightarrow i/D_{dor}) = 10 \times 2.05/5 = 4.1. \) Satie’s selection yields the least harmonic attraction value and contributes to the construction of a “perfect” cadence in the Dorian mode; simultaneously, it also enhances the element of non-anticipation (in tonal harmony context), which is characteristic of Satie’s music.

Another interesting issue, which relates to MPS, is that the two weak prolongations of PR levels c and b are not exactly of the same type, as shown in the tree structure. The chordal distance between the D major chord of bar 39 (ending of first part) and the D major chord of bar 5 (structural beginning) is: \( \delta(I/D_{mix} \rightarrow I/D_{ion}) = i + j + k = 1 + 0 + 1 = 2 \) (if the part had been entirely in D major tonality, the result should be 0). Furthermore, the chordal distance between the D minor chord of bar 78 (structural ending) and the D major chord of bar 5 (structural beginning) is: \( \delta(I/D_{dor} \rightarrow I/D_{ion}) = i + j + k = 2 + 0 + 3 = 5; \) if the piece ended in D minor tonality, the result would have been 7, which corresponds to the regional distance between major and minor. So, the use of the D Mixolydian and D Dorian modal regions as structural endings does not destroy the overall normative prolongational structure of the piece, although it does affect its content and the analogies between tension and relaxation.

The analyses in sections 5a and 5b present two different aspects of the analytical use of Modal Pitch Space. The first analysis (Bartók) focuses on the use of pitch space calculations for the construction of time-span and prolongational reductions, while the second (Satie) focuses on the use of attraction values’ calculations for the explanation and clarification of linear and chordal progressions. Both analyses illustrate the usefulness of modal region segmentation, a process essential for the structural comprehension of a modal piece; furthermore, they demonstrate how and to what extent Modal Pitch Space calculations may be incorporated in the GTTM methodology, thus enabling the use of the GTTM in modal music analysis.

6

Conclusions

The application of reductional-type analysis methodology to modal music is in no way new (for example see Salzer, 1962; Stern, 1981, pp. 5-39; Novack, 1983, pp. 113-133); it has mainly been conducted through the strengthening of the
prolongation concept and the weakening of the fundamental structure concept. The GTTM's flexibility and the substitution of the a priori fundamental structure with the tonally unspecialized and cognitively based normative structure enable its extensive use beyond its initial analytical range. The modal pitch space theory provides the analyst with a diatonic modal hierarchy, which facilitates the use of both time-span stability and prolongational connection preference rules within the modal context. Furthermore, the modal space theory clarifies the prolongational functions and connections by the calculation of the chordal, regional and attractional distances; therefore, it becomes possible to use the GTTM more efficiently in regard to modal music analysis. Although the GTTM has already been used for analysis of modal music (Auvinen, 1995; Tsougras, 1999, 2002) without the quantification that the modal pitch space provides — substituting it with references from semiotic analysis (Auvinen) or with intuitive classification of modal chords in folk tune context (Tsougras) — the model's potential use may lead to a more systematic methodology for the analysis of modal music.

The general criticisms of the pitch space theory, like its mathematical and geometrical approximations and its not yet complete empirical evidence, have already been mentioned. In addition to these, there is yet another point, regarding the extension of the tonal space towards modality and its use in the analysis process, which should be subjected to scrutiny: the distinction between chordal and regional space is highly ambiguous and this ambiguity interferes with the segmentation of a piece in modal regions. For example, the modal space regional distance \( \Delta(C_{dor} \rightarrow G_{aeo}) \) is equal to both the modal space chordal distance \( \delta(i/C_{dor} \rightarrow v/C_{dor}) \) and to the tonal space chordal distance \( \delta(iv/g \rightarrow i/g) \). This situation is similar to the distinction between modulation and tonicization in tonal music. What makes the distinction in these cases is the permanence of a chromatic factor, the nature of the melodic line and the prominence of the bass line. For example, a chord progression from C to G can be considered either I/C \( \rightarrow \) V/C or I/C \( \rightarrow \) I/G or I/C_{ion} \( \rightarrow \) I/G_{mix} — to name but few out of several other modulatory possibilities. If a new theme is introduced in G major, then the case is most likely a tonic modulation to G. If (after a secondary dominant chord) the C major tonality continues, then a tonicization has occurred. But, if a theme with strong melodic character is introduced that reinforces the role of G as tonic in the same key signature, the result may be considered a modal modulation to the region of G_{mix}; for example, the English horn solo in the third phrase of the Largo of Dvořák's "New World Symphony", which pertains to the Lydian mode of the IV chord, is often perceived as Lydian region IV_{lyd}, because of the unusual voice-leading of the leading tone.

Nevertheless, the difference between most diatonic modal regions is generally much more subtle than the one between tonicization and tonal modulation and the ambiguity factor is certainly much higher; the degree of proximity between pitch-space distances under different interpretations may provide a measure of the degree
of ambiguity between them. At this point, a short discussion of ambiguity in music analysis would not be out of place. Ambiguity is a prominent aspect of music and, consequently, a major issue in music analysis; apart from the importance of grammaticality (i.e. correctness and clarity), the diversity in the understanding (the cognition) of a work among the listeners is also significant in music. William Thomson (1983, p. 3) even considered it a functional aspect: “when a music event projects equivocation, implying no clear syntactic meaning or two or more potential meanings, I call this an instance of functional ambiguity”. Of course, all the ambiguous interpretations have to conform to all the well-formedness rules, as well as to a number of preference rules, pertaining to grammaticality; however, the significance and relative weight of preference rules in relation to the musical context is defined by the analyst, whose role is to dispel ambiguity. Within this context, what is most important about modal pitch space theory is that it reinforces the analyst’s interpretation potential of the music in question by providing explicitly defined stability conditions and preference rules, thus assisting the analyst to keep the ambiguity factor under control. To cite Kofi Agawu (1994, p. 107): “An analysis that terminates in undecidability represents a conscious or subconscious retreat from theory”; “to refrain from choosing between competing alternatives is to refuse to take advantage of the disambiguating functions of the theory”.

(8) Modal Pitch Space was conceived during my research at Columbia University as a visiting scholar for the 2000 spring semester and it was largely inspired by Fred Lerdahl’s “Tonal Pitch Space” class. I would like to thank Dr. Lerdahl for his help and advice during the development of the ideas presented in this paper.

Part of the material in this paper (a concise version of the mathematical and geometrical representation of the model) was presented at the 10th anniversary ESCOM conference on “Musical Creativity” in April 2002 in Liège.

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• "Modal Pitch Space":
  Un estudio teórico y analítico

Este trabajo presenta una aproximación al espacio tonal de los siete modos diatónicos. La teoría propuesta es una expansión del modelo de "Tonal Pitch Space" de Fred Lerdahl; su propósito es una descripción más cuidada de las situaciones implicadas en el análisis de la música diatónica modal. Se mantienen la metodología original y un conjunto de fórmulas de cálculo algebraico, pero son aplicadas en un espacio modal en lugar de tonal, basándonos en que lo tonal es un subconjunto de lo modal. En conexión con la teoría de Fred Lerdahl, el movimiento melódico, la atracción acoral y los tipos de cadencias se describen dentro del contexto modal. Además de calcular la altura y las representaciones algebraicas de espacio acoral y regional, se incluyen también representaciones geométricas de los tres niveles de "Modal Pitch Space". Finalmente, las condiciones de estabilidad que se derivan del nuevo modelo se emplean como criterios para construir la reducción interválica temporal y la reducción prolongacional, partes de los análisis de la música modal según la Teoría Generativa de la Música Tonal (GTTM). Se incluyen dos breves análisis según la GTTM, de obras de música modal del siglo XX, para ilustrar el empleo del nuevo modelo analítico.

• Lo spazio modale delle altezze:
  Uno studio teorico e analitico

Questo articolo presenta un accostamento allo spazio delle altezze nei sette modi diatonici. La teoria qui proposta è un’espansione del modello di Fred Lerdahl sullo spazio delle altezze tonali; il suo scopo è una descrizione più accurata delle condizioni legate all’analisi della musica diatonica modale. Si mantengono la metodologia originale e l’insieme delle formule di calcolo algebrico di Lerdahl, ma applicandoli allo spazio modale invece che a quello tonale, con l’assunto che il secondo sia un sottoinsieme del primo. Conformemente alla teoria di Fred Lerdahl, movimento melodico, attrazione armonica e differenti tipi di cadenza vengono descritti nell’ambito del contesto modale. Oltre al calcolo dell’altezza ed alle rappresentazioni algebriche dello spazio accordale e regionale, vengono qui incluse anche rappresentazioni geometriche dei tre livelli dello spazio modale. Infine, le condizioni di stabilità derivate dal nuovo modello vengono utilizzate come criteri per costruire la riduzione della distanza temporale e la riduzione prolongazionale, parti costituenti l’analisi della musica modale secondo la teoria generativa della musica tonale (GTTM). Vengono presentate due brevi analisi di musiche modali del Novecento secondo la GTTM, al fine di illustrare l’impiego analitico del nuovo modello.
Modal Pitch Space (espace modal des hauteurs) — étude théorique et analytique

On étudie ici l'espace de hauteurs des sept modes diatoniques. La théorie qui est proposée est une extension du modèle de l'espace tonal des hauteurs de Fred Lerdahl; elle vise une plus grande acuité de la description des situations rencontrées dans l'analyse de la musique modale diatonique. On conserve la méthodologie originale et l'ensemble des formules de calcul algébrique, mais on les applique à l'espace modal plutôt qu'à l'espace tonal, en se fondant sur le fait que celui-ci est un sous-ensemble du premier. Sur la base de la théorie de Fred Lerdahl, le mouvement mélodique, l'attraction des accords et les divers types de cadence sont décrits dans le contexte modal. À l'exception du calcul de la hauteur et des représentations algébriques de l'espace entre les accords et de celui de la région, les représentations géométriques de l'ensemble des trois niveaux de l'espace modal des hauteurs ont été prises en compte. Enfin, les conditions de stabilité découlant du nouveau modèle servent de critères dans la construction de la réduction de la trame temporelle et de celle de la prolongation de la Generative Theory of Tonal Music (GTTM) pour l'analyse de la musique modale. Deux brèves analyses GTTM de la musique modale du XXe siècle illustrent l'utilisation analytique du nouveau modèle.

Modaler Tonraum:
Eine theoretische und analytische Studie