A rigorous analysis of GPS data to detect crustal deformations. Application in the area of the Ionian Sea

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Abstract

An analysis of a deformation process in dynamic models is presented, where time is considered as the essential part. A rigorous adjustment model of unregularly spanned observations in space and time is proposed for the estimation of point velocities and strain rate parameters. The problem of the reference frame definition and choices of deformation models are discussed. Finally an application is given in order to illustrate the method, using real data from GPS observations at the geodynamically unstable area of the Ionian Sea. © 1998 Elsevier Science B.V. All rights reserved.

Keywords: crustal deformation; GPS; plate motions; tectonics

1. Introduction

The scientific purpose of the present work is twofold. First to stress the importance of the quality of the geodetic data used in a geodynamical investigation by proposing rigorous mathematical and statistical models, used here for the GPS data elaboration. Second to give a real application in the highly sensitive tectonic area of the Ionian Sea, estimating the displacement field and presenting reliable results for further investigations and comparisons with results from other geosciences.

It is well established that geodetic methods give reliable results for the monitoring of plate motions and the deformation of the earth’s surface. During the last years the acquisition of high-accuracy space data (baseline components or point coordinates) has become easier due to the extensive use of the GPS system. One of the most common methods that is followed for the above process is based on the repetition of observations concerning geodetic networks which are properly established in geodynamic areas under control and on the analysis of the results between different epochs of observation by means of appropriate models.

The assessment and interpretation of the geodetic results for the detection of possible spatial displacements and the estimation of deformation parameters have to be combined with a realistic geophysical model for the area. Usually, this study is carried out by fitting a polynomial function, which is considered sufficient to describe satisfactory the deformation pattern which is derived treating geodetic results. In terms of the computational steps needed, the fitting can be done either simultaneously with the analy-
sis of the geodetic measurements, in view of the approach followed in integrated geodesy (e.g. Vanicek et al., 1979; Bibby, 1982; Rossikopoulos, 1986; Dermanis and Rossikopoulos, 1988), or non-simultaneously (e.g. Livieratos, 1980a; Brunner et al., 1981; Dermanis et al., 1981; Chrzanowski et al., 1983; Dermanis and Grafarend, 1992).

The analytical functions usually used to smooth differential motions have a simple form and are expressed by easily conceived parameters such as rates and accelerations of displacements (Papo and Perelmuter, 1983; Welsch, 1986), tensors of deformations (Livieratos, 1980b; Brunner et al., 1981; Chrzanowski et al., 1983; Dermanis and Livieratos, 1983), rates of deformations (Bibby, 1982; Welsch, 1986). Sometimes more complex forms are used to include terms for episodic motions, e.g. due to a seismic event (Vanicek et al., 1979; Snay et al., 1983).

According to the form and type of the ‘original’ geodetic data and their precision, e.g. baseline components or point coordinates from net adjustment, the corresponding mathematical model may suffer some specific problems such as the inconsistency of the reference systems involved and the existence of non-positive covariance matrices.

In the present work, GPS measurements are used associated to a 9-point geodetic network connecting the Greek and Italian coasts in the Ionian and Adriatic seas (Figs. 1–5). The observations were carried out during campaigns in 1991, 1994 and 1995 and for about a week each year in the frame of the TyrGeonet Project (Anzidei et al., 1995) using dual frequency P-code receivers. As it is known, this area is under strong geodynamical activity due to its high seismicity.

Raw GPS data have been processed using the Bernese software (v. 3.5). In the first step code pseudo ranges were adopted in order to calibrate the receiver clocks with GPS time.

Detection and repair of cycle slips were performed simultaneously for L1 and L2 phase data at a triple and double difference level. A verification of systematic distortions was performed through the ionospheric free (L3) linear combination. Double difference phase measurements were used for the final baseline solution using an extrapolation model (Saastamoinen, 1972) to account for tropospheric refraction. Observations with an elevation angle greater than 20° were included in order to reduce the atmospheric noise.

Float L3 solutions were adopted because for some long baselines the percentage of solved integer ambiguities dropped to 50% or lower.

The Bernese solution consists in the computation of the independent baselines chosen on the basis of the maximum number of common observations and finally in a 3-D network adjustment per day of observations. Precise ephemerides were used from NOAA and IGS.

2. The type of analysis

The analysis which is followed here implies a specially developed algorithm which follows the following three main steps.

2.1. Network adjustment for each epoch

Adjustment of GPS observations for each day separately obtaining the estimation of point coordinates \((x, y, z)\) and their full covariance matrices. These results have been derived here by the Bernese software, v. 3.5 (Rothacher et al., 1993).
The 3-D network adjustment of each year follows, using as observations the results of the previous adjustments and considering the corresponding days as belonging to the same epoch (year). This is usually true since the time span in the same year is limited only to a few successive days.

2.2. Elimination of coordinate differences due to different datums definitions

Defining the first epoch/year as the reference epoch (1991), the coordinates of the next epochs (1994, 1995) are transformed in order to fit optimally those of the reference epoch. The model used is a 3-D similarity transformation. Finally, for each epoch new transformed coordinates are estimated with the associated covariance matrices.

2.3. Adjustment with a deformation model

The coordinates in the reference epoch and the transformed ones as described in the second step are adjusted taking into account a deformation model. New coordinates for the reference epoch and rates of displacements are estimated. A more complex model could be used, e.g. including accelerations and/or rates of deformations. Regarding our specific network it was meaningless to use such a complex model since, in order to do so, more than three epochs (years) are required for the estimation of the acceleration parameters and additionally, significant displacements in at least three points in each sub-tectonic area of homogeneous deformation have to be detected.

In the above steps a statistical analysis is proposed and applied in order to test the reliability of the results. The three main algorithmic steps are analysed below and some interesting results are presented and discussed.

3. Adjustment per epoch

The epoch-observations are the coordinates \( x^b_i \) (\( i = 1, \ldots, n \)) for each one of the \( n \) days in the same epoch \( t_a \). These coordinates were obtained by separating adjustments for each day and having a covariance matrix \( Q_i \). Since the days of observation in the same year are usually successive, the measurements can be seen as simultaneous. This hypothesis is tested by the data snooping technique applied in groups, where a group consists of the observations in each day.

The solution of the normal equations, for the \( t_a \) epoch and for \( n \) days of observations, is given by:

\[
\hat{x}_a = N_a^{-1} u_a = Q_a u_a
\]

where

\[
N_a = \sum_{i=1}^{n} Q_i^{-1} \quad \text{and} \quad u_a = \sum_{i=1}^{n} Q_i^{-1} x_i^b
\]

In the adjustment per day, the datums have to be defined. In order to simplify the algorithm in this part of the process and to avoid computation of generalized inverse covariance matrices \( Q_i \), the datums can be defined by means of constraints on the coordinates. In the case of GPS measurements the number of constant coordinates is three (one point kept fixed).

In order to access the results for each epoch the covariance matrix \( C^{(u)}_a \) of the estimation \( \hat{x}_a \), the global reference variance \( \hat{\sigma}_a^2 \) and the reference variance \( \hat{\sigma}_i^2 \) for each day are computed.

In addition, proper statistical tests are applied in order to detect possible systematic errors and outliers per group of observations of each day.

In the case that ‘all days’ pass successfully the above test, probably there is not any significant displacement within this small time period and therefore all the observations belonging to these days have to be considered as simultaneous. If the test fails, a reason has to be determined, usually being the existence of outliers.

In our application, two days have been rejected, the first and the third one of 1991, out of 10 days, because some points seemed to be unreliable. All the days in 1994 (6 days) and in 1995 (6 days) passed the test.

4. Best fitting of coordinates between two different epochs

The elimination of the difference between the coordinates of two epochs \( t_a \) and \( t_b \), due to their
different datums definition is obtained by the optimal fitting of $t_β$ coordinates to the corresponding $t_α$ (reference epoch), applying the well-known 3-D similarity transformation. Coordinates $\tilde{x}_β$ of $t_β$ epoch are transformed to new coordinates:

$$\tilde{x}_β = \tilde{x}_β - G(\tilde{x}_β - \tilde{x}_α)$$

which fit optimally in $O$ of the $t_α$ epoch. The new differences, after the elimination of the systematic part, are given by:

$$\tilde{u} = (I - G)(\tilde{x}_β - \tilde{x}_α)$$

In the above relations, $G = E(E^T E)^{-1} E^T$, where matrix $E$ has the form:

$$E = [E_1 E_2 \ldots E_N]$$

with $N$ the number of common points between two epochs. Considering small rotations and scale variation, each matrix $E_i$ is given by:

$$E_i = \begin{bmatrix}
1 & 0 & 0 & 0 & -z_i^β & y_i^β & x_i^β \\
0 & 1 & 0 & z_i^β & 0 & -x_i^β & y_i^β \\
0 & 0 & 1 & -y_i^β & x_i^β & 0 & z_i^β 
\end{bmatrix}$$

The covariance matrices for the new coordinates and the coordinate differences are given respectively by:

$$\hat{C}_{\tilde{x}} = (I - G) \hat{C}_{x}(I - G)$$

$$\hat{C}_{\tilde{u}} = (I - G)(\hat{C}_{x}(\alpha) + \hat{C}_{x}(\beta))(I - G)$$

where $\hat{C}_{x}(\alpha), \hat{C}_{x}(\beta)$ are the covariance matrices of the coordinates for the epochs $t_α$ and $t_β$ as obtained by the separating adjustments (second step).

A global test for the existence of systematic errors, except those due to datum definition, is based on the comparison of the so-called 'mean square displacement' to the 'global reference variance' of the two separating adjustments (Livieratos and Rossikopoulos, 1984).

In case the global test is successful, the network has probably remained stable within the period $t_α - t_β$. Otherwise, either the results of the separating adjustments are unreliable or some of the points or all of them have been moved. In the last case the testing of each point is necessary.

Thus, for each point the statistic $F$ is computed by (Heck, 1985; Kaltsikis et al., 1993):

$$r_i^2 = \frac{\hat{u}_i^T (I_3 - E_i (E_i^T E_i)^{-1} E_i^T)^{-1} \hat{u}_i}{3\sigma^2}$$

$$F = \frac{r_i^2}{\frac{2N - 7}{2N - 10 - 3r_i^2}}$$

where

$$\sigma^2 = \frac{1}{2N - 7} \tilde{u}_i \hat{u}_i = \frac{1}{2N - 7} \sum_{i=1}^{N} [\hat{u}_i^2 + \hat{v}_i^2 + \hat{w}_i^2]$$

and the test is accepted if $F \leq F_{\alpha, \beta_{i-1}} (f = 3N - 7)$.

The above testing process is applied by means of the data snooping technique: the point which fails and has the greater $F$-value is excluded. Then the adjustment is repeated until all points pass the test. In this way the points can be distinguished as stable or unstable.

The $F$-values given in Tables 1 and 2 are compared with $F_{0.05}^{0.05} = 3.59$ and $F_{0.15}^{0.05} = 3.40$, respectively. From this test only point 18, for the epoch 1994, seems to have moved.

### Table 1
The transformed coordinates of 1994 to 1991 and their differences

<table>
<thead>
<tr>
<th>$i$</th>
<th>$\tilde{x}_i$ (m)</th>
<th>$\tilde{y}_i$ (m)</th>
<th>$\tilde{z}_i$ (m)</th>
<th>$\tilde{x}_β - \tilde{x}_α$ (cm)</th>
<th>$\tilde{y}_β - \tilde{y}_α$ (cm)</th>
<th>$\tilde{z}_β - \tilde{z}_α$ (cm)</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>42</td>
<td>4761340.1775</td>
<td>1419704.9611</td>
<td>3986901.6564</td>
<td>-1.40</td>
<td>1.80</td>
<td>-0.13</td>
<td>0.51</td>
</tr>
<tr>
<td>24</td>
<td>4621781.2166</td>
<td>1705359.9396</td>
<td>4037659.0762</td>
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<td>-0.70</td>
<td>0.90</td>
<td>0.14</td>
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<tr>
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<td>4636898.4653</td>
<td>1547667.3233</td>
<td>4083410.9077</td>
<td>0.25</td>
<td>2.86</td>
<td>5.37</td>
<td>4.76</td>
</tr>
<tr>
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<td>3895980.8469</td>
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<td>-1.73</td>
<td>-1.16</td>
<td>1.36</td>
</tr>
<tr>
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<td>-0.23</td>
<td>0.25</td>
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<td>0.04</td>
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Table 2
The transformed coordinates of 1995 to 1991 and their differences

<table>
<thead>
<tr>
<th>i</th>
<th>(x_{1995}) (m)</th>
<th>(y_{1995}) (m)</th>
<th>(z_{1995}) (m)</th>
<th>(x_{1991}) (m)</th>
<th>(y_{1991}) (m)</th>
<th>(z_{1991}) (m)</th>
<th>(\Delta x) (m)</th>
<th>(\Delta y) (m)</th>
<th>(\Delta z) (m)</th>
<th>(\Delta x) (m)</th>
<th>(\Delta y) (m)</th>
<th>(\Delta z) (m)</th>
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<tbody>
<tr>
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<td>1419704.9628</td>
<td>3986901.6561</td>
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<td>1.97</td>
<td>-0.16</td>
<td>0.33</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>0.42</td>
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<td>2.82</td>
<td></td>
<td></td>
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<td>4083410.8976</td>
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<td>4.37</td>
<td>2.30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>3895980.8257</td>
<td>0.01</td>
<td>-2.58</td>
<td>-3.28</td>
<td>2.82</td>
<td></td>
<td></td>
<td></td>
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<tr>
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<td>1735712.6163</td>
<td>3979543.4263</td>
<td>0.03</td>
<td>0.63</td>
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<td>0.25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>1310783.2512</td>
<td>4222005.8141</td>
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<td>4255762.5369</td>
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<td>-0.04</td>
<td>-1.00</td>
<td>0.89</td>
<td></td>
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</tr>
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</table>

A more strict and descriptive test for the significance of the displacements can be done by means of their confidence error ellipses (e.g. Heck et al., 1977). Figs. 1–3 show the error ellipses of coordinate variations (differences) for a level of significance \( \alpha = 0.05 \) and the displacement vectors as derived by a proper transformation in 2-D. The corresponding numerical values and those for the heights as well, are given in Table 3.

From Table 3 and the corresponding Figs. 1–3 the coordinate variations are clearly shown. The displacement tendencies between 1991 and 1994 and

Table 3
Horizontal and vertical displacements, confidence error ellipses and intervals for \( 1 - \alpha = 0.95 \)

<table>
<thead>
<tr>
<th>i</th>
<th>Horizontal displacement (cm)</th>
<th>Error ellipses (cm)</th>
<th>Vertical displacement (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(dS_x)</td>
<td>(dS_y)</td>
<td>(a)</td>
</tr>
<tr>
<td>42</td>
<td>2.1</td>
<td>0.4</td>
<td>0.9</td>
</tr>
<tr>
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<td>0.0</td>
<td>-0.2</td>
<td>1.2</td>
</tr>
<tr>
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<td>-0.3</td>
<td>1.1</td>
<td>2.7</td>
</tr>
<tr>
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<td>0.8</td>
</tr>
<tr>
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<td>0.8</td>
<td>0.5</td>
<td>1.1</td>
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<td>41</td>
<td>2.6</td>
<td>3.4</td>
<td>1.1</td>
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<td>0.7</td>
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<td>-0.1</td>
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<tr>
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<td>-2.1</td>
<td>1.2</td>
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</tr>
<tr>
<td>1.6</td>
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<td>1.0</td>
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<td>-0.4</td>
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</tr>
<tr>
<td>-1.2</td>
<td>-0.6</td>
<td>1.2</td>
<td>2.9</td>
</tr>
<tr>
<td>-1.3</td>
<td>0.1</td>
<td>1.1</td>
<td>2.7</td>
</tr>
<tr>
<td>19</td>
<td>0.6</td>
<td>0.9</td>
<td>1.2</td>
</tr>
<tr>
<td>44</td>
<td>0.1</td>
<td>1.4</td>
<td>1.1</td>
</tr>
</tbody>
</table>

\( k = \sqrt{F_{x,\alpha} + f_{\alpha}^{2.05}} \), \( f_{91} = 99 \), \( f_{94} = 90 \), \( f_{95} = 114 \), \( k_1 = 2.45 \), \( k_2 = 1.96 \).
5. Simultaneous adjustment of three epochs for the velocity field estimation

Under the hypothesis of a smooth motion in the deformation area, the vector of the displacements is a function of the velocity displacement and the acceleration, according to the model:

\[ x_a = x_0 + \delta t_a \dot{x} + \frac{1}{2} \delta t_a^2 \ddot{x} + \ldots \]

where \( \delta t_a = t_a - t_0 \). Applications of this model are given by Papo and Perelmuter (1983).

Considering a homogeneous displacement field in time, we use only the linear part of the above equation. Thus, the system of observation equations for the \( m \) epochs is formulated by:

\[
\begin{align*}
    x_0^b &= x_0 + v_0 \\
    x_1^b &= x_0 + \delta t_1 \dot{x} + v_1 \\
    & \vdots \\
    x_m^b &= x_0 + \delta t_m \dot{x} + v_m
\end{align*}
\]

where \( x_0^b \) is the vector of coordinates of the reference epoch (1991) derived from the network adjustment in that epoch (first algorithmic step), \( x_1^b, x_2^b \) are the vectors of the transformed coordinates \( \dot{x}_1 \) and \( \dot{x}_2 \) of the epochs 1994 and 1995 respectively, \( x_0 \) is the vector of the new coordinates of the reference epoch, \( \dot{x} \) is the vector of the velocities and \( v_0, v_1, v_2 \) are the vectors of the observation errors.

The system of normal equations for the \( m \) epochs (in our application \( m = 2 \)) is given by:

\[
\begin{bmatrix}
    \tilde{N} & \tilde{N} \\
    \tilde{N}^T & \tilde{N}
\end{bmatrix}
\begin{bmatrix}
    \dot{x}_0 \\
    \ddot{x}_0
\end{bmatrix}
= 
\begin{bmatrix}
    \dot{u} \\
    \ddot{u}
\end{bmatrix}
\]

where

\[
\begin{align*}
    \tilde{N} &= \sum_{a=0}^{m} W_a, \quad \tilde{N} = \sum_{a=1}^{m} \delta t_a W_a, \quad \tilde{N} = \sum_{a=1}^{m} \delta t_a^2 W_a \\
    \dot{u} &= \sum_{a=0}^{m} W_a x_a^b, \quad \ddot{u} = \sum_{a=1}^{m} \delta t_a W_a x_a^b
\end{align*}
\]

The covariance matrix for the reference epoch (1991) is a positive definite matrix since we have performed the initial adjustment with one point kept...
fixed. Therefore, the weight matrix can be taken as the inverse:

\[ W_0 = \left\{ \hat{\mathbf{C}}_x^{(0)} \right\}^{-1} \]

For the next epochs, i.e. 1994 and 1995, the corresponding covariance matrices \( \hat{\mathbf{C}}_x^{(1)} \) and \( \hat{\mathbf{C}}_x^{(2)} \) derived by the transformations, are non-positive definite matrices. In this case the weight matrix can be given by the generalized inverse:

\[ W_a = \left( \hat{\mathbf{C}}_x^{(\alpha)} + \delta^2 \mathbf{I} \right)^{-1}, \quad \alpha = 1, 2 \]

which, according to Bjerhammar (1973) and Uotila (1974), results in the best unbiased estimations for the parameters \( x_0 \) and \( \hat{x} \) since the system of normal equations has not a rank defect.

The above choices of the weight matrices result in the following estimations and covariance matrices:

\[ \hat{\mathbf{v}} = \left[ \begin{array}{c} \hat{v}_0 \\ \vdots \\ \hat{v}_n \end{array} \right], \quad \hat{\mathbf{v}} = x_0 - \hat{x} \\
\hat{\mathbf{v}} = \hat{\psi} / \hat{\mathbf{J}} \]

where

\[ \hat{\psi} = \sum_{\alpha=0}^{m} \hat{\mathbf{v}}_\alpha W_a \hat{\mathbf{v}}_\alpha \]

\[ \hat{\mathbf{C}}_0 = \hat{\sigma}^2 (\hat{\mathbf{Q}} - \delta^2 \mathbf{I}), \quad \hat{\mathbf{C}}_x = \hat{\sigma}^2 (\hat{\mathbf{Q}} - \delta^2 \mathbf{I}), \quad \hat{\mathbf{C}}_{\hat{x}} = \hat{\sigma}^2 \hat{\mathbf{Q}} \]

and

\[ \begin{bmatrix} \hat{\mathbf{Q}} & \hat{\mathbf{Q}}^T \\ \hat{\mathbf{Q}} & \hat{\mathbf{Q}} \end{bmatrix} = \left[ \begin{bmatrix} \hat{\mathbf{N}} & \hat{\mathbf{N}}^T \\ \hat{\mathbf{N}} & \hat{\mathbf{N}} \end{bmatrix} \right]^{-1} \]

In order to access the fitting of the used velocity model to the coordinate differences from epoch to epoch, proper statistical tests were applied and resulted in the acceptance of the model.

In Table 4 the velocities of the coordinates and their error ellipses are presented. Looking at the velocities, the displacements per year are in general smaller than 0.5 cm. This means that it is not necessary to repeat the observations every year. The same conclusion has also been drawn from Fig. 3.

The displacements for a specific epoch \( t_a \) in relation to the reference epoch \( t_0 \) and their covariance matrix are given by:

\[ \hat{\mathbf{u}}_a = \delta t_a \hat{x}, \quad \hat{\mathbf{C}}_{\hat{\mathbf{u}}_a} = \delta t_a^2 \hat{\mathbf{C}}_x \]

Figs. 4 and 5 show the displacements with their confidence ellipses for the epochs 1994 and 1995, starting from the reference epoch 1991. The corresponding numerical values, including those for the heights, are given in Table 5. It should be noted that the vertical displacements in the final adjustment are statistically insignificant (Table 5).

| Table 4 |
| Velocities of the coordinates and their error ellipses |
| i | Velocities (cm/year) | Error ellipses |
| | | (x, y) plane | (x, z) plane |
| | u | v | \( a \) (cm) | b (cm) | \psi (grad) | a (cm) | b (cm) | \psi (grad) |
| 42 | 0.03 | 0.39 | 0.06 | 0.58 | 0.45 | 152.41 | 0.55 | 0.30 | 70.73 | 0.53 | 0.36 | 90.46 |
| 24 | 0.07 | -0.08 | 0.79 | 0.58 | 0.43 | 136.11 | 0.58 | 0.30 | 73.25 | 0.48 | 0.36 | 91.73 |
| 18 | -0.61 | 0.34 | 0.80 | 0.59 | 0.46 | 140.24 | 0.59 | 0.34 | 68.91 | 0.52 | 0.40 | 83.89 |
| 41 | -0.20 | -0.56 | -1.01 | 0.56 | 0.43 | 126.71 | 0.59 | 0.32 | 67.53 | 0.46 | 0.39 | 75.40 |
| 43 | 0.24 | 0.28 | 0.16 | 0.60 | 0.46 | 129.33 | 0.62 | 0.34 | 68.71 | 0.0 | 0.41 | 75.00 |
| 10 | 0.28 | -0.40 | -0.54 | 0.63 | 0.42 | 133.48 | 0.63 | 0.34 | 72.27 | 0.49 | 0.40 | 85.13 |
| 1 | 0.11 | -0.19 | -0.14 | 0.34 | 0.34 | 0.00 | 0.34 | 0.34 | 0.00 | 0.34 | 0.34 | 0.00 |
| 19 | -0.47 | 0.15 | -0.17 | 0.64 | 0.51 | 142.58 | 0.65 | 0.34 | 67.74 | 0.56 | 0.43 | 89.50 |
Table 5
Horizontal and vertical displacements, confidence error ellipses and intervals for $1 - \alpha = 0.95$ as derived from the estimation of velocities

<table>
<thead>
<tr>
<th>$i$</th>
<th>Horizontal displacement (cm)</th>
<th>Error ellipses (cm)</th>
<th>Vertical displacement (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$dS_x$</td>
<td>$dS_y$</td>
<td>$dS$</td>
</tr>
<tr>
<td>-----</td>
<td>--------</td>
<td>--------</td>
<td>------</td>
</tr>
<tr>
<td>42</td>
<td>1.1</td>
<td>-0.1</td>
<td>1.1</td>
</tr>
<tr>
<td>1.5</td>
<td>-0.2</td>
<td>1.5</td>
<td>1.6</td>
</tr>
<tr>
<td>24</td>
<td>-0.3</td>
<td>1.8</td>
<td>1.8</td>
</tr>
<tr>
<td>-0.4</td>
<td>2.3</td>
<td>2.3</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>1.5</td>
<td>2.7</td>
<td>3.1</td>
</tr>
<tr>
<td>2.1</td>
<td>3.7</td>
<td>4.2</td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>-1.4</td>
<td>-1.7</td>
<td>2.1</td>
</tr>
<tr>
<td>-1.8</td>
<td>-2.2</td>
<td>2.8</td>
<td></td>
</tr>
<tr>
<td>43</td>
<td>0.5</td>
<td>-1.0</td>
<td>1.1</td>
</tr>
<tr>
<td>0.7</td>
<td>-1.3</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>10</td>
<td>-1.4</td>
<td>-1.5</td>
<td>2.1</td>
</tr>
<tr>
<td>-1.8</td>
<td>-2.0</td>
<td>2.7</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-0.6</td>
<td>-0.4</td>
<td>0.7</td>
</tr>
<tr>
<td>-0.9</td>
<td>-0.6</td>
<td>1.1</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>0.8</td>
<td>0.5</td>
<td>0.9</td>
</tr>
<tr>
<td>1.1</td>
<td>0.6</td>
<td>1.3</td>
<td></td>
</tr>
</tbody>
</table>

$k = (1 - \alpha)^{0.5}$, $f = 21$, $k_1 = 2.63$, $k_2 = 2.07$.

6. Discussion

Figs. 4 and 5 show almost the same behaviour of the displacement field in comparison to the corresponding Figs. 1 and 2 derived from the transformation process. Strictly speaking, the results of the final adjustment (Tables 4 and 5, Figs. 4 and 5) are considered more reliable than the corresponding...
ones derived by the transformation process, as can be also seen by comparing the error ellipses of Figs. 1 and 2 with those of Figs. 4 and 5.

With respect to the period 1991–1995, we can come to some distinct conclusions: The relative horizontal displacements of the greek sites Igoumenitsa and Kastro I. are of the order of 1–3 cm (velocity ~7 mm/year) with quite different directions. The site of Lefkas located at the centre of the Ionian Islands and halfway between Igoumenitsa and Kastro I. shows a displacement of 1.5 cm, also with a quite different direction. Looking at the error ellipses, only the site of Kastro I. has probably moved significantly during 1991–1995. The smaller magnitude of the displacement of the Lefkas site and its direction may be explained by the spreading of the Gulf of Corinth (Rigo et al., 1992) and the activity of the Kephalonia Fault System being located in a transition zone (Anzidei et al., 1995; Kahle et al., 1995; Kahle et al., 1996).

Concerning the Italian sites the distinct conclusion is the opposite directions of the displacement vectors between Specchia C.–M.S. Angelo and Tremiti Is.–M.S. Angelo. In addition, the sites Specchia C. and M.S. Angelo seem to have moved significantly, with a magnitude of the order of 3–4 cm (velocity ~1 cm/year). Moreover we notice that the southern Apulia (Specchia C.) platform shows the greatest relative displacement compared to the other sites. The different behaviour in northern Apulia (Tremiti Is.–M.S. Angelo) could be explained by the difference in stress regime produced by the left lateral fault of Mattinata close to M.S. Angelo (Anzidei et al., 1995). A different behaviour is evident for the site of Catanzaro, located at the Calabrian Arc with different tectonics. In addition, Catanzaro does not show any significant motion.

7. Conclusions

The presented algorithm is developed rigorously in a sequential order: starting from the network epoch adjustments and proceeding with the elimination of the datum effects between different epochs and an initial epoch, the algorithm is completed in a final adjustment, fitting a smooth velocity displacement deformation model to the transformed results, free of systematic biases. Throughout the process statistical tests are applied in order to assess the reliability of the results.

The available geodetic GPS data, used here in order to illustrate the method, are the cartesian coordinates of a 9-point network established in the area of the Ionian Sea. The data with their precision (full covariance matrices) are obtained initially from a network adjustment using the Bernese software (daily adjustment for each campaign) and are elaborated further on. Three GPS campaigns have been carried out during 1991, 1994 and 1995 in the frame of the Tyrgeonet Project. From the first daily adjustments, accurate data sets were derived in order to compose a unique set for each year. The complete observation sets of two days in 1991 have been rejected.

The transformation results between 1994 and 1991 and 1995 and 1991 indicate a trend of probable motion for five points (10, 18, 24, 41, 42). Fitting a simple deformation model to the transformed results, by introducing velocity displacements, three of the above points (10, 18, 41) show significant horizontal displacements. The total displacement field looks compatible to the tectonic features of the area and the results of other authors. It is evident that more observation epochs are needed in order to come to stronger conclusions and to apply more complex deformation models. For the presented application the used deformation model proved efficient.

The results given here are similar to the results of other investigations for the same area.

This algorithm is a part of a more general algorithm which has been developed for the dynamic adjustment of observations of angles, spatial distances, height differences and baseline components, including velocity or strain rate deformation models.

The incorporation of data of a physical type, such as gravity and dynamic height observations, involves the integrated approach, where the main advantage is a more reliable estimation of the vertical component of the deformation field.

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References


